Braids in algebra, geometry and topology

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Abstracts

Berger, Mitchell

Applications of braids in dynamics and solar physics

The solar atmosphere experiences magnetic storms (flares) which both heat up the atmosphere and occasionally eject material into interstellar space. A substantial fraction of the smaller flares are thought to arise from reconnection of braided magnetic fields. Endpoint motions in the surface of the sun add to the braid complexity, while flares reduce the complexity by removing crossings. This process will be described and some models of self-organized criticality introduced. Secondly, I will describe a method of generating geometrical braids using Hamiltonians based on higher order invariants.

Birman, Joan

Computing distance in the curve graph

The curve complex C(S) of a surface S is a simplicial complex whose vertices are isotopy classes of simple closed curves on S. Vertices x and y are joined by an edge if they have disjoint representatives. Assigning length 1 to each edge makes the 1-skeleton $C^1(S)$ a metric graph. The group Aut C(S) is the mapping class group of S. We will discuss joint work with Margalit and Menasco on an effective algorithm, implemented with a computer program, for computing distance in C(S). The algorithm replaces the 'tight geodesics' that have been used widely in studies of C(S) by a new class, called 'efficient geodesics'. Like tight geodesics, there are at most finitely many efficient geodesics that join arbitrary vertices v,w of C(S).

Boston, Nigel

Random Galois groups and pro-p braid groups

In 2011, Ellenberg and I gave a heuristic prediction for the distribution of Galois groups of maximal pro-p extensions of the rationals unramified outside a 'random' set of primes. I shall review this for a general audience and then describe subsequent work generalizing it.

Bridson, Martin

Kahler groups, RAAGs and subdirect products of surfaces

It remains a mystery as to which finitely presented groups are fundamental groups of compact Kahler manifolds (or complex projective varieties) – ie, are Kahler groups. I'll describe recent work of myself and others that provides new examples of Kahler groups, on the one hand, and new obstructions to being Kahler, on the other hand.

Farb, Benson

Braids, polynomials and Hilbert's (still open) 13th problem

There are still completely open fundamental questions about polynomials in one variable. One example is Hilbert's 13th Problem (widely believed (incorrectly) to have been solved in 1957). In this talk I will explain how braid groups are central to understanding such problems. Along the way we will see some beautiful classical objects – the space of monic, degree d square-free polynomials, hyperplane complements, algebraic functions, discriminants, braid groups and configuration spaces – all intimately related to each other, all with mysteries still to reveal.

Ghrist, Robert
Morse theory on spaces of braids
This will overview work of the speaker with Rob Vandervorst and others on doing Morse theory on the space of braids. Starting from simple parabolic PDEs, I'll explain how to define a Morse homology as a forcing index for pairs of braids, culminating in a computable(?) Floer theory.

Grigsby, Julia Elisenda
Braids and homology-type invariants
Categorified homology-type invariants have proved quite useful for studying objects in low-dimensional topology. One such invariant, an annular version of Khovanov homology, first introduced by Asaeda-Przytycki-Sikora, is particularly well-suited to studying questions about braids and their closures. I'll survey what we know, and say a few words about what we wish we knew.

Hepworth, Richard
The edge of the stable range
A sequence of groups, each one a subgroup of the next, is said to satisfy "homological stability" if, in each fixed degree, the induced maps on homology are isomorphisms once you are far enough along the sequence. Stability holds in many cases, for example symmetric groups, general linear groups of fields, and automorphism groups of free groups. Stability often leads to complete computations within the "stable range" where the maps are all isomorphisms. I will explain a new approach to homological stability that gives new information about what happens just outside the stable range (i.e. on the "edge" in the title). I will explain applications to automorphism groups of free groups and to the general linear groups of the field of two elements.

