

**Report on the ICMS Workshop: 3-Manifolds after Perelman**  
**Heriot-Watt University, March 13-17 2006**

**1.** The original proposal called for 40 participants. After receiving the referees' comments it was decided to raise that number, and to hold the workshop at Heriot-Watt University.

Because of a death in his family a week before the workshop, David Gabai, one of the original five principal speakers, was unable to attend. In his place we added two 1-hour talks, by Cyril Lecuire and Stephan Tillmann.

**2.** The goal of 3-dimensional topology is to understand all mathematically possible "universes", i.e. spaces (3-manifolds) that are locally like ordinary 3-dimensional Euclidean space but whose global structure might be quite complicated. Since the mid 1970's, the guiding light of the subject has been Thurston's Geometrization Conjecture, which asserts that every 3-manifold is built up from pieces that have a geometric (as opposed to a merely topological) structure. In 2003 a proof was announced by Perelman, using analytic techniques.

The aim of the workshop was to have leading researchers describe the current developments in 3-dimensional topology that are more or less independent of Perelman's work, and what they see as the important directions and problems for the future, now that the Geometrization Conjecture has been proved. The main focus of the program was four series of three lectures given by four leading researchers, who have been responsible for some of the most exciting recent developments in the subject: Jeffrey Brock, Marc Lackenby, Peter Ozsvath and Peter Shalen. There were an additional 11 1-hour lectures given by other internationally recognized experts. The meeting also provided a useful forum for discussion among the participants. We summarize some of the mathematical content below.

**3.** The theme of the workshop was current developments and future directions in 3-dimensional topology, and the 4 principal speakers Brock, Lackenby, Ozsvath and Shalen gave series of lectures that reflected this. Brock discussed the proof of the Ending Lamination Conjecture and how techniques in the proof of the ELC are now starting to be applied to understand the geometry and topology of closed 3-manifolds. Lackenby's talks focused on approaches to the virtual Haken conjecture (and strengthenings) using methods that he has developed. These methods bring together a wonderful diversity of mathematics; including group theory, combinatorics, topology and arithmetic. Ozsvath discussed applications of Heegaard Floer homology, developed by himself and Z. Szabo. Of particular relevance to the theme of the workshop, he discussed the recent impact of Heegaard Floer homology on some old questions in low-dimensional topology and knot theory. Shalen's lectures described his recent joint work with M. Culler on connecting the hyperbolic geometric structure on a closed hyperbolic 3-manifold to the topology of the 3-manifold. This work also advertized work of Agol, Storm and Thurston that uses some of Perelman's work in providing volume estimates for hyperbolic manifolds.

The remaining 11 one hour talks covered a variety of topics that fitted well with the theme of the workshop. For example, Bowditch gave a talk on an alternative approach

to constructing the model manifolds used by Brock, Canary and Minsky in the proof of ELC. The impact of low-dimensional topology in geometric group was illustrated by the talks of Bridson, Sela and Weidmann. Since Thurston's work, experimental methods have always been an important tool in exploring 3-manifolds, and Hodgson's talk illustrated a new computer tool for exploring geometric structures on orbifolds.

There were 93 participants: 4 principal lecturers, 11 1-hour speakers, 24 graduate students, 26 postdocs, 25 senior researchers, and the 3 organizers. We were pleased by the large number of young researchers in attendance. There was a strong international flavour to the workshop, the countries represented, and the corresponding numbers of participants, being as follows: USA (38), UK (25), Germany (7), France (6), Switzerland (3), Australia (2), Israel (2), New Zealand (2), South Korea (2), Spain (2), Canada (1), Italy (1), Japan (1), and Poland (1). We also note that 12 of the participants were women.

The responses to the questionnaires were uniformly positive. They confirm our feeling that the workshop was very successful, and that its success was mainly due to the unusually high quality of the talks, especially the lecture series given by the four principal speakers. Over half the participants were graduate students and postdocs, and with three lectures at their disposal the principal speakers were able to aim their talks at this audience. To quote one of the questionnaire responses:

"Without a doubt the highlight was the excellent program of talks...I think giving selected speakers three one hour slots was a good move; it forced them to concentrate on exposition rather than rattling off about their achievements."

Other comments in the same vein on the highlights of the workshop are:

"There were an unusually large number of truly excellent talks. The organizers did a fantastic job in selecting people capable of giving straightforward and entertaining lectures. The fact that some of these lectures were in 3-lecture series was also very beneficial to people without a great deal of familiarity with the given subject area."

"I think each of the speakers who delivered three 1-hour lectures did a fantastic job discussing their topics. Each of these lectures was a major highlight."

"The highlights were the series of lectures given by the principal speakers. All of them were a good choice and gave very nice introductions to a difficult field of mathematics."

"The highlight of the workshop was the series of lectures given by the main speakers, Jeffrey Brock, Marc Lackenby, Peter Ozsvath, Peter Shalen. They presented different points of view on the study of 3-manifolds and gave a very deep and interesting picture of the subject."

Similar sentiments were expressed regarding the overall academic value of the workshop:

"Very high level conference on 3-manifolds. Good combination of speakers and participants."

"I think it was an excellent meeting. There were lots of discussions."

"I was extremely impressed by the number and quality of the participants. The talks provided good summaries of work done over the last couple of years and suggested directions for future research."

"Excellent overview of recent developments and directions for the future."

"Very high. The speakers were each leaders in their fields. In addition to giving overviews of the major techniques in 3-manifold topology, they also provided a variety of questions in the subject. As a graduate student near completion, I found the talks as a nice roadmap of research trends."

"I felt the overall academic value of the workshop was exceedingly high, in particular because this is a transitional time for the field of 3-manifolds, and a great deal of leadership was provided on the question of how to proceed from here. I can imagine that this was particularly beneficial to younger researchers (of which there was an inordinately high number in attendance) who are seeking guidance and direction."

The workshop also seems to have been useful in enabling the participants to develop or sustain contacts, and to acquire new techniques:

"I was able to talk with 2 co-authors and develop new ideas."

"Had fruitful discussions with two of my collaborators."

"I had a chance to discuss a lot with my colleagues in the USA who I don't get to meet very often."

"A new collaboration is likely to result."

"For me the workshop was also a good occasion to talk with people I am already working with. So it helped us to make some advances in our joint work."

"It was particularly good to have researchers there from Australia and North America,..."

"During the meeting I got to know mathematicians I only had email contact with before. I also met a collaborator and we had the opportunity to further one of our projects."

"The stimulations provided by the participants and talks suggested to me some new avenues to explore."

"I have made new progress in my research based on thoughts developed at the workshop."

Our feelings about the success of the workshop are summed up by another questionnaire response:

"It was widely acknowledged by the participants that this conference was one of the first in quite a while to be so engaging. The participants, the talks, and the subject matter were all of a very high calibre. I will take it as a model for organizing my own conferences, of which I have a few coming up. I was delighted to be invited."

Cameron Gordon

Jim Howie

Alan Reid