Matrix factorisations and related topics

24 – 28 July 2017

International Centre for Mathematical Sciences, Edinburgh

Abstracts

Baur, Karin
Dimer models with boundary and Grassmannian cluster categories
In this talk, I will illustrate the cluster structure on a coordinate ring of a Grassmanniann. Then I will explain how to get a categorical version of the cluster structure on the Grassmannian, using dimer models. A dimer model can be defined as a quiver embedded into a surface in such a way that the complement is a disjoint union of disks with oriented boundaries. Such models can also be considered in the case of a surface with boundary. We use Postnikov diagrams of Grassmannian type as source for such dimer models. This allows us to define a natural (Jacobian) algebra. Its boundary algebra is the idempotent subalgebra with respect to the boundary vertices. The latter gives rise to a categorical version of Scott's cluster structure on the Grassmannian. This is joint work with A. King and R. Marsh.

Brown, Michael
Topological K-theory of matrix factorisation categories
This is a report on joint work with Tobias Dyckerhoff. Topological K-theory of complex-linear dg categories is a notion introduced by Blanc in his recent article "Topological K-theory of complex noncommutative spaces". In this talk, I will discuss a calculation of the topological K-theory of the dg category of graded matrix factorisations associated to a quasi-homogeneous polynomial with complex coefficients in terms of a classical topological invariant of a complex hypersurface singularity: the Milnor fiber and its monodromy. Along the way, I will discuss a compatibility between Knörrer periodicity and equivariant Bott periodicity at the level of topological K-theory spectra.

Buchweitz, Ragnar-Olaf
Matrix factorisations of cubics
I will review what is known about matrix factorisations as in the title. Among the results we exhibit the graded Betti tables of indecomposable such factorisations over a smooth plane cubic and use that knowledge to find all matrix factorisations with linear entries. We then show how this can be used to find a small resolution of singularities of a certain singular Calabi-Yau threefold. (Based on work of A. Pavlov and collaborations/discussions with him.)

Burban , Igor
Cohen-Macaulay modules and Calogero-Moser systems
In my talk, I am going to speak about the ring of (generalized) two-dimensional quasi-invariant polynomials proving that it is Cohen-Macaulay and Gorenstein in codimension one. Using the technique of matrix problems, I shall classify all maximal Cohen-Macaulay modules of rank one over such a ring and determine its Picard group. In terms of this classification, I shall describe the spectral module of a rational Calogero--Moser system of dihedral type, proving that it is actually projective. Finally, using methods of the higher dimensional Krichever correspondence, I am going to present a new example of a deformed Calogero-Moser system arising from a non-projective Cohen-Macaulay module. This is a joint work with Alexander Zheglov.

Dao, Hailong

On rigid MCM modules

A maximal Cohen-Macaulay module M is called rigid if $\mathrm{Ext}^1(M,M)=0$. In this talk I will explain how matrix factorisations help us understand rigid MCM modules over hypersurface singularities. I will also explain how to generalize the results to a complete intersection R using some a suitably extended version of the Hochster's pairing on the Grothendieck group of $MCM(R)$.

Donovan, Will

Perverse sheaves of triangulated categories and Bridgeland stability

Kapranov and Schechtman have initiated a program to study perverse sheaves of triangulated categories, or perverse schobers. I explain how natural examples arise from certain variations of GIT quotient, and flopping contractions. For the Atiyah flop case, I relate these to a schober on a compactification of the stringy Kaehler moduli space, and to a local system of categories on the Bridgeland stability manifold.

Faber, Eleonore

Matrix factorizations of discriminants of reflection groups

Let G be a finite subgroup of $GL(n,\mathbb{C})$. Then G acts linearly on the polynomial ring S in n variables over $\mathbb{C}$. When G is generated by reflections, then the discriminant D of the group action of G on S is a hypersurface with singular locus of codimension 1, a so-called free divisor. In this talk I will present certain matrix factorizations that arise from our construction of a noncommutative resolution of singularities of D as a quotient of the twisted group ring $S*G$. This is joint work with R.-O. Buchweitz and C. Ingalls.

Hara, Wahei

Mukai flops and P-twists via non-commutative crepant resolutions

In this talk, we will discuss the derived category $D(T^{*P})$ of the cotangent bundle T^{*P} on a projective space P from the point of view of non-commutative crepant resolutions. More precisely, we describe P-twists on $D(T^{*P})$ and Kawamata-Namikawa's equivalences for Mukai flops in terms of mutations of non-commutative crepant resolutions.

Hirano, Yuki

Relative singular locus and matrix factorisations

We introduce the notion of relative singular locus for any closed immersion of Noetherian schemes, and we classify certain thick subcategories of derived matrix factorisation category of a Landau-Ginzburg model by means of specialisation-closed subsets of the relative singular locus of the zero fiber of the potential of the Landau-Ginzburg model. Moreover we discuss the Balmer's spectrum of derived matrix factorisation categories.

Ingalls, Colin

Noncommutative Curves of Finite Representation Type

We classify noncommutative curves of finite representation type which are quotients of quantum planes at roots of unity. We use noncommutative matrix factorisations and general results of Greuel-Knörrer. We also compute their AR quivers using the classification of orders of finite representation type of dimension two by Artin and Van den Bergh, Reiten. Joint work with Daniel Chan.