Hironaka, Eriko
Braid actions on rational maps
Given a branched covering $f : (S^2, A) \to (S^2,B)$, where B is a finite subset containing the critical values of f, and A is a subset of B, what can we say about the space D_f of rational maps $(P^1,A') \to (P^1,B')$ that realize this covering? Aside from a family of counter-examples known as maps of Lattes type, W. Thurston's rigidity theorem for rational maps shows that if A=B the space D_f is either empty or a single point, and A. Epstein proved that D_f (which naturally sits in $Teich(S^2,B)$) is either empty or a complex submanifold of dimension $|B-A|$. So far, little is known about the global topology of D_f. In this talk we sketch a proof that the deformation space of rational maps need not be connected, by exhibiting an example with infinitely many connected components. The talk will be based on joint work with Sarah Koch.

Margalit, Dan
Hyperelliptic Torelli Groups: problems and progress
The hyperelliptic Torelli group for a surface with one boundary component is isomorphic to the kernel of the integral Burau representation of the braid group. We will discuss recent progress on the algebraic structure of this group, including work with Brendle and Putman in which we determine a natural generating set. We will then discuss open problems and new directions in the field.

Marin, Ivan
Braids, ties, and Hecke algebras
The (Iwahori-)Hecke algebra of type A is well-known to provide interesting representations of the braid groups. More generally, the Hecke algebra associated to a Coxeter group is a natural quotient of the group algebra of the corresponding Artin group. We introduce another, larger, quotient of it, which is an extension of the Hecke algebra and specializes in the type A case to an algebra of `braids and ties' introduced earlier by Aicardi and Juyumaya. Our construction provides new insight on this special case, too. When the Coxeter group is a Weyl group, this algebra is related to the so-called Yokonuma-Hecke algebra of the corresponding root system. If time permits, we will also sketch a generalization of this construction to the complex reflection groups setting.

Masbaum, Gregor
An application of TQFT to modular representation theory
Let G=Sp(2n,k) be the symplectic group defined over an algebraically closed field k of characteristic $p > 0$. Simple G-modules in the natural characteristic can be classified up to isomorphism by highest weights, but their dimensions are largely unknown in general. In this talk, we will present a new family of highest weights for G whose dimensions we can compute explicitly. The corresponding G-modules appear as a by-product of Integral Topological Quantum Field Theory, and their dimensions are given by formulae similar to the Verlinde formula from Conformal Field Theory.

Paris, Luis
Endomorphisms of the mapping class group of a non-orientable surface
Let N be a non-orientable surface with possibly a finite number of boundary components. Recall that the mapping class group of N, denoted by M(N), is the group of isotopy classes of homeomorphisms of N. We consider a finite index subgroup G of M(N), and an injective homomorphism $\varphi : G \to M(N)$. The talk will turn around the following result (in writing process) in collaboration with Elmas Irmak. Theorem: there exists f in M(N) such that $\varphi(g) = fgf^{-1}$ for all g in G. In other words, φ is defined by a homeomorphism of N.

Roberts, David
Monodromy of Hurwitz-Belyi maps
A Belyi map is a map from a compact Riemann surface to the Riemann sphere which is ramified only above 0, 1, and infinity. Particularly interesting Belyi maps arise as solutions to Hurwitz moduli problems. I will discuss these Hurwitz-Belyi maps, including explicit equations in some large degree examples. The emphasis will be on how braid group techniques can be used to compute their monodromy.

Rolfsen, Dale
Braids, Aut(F_n) and small volume hyperbolic 3-manifolds
As observed by E. Artin, the braid group has a faithful representation on the automorphism group Aut(F_n). Since F_n is an orderable group, we study which braids give rise to automorphisms which preserve some invariant ordering of F_n. This is applied to study orderability of the fundamental groups of minimal volume cusped hyperbolic 3-manifolds.

Suciu, Alexandru
Algebraic invariants of pure braid-like groups
I will discuss the resonance varieties, the lower central series ranks, and the Chen ranks, as well as the residual and formality properties of several families of braid-like groups: the pure braid groups P_n, the welded pure braid groups wP_n, the virtual pure braid groups vP_n, as well as their `upper' variants, wP_n^+ and vP_n^+. This is joint work with He Wang.

