Report on ICMS meeting
Applied Asymptotics and Modelling
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1 Short description of the meeting

A key aim of the meeting was to bring together a significant number of different groups of very active, yet often disconnected (geographically and ideologically) workers both in exponential asymptotics and beyond. A meeting covering such a broad range of topics and applications had not been attempted for a decade in the UK, since when the state of the subject has advanced dramatically.

To that end we planned the meeting around themed days.

- Monday: Recent developments in exponential asymptotics for PDEs; zeta functions.
- Tuesday: New challenges in asymptotic applications I: biological and stochastic systems to the onset of turbulence through coupled exponential scales.
- Wednesday: Applications of Riemann Hilbert methods.
- Thursday: New challenges in asymptotic applications II: from field theory to nanotechnology and elasticity.
- Friday: Asymptotics applications to dynamical systems, Painlevé analysis. Recent significant advances in integral asymptotics.

The meeting attracted over 40 researchers worldwide, spanning pure and applied mathematics, and from physics to engineering.

The informal atmosphere of ICMS and the facilities of the surrounding town were ideal for new contacts that were forged through planned and spontaneous discussion sessions which has resulted in several new concrete collaborations.

2 Comprehensive report on the meeting

A key aim of the meeting was to bring together a significant number of different groups of very active, yet often disconnected (geograhically and ideologically) workers both in exponential asymptotics and beyond. A meeting covering such a broad range of topics and applications had not been attempted in the UK for a decade, since when the state of the subject has advanced dramatically.

The meeting attracted over 40 researchers worldwide, spanning pure and applied mathematics and from physics to engineering. Recent advances in asymptotic theory were presented with a view to meeting new challenges in emerging applications. The applications discussed included biomathematics, turbulence, atmospheric dynamics, nanotechnology, random matrix theory, fluid flows, elasticity and dislocation and dynamical systems. The common thread of the novel use of asymptotics gave a common language that facilitated understanding between groups working in different research areas.

The original proposal linked the ICMS workshop with a preceding LMS summer school. Due to delays in obtaining referee reports, as late as March 2006, we took the regrettable decision to cancel the latter. Despite this a significant number of active postgraduate students did manage to attend the ICMS meeting. One key participant was unable to attend at the last minute, due to a family illness. The meeting proved to be very popular and we hosted over 40 people (rather than the planned 35) at India Street, having to turn several applicants away.

The meeting was remarkable for several concrete connections that were established by the mix of participants. Space prevents us listing all the highlights of the meeting, so we include but a few of these below.

Dorey has been working with exactly solvable models, the Bethe ansatz and their relation to PT-symmetric hermitian systems. During the meeting he discovered that a significant outstanding problem had been solved using exponential asymptotics in an entirely different context in unpublished work by Chapman. This discovery has spawned a new collaboration that is now underway.

In a highly entertaining survey talk Baumberg, a nanotechnologist, threw down a gauntlet to mathematicians to develop sub-wavelength asymptotics, or uniform asymptotics techniques that span the sub- and super- wavelength barrier in multicomponent nano-systems. This has opened up a wide range of opportunities for future collaboration in the rapidly emerging area of nanotechnology and collaborative projects are currently being actively explored.

The idea of the coupling of exponential scales as a mechanism for physical processes has been a strong undercurrent in recent asymptotic developments, with several groups working in parallel. This meeting brought groups actively together for the first time. Ideas that emerged from discussions included an exponential-asymptotic approach to coupled scales, for example as a toy model for onset of turbulence. This was examined from both a stochastic and deterministic view. Future inter-group collaborations will follow.

One day focussed on a detailed exposition of emerging use of Riemann-Hilbert methods in asymptotics, in particular as applied to random matrix theory (RMT). The asymptotic use of these

techniques in this area are little known outside a small group of connected researchers. The meeting exposed the power of these ideas to obtain rigorous results in RMT to a much wider audience. Participant feedback suggests that several will privately explore these techniques further.

A major new unified approach to the development of asymptotics for integrals, unifying Laplace, Mellin and distributional methods was presented by Lopez and his school from Spain. This is an important comprehensive new theory that will certainly be followed up in new links established through this meeting.

Bender exemplified quantum effects arising from transformations of PT-symmetric field theories. His results pose tantalising conjectures for the Higgs boson.

Movchan provided a comprehensive account of the uniform asymptotics for Green functions in singularly perturbed elastic domains.

Applications of special functions to novel problems was a recurring theme in several talks including classical hypergeometric functions, generalised zeta functions, higher dimensional error functions and Painlevé transcendents.

The 45 minute discussion sessions led to many informal contacts and research discussions which continued on into other venues.

Overall the feedback from participants was extremely positive. Participants cited as highlights of the meeting the range of new ideas and topics, the variety and quality of people attending, the discussion with new contacts, across discipline.

The rating of the academic content by participants ranged from a good to excellent with the majority regarding the meeting as very valuable.

The majority of the participants stated that the workshop helped to sustain and develop new contacts. Almost all said that the workshop had resulted in new ideas or exposure to new techniques.

According to the participants, the informal atmosphere of ICMS and the facilities of the surrounding town were ideal for new contacts to be forged through planned and spontaneous discussion sessions.

There was overwhelming participant support for the efficiency of the staff and administration at the ICMS. We echo this: due to the ICMS staff this has been one of the easiest meetings to plan and execute that either of us have ever organised. The facilities in India street (especially the wireless internet) were also praised.

The apparent overall impression of the majority of participants can be best summarised by the following direct quotation:

"I came home with several ideas worth pursuing. I'll probably be contacting some of the other participants to get more references and learn more. Almost every talk was worth attending and that's unusual.... The workshop was more useful than an Oberwolfach gathering the previous week that was less focused and more confrontational!"