Sampling numbers of embeddings of Besov spaces

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Assume that we want to recover a function $f : \Omega \to \mathbb{C}$ from the unit ball of a Besov space $B^{s_1}_{p_1 q_1}(\Omega)$ by a linear sampling method

$$S_n f = \sum_{j=1}^{n} f(x_j) h_j,$$

in the norm of another Besov space, say $B^{s_2}_{p_2 q_2}(\Omega)$. Here h_j are fixed functions from $B^{s_2}_{p_2 q_2}(\Omega)$, $x_j \in \Omega$ and Ω is a bounded Lipschitz domain in $\mathbb{R}^d$. We prove that the optimal rate of convergence of linear sampling method is

$$n^{-\frac{s_1 - s_2}{d} + (\frac{1}{p_1} - \frac{1}{p_2})_+}$$

if $s_2 > 0$ and

$$n^{-\frac{s_1}{d} + (\frac{s_2}{d} + \frac{1}{p_1} - \frac{1}{p_2})_+}$$

if $s_2 < 0$. The same optimal rate is achieved when considering the class of nonlinear sampling methods. In the case $s_2 > 0$ we prove the result only for function spaces on unit cube $\Omega = (0, 1)^d$. On the other hand, this allows to describe the sampling operator more explicitly. Finally, we point out, that the result may be simply carried over to the scale of Triebel-Lizorkin spaces, which includes as a special case also Sobolev spaces.