Thiffeault, Jean-Luc
Topological optimization with braids
The close relationship between braids and the mapping class groups of punctured disks can be used to enumerate ways of stirring a fluid, or of stretching a piece of taffy. This presents a natural optimization problem: what is the braid that leads to the most efficient method of stirring? In mathematical language, we are asking for braids corresponding to mapping classes with large topological entropy. Such an optimization problem requires some kind of cost function, and a natural choice is to use the minimum word length of the braid in terms of Artin generators. We show that in that case the optimal braid leads to an entropy related to the Golden ratio. Using a different and more realistic cost function leads to an entropy based on the lesser-known Silver ratio.

Venkataramana
Burau representation and arithmetic groups
We show that the monodromy group associated to the family of cyclic coverings of the projective line with prescribed ramification data, is an arithmetic group provided the number of ramifications exceeds the order of the cyclic covering.

Vogtmann, Karen
Homology stability for braid groups and mapping class groups
In this talk I will use a complex of tethers to give a simple proof of homological stability for braid groups, then show how this can be adapted to give a newly streamlined proof of homology stability for more general mapping class groups. This is joint work with Allen Hatcher.

Wahl, Nathalie
Polynomiality for braid group representations
Through a braided version of the category FI of finite sets and injections, I will explain what "polynomial" braid group representations are, and state a stability theorem for the homology of the braid groups with polynomial and abelian coefficient systems.

Westerland, Craig
Fox-Neuwirth/Fuks cells, quantum shuffle algebras, and Malle's conjecture for function fields
In 2002, Malle formulated a conjecture regarding the distribution of number fields with specified Galois group. The conjecture is an enormous strengthening of the inverse Galois problem; it is known to hold for abelian Galois groups, but for very few non-abelian groups. We may reformulate Malle's conjecture in the function field setting, where it becomes a question about the number of branched covers of the affine line (over a finite field) with specified Galois group. In joint work with Jordan Ellenberg and TriThang Tran, we have shown that the upper bound in Malle's conjecture does hold in this setting. The techniques used involve a computation of the cohomology of the (complex points of the) Hurwitz moduli spaces of these branched covers. Surprisingly (at least to me), these cohomology computations can be rephrased in terms of the homological algebra of certain braided Hopf algebras arising in combinatorial representation theory and the classification of Hopf algebras. This relationship can be leveraged to provide the upper bound in Malle's conjecture.

Lightning talks

Boyd, Rachael
Homological stability for families of Artin groups
My current research is on homological stability for families of Artin groups. The aim is to extend the known cases of stability for A_n, B_n and D_n to more general families of Artin groups. I will explain the problem and the approach I am using.

Bregman, Corey
Hyperelliptic graphs and the Jacobian map on outer space
The Jacobian map on Culler-Vogtmann's outer space associates to each marked, metric graph a positive definite quadratic form, and is the free group analog of the classical Abel-Jacobi map on Teichmuller space. We compute the fibers of this map, showing them to be contractible subspaces with natural stabilizers. Metric graphs admitting a 'hyperelliptic involution' play an important role in the structure of the Jacobian map, leading us to define the hyperelliptic Torelli group, $ST(n) < Out(F_n)$, for which we also determine normal generating set.

Chen, Lei
Surjective homomorphisms between surface braid groups
Let PBn(Sg,p) be the pure braid group of a genus g>1 surface with p punctures. In this paper we prove that any surjective homomorphism PBn(Sg,p)→PBm(Sg,p) factors through one of the forgetful homomorphisms. We then compute the automorphism group of PBn(Sg,p), extending Irmak, Ivanov and McCarthy's result to the punctured case. Surprisingly, in contrast to the n=1 case, any automorphism of PBn(Sg,p), n>1 is geometric.

Chen, Weiyan
Homology of braid group with coefficients in the Burau representation
I will describe the homology of the braid groups with twisted coefficients in the rational reduced Burau representations, and explain why we care.

Damiani, Celeste
Loop braid groups: ubiquitous objects opening the door to a plethora of questions through different fields
The study of loop braid groups has been widely developed during the last twenty years, in different domains of mathematics and mathematical physics. They have been called with several names such as motion groups, groups of permutation-conjugacy automorphisms, braid-permutation groups, welded braid groups and untwisted ring groups. We will give a glance on how every equivalent definition carries open questions in the domain it belongs to.