Iyama, Osamu

Geigle-Lenzing complete intersections and Tate resolutions

The stable category of Cohen-Macaulay modules over a Geigle-Lenzing complete intersection ring is triangle equivalent to the bounded derived category of a certain finite dimensional algebra A. We discuss properties of A including explicit value of its global dimension as well as their applications. A key ingredient is Tate's DG algebra resolution of complete intersection rings. This is a joint work with Herschend, Minamoto and Oppermann

Karmazyn, Joseph
Knörrer periodicity for cyclic quotient surface singularities
The singularity category of the type A_n Kleinian surface singularity is equivalent to that of the finite dimensional algebra $k[x]/x^n$ and this equivalence can be realised via Knörrer periodicity. I will discuss joint work with Martin Kalck that extends a similar phenomenon outside of the Kleinian case to produce an equivalence between the singularity category of a cyclic quotient surface singularity and a certain finite dimensional algebra.

King, Alastair
Quivers and Conformal Field theory: preprojective algebras and beyond
I will describe the appearance of ADE preprojective algebras in Conformal Field Theories arising as $SU(2)$ WZW models and explain how it generalises to the $SU(3)$ case, where 'almost $CY3$' algebras appear.

Leuschke, Graham
A generalization of Knörrer periodicity, with applications to noncommutative hypersurfaces
This talk will describe joint work with Alex Dugas, relating matrix factorizations of a polynomial f and of $f + z^n$, which gives for each $k = 1, \dots, n$ a covariantly finite subcategory of matrix factorizations of the latter. The case $k = 1$ and f an ADE singularity leads to a class of noncommutative rings over which every minimal projective resolution is eventually periodic of period 2, suggesting that they might be called noncommutative hypersurfaces.

Ploog, David
Tilting chains of negative curves on surfaces
I present an abstract result when which ordinary tilting theory has a peculiar, special property: it preserves the hearts of the derived categories. This result applies to the exceptional sequence of line bundles of a chain of negative curves. (joint work with Lutz Hille)

Raedschelders, Theo
Frobenius pushforwards on Grassmannians $G(2,n)$
Let R be the Plücker coordinate ring of the Grassmannian $G(2,n)$ over an algebraically closed field of characteristic $p > 0$. In this talk I will discuss the geometric interpretation of recent work with Špela Špenko and Michel Van den Bergh which provides an explicit and characteristic free decomposition of R as graded R^p-module. If time permits I will compare these results to work of Bezrukavnikov and Mirković on Frobenius decompositions for general partial flag varieties.

Ros Camacho, Ana
Matrix factorisations and the Landau-Ginzburg/conformal field theory correspondence
We will review the Landau-Ginzburg/conformal field theory correspondence, a result conjectured in the theoretical physics literature around late 80s and early 90s which in particular relates categories of representations of vertex operator algebras and categories of matrix factorisations - but which to date lacks a precise mathematical statement. We will present some recent progress in this topic, based on joint work with N. Carqueville, A. Davydov, R. Newton and I. Runkel.

Segal, Ed
Hori Duality
Hori has argued that there is a physical duality relating certain non-abelian gauged linear sigma models. I will explain how we can interpret this duality as predicting an equivalence between two categories, whose definitions involve matrix factorizations and non-commutative resolutions. I will also (hopefully) explain how we proved this equivalence. This is joint work with Joergen Rennemo.

Schnuerer, Olaf
Motivic measures via matrix factorizations and motivic vanishing cycles
We explain and compare two motivic measures on the Grothendieck ring of varieties over the affine line. These two measures are of completely different nature: the first is of categorical nature, using dg enhancements of categories of matrix factorizations; the second is of geometric nature and uses the motivic vanishing cycles introduced by Denef and Loeser.

Špenko, Špenka

Frobenius morphism in invariant theory

This is a joint work with Theo Raedschelders and Michel Van den Bergh. The coordinate ring R of the Grassmannian $\mathbb{G} = \mathrm{Gr}(2, n)$ admits a "standard" non-commutative resolution given by Cohen-Macaulay modules of covariants. In the talk we will discuss new non-commutative resolutions of R arising from the R-modules $R^{p^{-r}}$ for $r \geq 1$ defined over an algebraically closed field of characteristic $p > 0$. We will give an explicit decomposition of $R^{p^{-r}}$, which, in particular, shows that not all indecomposable summands are modules of covariants. Moreover, the number of modules occurring as summands when varying r is finite (in a suitable sense), showing that R has finite F-representation type (FFRT). Passing to the geometric side, we will note that however the Frobenius pushforward $\mathrm{Fr}_*^r \mathcal{O}_{\mathbb{G}}$ is not a tilting bundle on $\mathbb{G}$.

Stevenson, Greg

Thick subcategories for graded hypersurfaces

I'll discuss lattices of thick subcategories for homotopical gadgets associated to hypersurfaces, e.g. homotopy categories of matrix factorizations. The focus will be on the importance of understanding the graded case; a major subtheme will be the current wealth of our ignorance. Motivation and examples will be drawn from joint work with Jesse Burke and recent joint work with Sira Gratz.

Takahashi, Atsushi

Matrix factorisations and orbifold Jacobian algebras

We give a natural generalisation of the Jacobian algebra to the pair of a polynomial and a finite abelian group preserving the polynomial. Its relationship with the Hochshild cohomology group of the category of equivariant matrix factorisations will also be explained.