Report on
Further Developments in Quantitative Finance

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(on behalf of the organisers)

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Scientific Organisers

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The meeting took place at ICMS from 9–13 July 2007 and was attended by 36 participants, almost all attending for the entire workshop. There were 32 talks, each of 45 minute duration, from established leaders in the field of Financial Mathematics (FM) and from younger researchers of outstanding promise.

The aims of the meeting were to build on the successes of the *Developments in Quantitative Finance* programme at the Isaac Newton Institute for Mathematical Sciences, Cambridge, in 2005, by inviting some of the participants of that programme to reconvene, to further the advances and achievements of that programme and to describe more recent progress.

These aims were met with resounding success. The comments from the participants were extremely positive in virtually every respect.

Short description of the meeting

Much of the research presented fell naturally into broad themes, such as *illiquid markets*, *transaction costs* (TC), *incomplete information*, *credit risk*, and analysis of risk preferences such as *forward utilities* and *risk measures*. Talks on the same theme were often grouped together. Many delegates commented that they liked this format.

There were talks on other areas, such as problems related to *optimal stopping*, and on modelling the observed *implied volatility* surface of market option prices.

The common theme was the extension of classical, perfect models of financial markets to incorporate one or more realistic and important market imperfections. The practical importance of such research is that it is essential to understanding how market failures can arise, and how to deal with the resulting risks. The recent turmoil in financial markets has arisen due to defaults on home loans in the United States (and is thus an instance of

credit risk), and has resulted in lack of liquidity in interbank lending markets. The practical importance of the research therefore cannot be overstated.

The mathematical challenges involve techniques of functional and stochastic analysis, new techniques of stochastic control such as backward stochastic differential equations (BSDEs), and techniques of singular control, impulse control, and numerical and asymptotic analysis of partial differential equations (PDEs), often in high dimensions.

It is difficult to pick a single highlight, but the research on transaction costs, by Jan Kallsen, and on the dynamics of risk preferences, by Thaleia Zariphopoulou, generated a great deal of discussion and potential for future collaboration.

Comprehensive report on the workshop

The workshop was relatively small in size, and this allowed for an intimate atmosphere in which interaction between delegates was easy to initiate. Many delegates commented positively on this aspect, and also complimented the venue and organisation, both academic and administrative. There were hardly any negative comments, but it is worth remarking that some delegates would have liked accommodation closer to ICMS.

Delegates made very constructive comments on many talks. Dmitry Kramkov made extremely pertinent remarks on a model of optimal hedging under parameter uncertainty presented by Michael Monoyios. These centred on how to interpret the results of a filtering algorithm to learn about unknown parameter values in a Bayesian setting. The suggestions will help to resolve the crucial question of whether utility-based methods, by now a classical technique in FM, are of any practical use in managing risk from selling options and other contingent claims.

Hans Föllmer, after giving a very interesting talk on the dynamics of convex risk measures, gave an impromptu short presentation which commented on a curious feature of an optimal investment model presented by David Hobson. In Hobson's model, an apparently counter-intuitive result was presented, in which a risk-averse agent entered into risk-seeking behaviour. Föllmer made very insightful comments on how this apparent anomaly could arise, leading to future collaboration which hopes to understand this feature in detail.

There were extremely interesting talks on other problems related to optimal stopping. Xin Guo elucidated the link between switching controls and optimal stopping, which arise when one has the choice to increase or decrease the production rate of a real investment such as copper mine or oil field. This was inspired in part by earlier work of Mihail Zervos, and the two delegates were able to engage in fruitful discussions on this topic.

Some of the major themes of the meeting are described below.

Illiquidity Research on illiquid markets addresses the fact that agents cannot effect trades of arbitrarily large size without a concomitant effect on prices in financial markets. This generates corrections to classical results on derivative pricing theory, such as the celebrated Black-Scholes-Merton (BSM) results, allowing for price bounds which reflect the risk from finite liquidity. Talks by Dmitry Kramkov, Dirk Becherer, Huyen Pham and Chris Rogers addressed this important issue, which has been at the forefront in recent weeks due to the marked absence of liquidity in interbank lending markets.

Credit risk This work attempts to understand how to value risk arising from potential defaults in financial transactions. This is a difficult and important topic, as much default risk has been bundled into complex contracts such as collaterised debt obligations (CDOs). Much of the recent nervousness in financial markets stems from the uncer-

tainty in the true values and risks of such instruments. Advances in this area based on time changed Brownian were reported by Tom Hurd.

