Geometry and Algorithms

Report of Meeting

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1 Deviations from the Proposal

There were no significant deviations. Most key invitees attended although sadly one had to cancel at the last minute. Almost all the younger invitees attended. Of the topics listed in the original proposal only the last (rapid mixing) was represented less than expected, in part because the talks by Rademacher and Vempala focussed on a lower bound for the problem.

2 Short description of the meeting

During the last 15 years or so, it has become clear that many problems in theoretical computer science are actually geometric problems in disguise. In other cases, the problems are so complex that the best available approximation algorithms were devised by embedding the underlying structure into a familiar geometry so that it becomes "geometrically obvious" what to do. This general philosophy led to the best currently available algorithms for tackling a long list of computationally hard problems and in consequence the embedding method is one of the hottest topics in theoretical computer science. This meeting brought together mathematicians and computer scientists working in this fertile area to enable them to see both sides of the picture. Its second aim was to advertise the field to British researchers (especially young ones). The expository talks, which were intended to enable both groups to understand each other's language, were extremely well prepared and clear.

3 Full Report

The workshop comprised 27 one hour lectures and an open problem session. There were 6 expository lectures by Kindler, Arora and Linial in Computer Science and by Schechtman, Kleiner and Cheeger in Maths, whose purpose was to explain topics in a way that would be accessible to participants from the other background. These were all exceptionally well prepared and clear. Most attendees, especially the younger ones, commented upon how useful they were. Kindler's lecture in particular, was masterly. He managed to define the majority of the technical terms used by the CS group in a completely clear way in the course of an interesting discussion of a suitably chosen representative problem. Kleiner and Cheeger presented a difficult topic in differential geometry in a way that was intelligible to everyone and extremely informative.

The lectures of Kleiner and Cheeger and also those of Fefferman, Lafforgue and Weinberger were special "guest" lectures by distinguished mathematicians who do not work directly on metric embedding but whose recent work on geometric group theory and the Whitney extension problem has impinged upon the area. For the computer scientists these lectures gave a very different context to the tools they use and have been instrumental in developing.

The remaining 18 lectures were regular research lectures in which speakers presented new results on the major questions in the field. Many participants stated in their questionnaires that they couldn't pick out the best because there were so many good ones. Highlights included the following talks:

Indyk spoke about his recent work which provides the first explicit embedding of L_2 into L_1 of essentially the same dimension. This problem attracted a lot of attention in the late 80's from the analytic geometry community but there was essentially no result of note. It was known that random embeddings gave the same dimension but the only explicit embeddings went via L_4 and produced a quadratic increase in dimension. Indyk used extractors from computer science coupled with a delicate iterative scheme.

Naor and Mendel spoke about uniform convexity properties in metric spaces. Their work on metric cotype has produced an astonishing structure theorem for metric spaces. Despite work in the 80's and before by Enflo and Bourgain on metric type and in the 90's by Ball on Markov versions of these invariants, there was no satisfactory way to extend the geometric properties of cotype to metric spaces. Mendel and Naor have produced an invariant with all the properties one could hope for and established a subtle non-linear analogue of the Maurey-Pisier Theorem, guaranteeing the presence of large ℓ_∞ grids in any metric space without the property.

O'Donnell gave a very attractive talk showing that the parallel repetition problem from computer science strengthens several geometric problems to do with

foams (minimal surface area lattice tilings of space). It is striking that the gap between known upper and lower bounds on this problem is essentially as large as it can be: nothing non-trivial is known.

Lee presented some results on planar graphs and recalled the intriguing conjecture that planar graphs can be embedded with bounded distrotion in L_1. His talk was presented in a particularly relaxed and accessible way and several younger attendees said how much they enjoyed it.

Krauthgamer showed that edit distance is hard to approximate using Foruier analytic techniques. One intriguing aspect of the talk is that he thereby obtained a lower bound for the embedding dimension of the edit distance metric into L_1: it is much more usual to use upper bounds for embedding to imply algorithmic bounds. Here the computer science is yielding consequences for the mathematical theory.

The main aim of the meeting was to encourage collaboration between mathematicians and computer scientists and to kick start another push to solve outstanding problems. The problem session was thus one of the most important parts of the programme. The enthusiasm of the participants for this session can be gauged from the fact that we ran over the two hour slot allocated and the organisers still had to beg the participants to go out for tea. As explained in the original proposal the problem session will be recorded on the problems page at

`http://kam.mff.cuni.cz/~matousek/`

It is very likely that new collaborations will emerge from the meeting. Several participants stated in questionnaires that they expected this to occur. Tim Austin, one of the student participants, mentioned problems that he will work on with Naor: in fact their article has just been submitted to JFA.

Of the 50 participants 6 were female and 11 were from British Universities.

Several of the questionnaire responses contained phrases such as "This was the best workshop I ever attended." While vague, such comments are at least very gratifying. The question on organisation also elicited response like "Only superlatives, such as perfect organisation...." The scientific organisers would like to say a huge thankyou to all the staff of ICMS and especially to Audrey Brown.