Fullarton, Neil
Top-dimensional cohomology in the mapping class group
A basic question about any group or space is what its (rational) cohomology groups look like. I will give a summary of what is known about such cohomology of the mapping class group of an oriented surface, and then discuss the vast amount of cohomology Putman and I discovered for some of its finite index subgroups. This has consequences for the coherent cohomological dimension of the moduli space of curves, which I will outline. This is joint work with Andrew Putman.

Ghaswala, Tyrone
Braids embedded in mapping class groups
A non-geometric embedding of a braid group in a mapping class group is an embedding where each standard braid generator is not mapped to a Dehn twist. In this talk I will present new examples of non-geometric embeddings of braid groups in mapping class groups. This is joint work with Alan McLeay.

Hubbard, Diana
An invariant of closed braids from the Khovanov setting
I will discuss an invariant of closed braids, κ, that arises from the settings of Khovanov homology and annular Khovanov homology. This invariant can be used to obstruct braid destabilization, detect negative flypes, and it yields a new solution to the world problem in the braid group. This work is joint with Adam Saltz.

Knudsen, Benjamin
Homology of surface and graph braid groups
We give explicit formulas for the Betti numbers of the configuration spaces of an arbitrary finite surface, and we present a small complex computing the homology of the configuration spaces of an arbitrary finite graph. This is joint work with Byunghee An and Gabriel Drummond-Cole.

Maguire, Megan
Cohomology algebras of conguration spaces
For a manifold X with finite-dimensional cohomology, we know that the cohomology algebra of each unordered conguration space of X is finitely generated, but can we say something stronger about its generators? More precisely, does there exist a D (depending only on X) so that the cohomology algebra of each unordered conguration space of X can be generated in degree at most D? We will answer this question in some cases.

McLeay, Alan
Ivanov's metaconjecture and complexes of regions
It is a well-known and fundamental result of Ivanov that the curve complex (or curve graph) of an orientable surface with punctures has automorphism group isomorphic to the extended mapping class group of the surface. Similar results are true for a number of other complexes related to a surface, leading Ivanov to make a metaconjecture: all sufficiently rich objects related to the surface will have automorphism group isomorphic to the extended mapping class group. I will discuss a

generalisation of the work done by Brendle-Margalit which resolves this metaconjecture for a broad class of complexes.

Salter, Nick
Mapping class groups and monodromy of families of plane curves
We will discuss a problem at the intersection of complex algebraic geometry and low-dimensional topology. Associated to the family of embedded smooth plane curves of degree d is the monodromy representation; this is a subgroup of the mapping class group of the underlying topological surface. A folklore theorem states that there are invariants known as "n-spin structures" preserved by the monodromy; consequently the monodromy group can be at most finite index. We will discuss recent progress towards showing that these monodromy groups are as large as possible given this restriction.

Strenner, Balazs
Fast Nielsen-Thurston classification
The Nielsen-Thurston Classification Theorem classifies every element of the mapping class group as finite order, reducible or pseudo-Anosov. Calvez has shown that for braid groups there is fast algorithm (quadratic time in the word length) that classifies an element as one of the three types. In joint work with Dan Margalit and Oyku Yurttas, we give a new quadratic time algorithm that also works for higher genus surfaces.

Winarski, Becca
Lifting homeomorphisms of branched covers of spheres
Birman and Hilden ask: given finite branched cover X over the 2-sphere, does every homeomorphism of the sphere lift to a homeomorphism of X? For covers of degree 2, the answer is yes, but the answer is sometimes yes and sometimes no for higher degree covers. In joint work with Ghaswala, we completely answer the question for cyclic branched covers.

Wolfson, Jesse
Coincidences of homological densities, predicted by arithmetic
Motivated by analogies with basic density theorems in analytic number theory, we introduce a notion of the homological density of one space in another. We use Weil's number field/function field analogy to predict coincidences for limiting homological densities of various sequences of spaces of 0-cycles on manifolds X, which we then establish. This is joint work with Benson Farb and Melanie Matchett Wood.