Transaction costs This research incorporates commissions and other frictions on stock trading, a realistic effect that is absent from classical models of perfect markets. This is a much studied area, important because it attempts to generate realistic discrete trading strategies, as opposed to the idealised continuous trading strategies of classical (frictionless) models. It has proved difficult to establish many concrete results, but recent advances have led to exciting possibilities for both establishing fundamental valuation theorems, and also for new ways of computing optimal portfolios in this notoriously difficult field. The talks in this area were one of the highlights of the meeting, and look set to lead to productive future collaboration.

Walter Schachermayer and Miklos Rasonyi presented advances in establishing fundamental theorems of asset pricing in continuous time models with small transaction costs. This work was based on using duality theory, relating the martingale properties of asset prices to generalisations of no arbitrage conditions (known as *consistent price systems*) in models with TC. Jan Kallsen gave a beautiful and surprising talk on TC, in which he exploited such duality theory to give a new way of computing optimal portfolios with TC. This was a deviation from the published programme, and constituted a very recent and extremely original development that generated a great deal of fruitful discussion.

Stochastic and implied volatility models This is another area which has received much attention in the past, in an attempt to understand the patterns of implied volatilities of market option prices across strikes and maturity. Despite many advances, it has proved quite difficult to generate models that capture the observed dynamics of the implied volatility surface, and that also respect fundamental tenets of finance theory such as absence of arbitrage. The workshop featured some new approaches to the problem, using new parametrisations to encode the information in market option prices. Martin Schweizer and Jan Obloj gave two excellent talks in this area, and Michael Monoyios was able to alert Professor Schweizer to recent advances by Rene Carmona in Princeton on this topic. This has fed a future possible collaboration, which Schweizer and Monoyios will pursue at a meeting in Oberwolfach in January 2008.

Parameter uncertainty and incomplete information This is an important and relatively recent development in financial mathematics, which relaxes the unrealistic assumption of classical models that agents have perfect knowledge of the values of parameters appearing in the stochastic equations describing asset prices. This is one of the main reasons that highly stylised and beautiful mathematical models of optimal investment, pioneered by Merton in the 1970s, have failed to have a significant impact on practical investment decisions. The work presented in the workshop shows promise in remedying this situation, allowing sophisticated mathematical models to play a more meaningful role in investment decisions, and in the hedging of claims in incomplete markets, in a similar manner to their role in the pricing of derivative securities in complete markets.

Andrew Lim gave a very interesting presentation on how to make decisions that are robust to such model uncertainty, and contributed valuable suggestions to Michael Monoyios' talk, as did Dmitry Kramkov.

As well as Hans Föllmer's talk, Giacomo Scandolo presented a very good talk on robustness of risk measurement procedures, that emphasised the role of the statistical

estimation procedure implicit in risk measurememt. There were clear links to the talks by Lim and Monoyios, and possibilities for future collaboration.

Modelling of preferences via dynamic utility Real markets, as opposed to classical perfect markets, are incomplete, meaning that risk associated with derivative contracts cannot be entirely eliminated by judicious trading of assets underlying the derivative contracts. The preferences of agents toward such risks must therefore be taken into account, and utility functions are one way of specifying such preferences. Traditionally, such preferences are assumed to be constant in time, but in reality are adjusted to take into account market dynamics. Moreover, the preferences have traditionally been set into a fixed final horizon, so it is unclear how to deal with problems of random horizon.

Important new developments in this area were presented by Thaleia Zariphoulou and Mike Tehranchi, and this led to an extremely lively discussion. Professor Zariphopoulou's talk, and the ensuing discussion, over-ran the allotted time by some 45 minutes. David Hobson pointed out that he and Vicky Henderson has produced work of a similar nature, and there is great promise for future application of these ideas.

Utility based methods for asset valuation The use of utility theory to price and hedge derivative contracts in incomplete markets has become a classical paradigm in financial mathematics, but there are relatively few explicitly solved models. The workshop featured some new approaches to the problem, involving asymptotic methods (by Mihai Sirbu and Michael Monoyios), and also using modern techniques of stochastic control based on backward stochastic differential equations (by Stefan Ankirchner). This opens up the possibility of the use of these techniques in realistic situations, as well as being a rich source of interesting mathematical control problems.

There were other highlights that gave the possibility of further progress and collaboration. Mihail Zervos gave a talk on understanding when a set of discrete data can be assumed to be generated by a diffusion model, and both David Hobson and Jan Obloj pointed out relations to the Skorohod embedding problem, on which they are experts.

In summary, this was an extremely successful meeting, and it is hoped that a further such workshop can be organised in future. Many delegates said they would be interested in attending such an event again.