

# Open Problems for ICMS Workshop

Michael Elkin \*

**Open Problem 1:** Is it possible to embed a constant-degree expander with  $n$  vertices in  $\ell_\infty$  with distortion strictly smaller than 3 and dimension  $o(n)$ ?

**Open Problem 2:**

**Definition 0.1** *Given an undirected graph  $G = (V, E, \omega)$ , and a set  $S \subseteq V$  of vertices, a subgraph  $G' = (V, E', \omega)$  is called a source-wise distance preserver with respect to  $S$  if for every  $u, w \in V$ ,  $\text{dist}_{G'}(u, w) = \text{dist}_G(u, w)$ .*

It is known [Coppersmith, Elkin, SODA'05] that for every  $N$ -vertex undirected possibly weighted graph  $G = (V, E, \omega)$ , and a subset  $S \subseteq V$  of  $O(N^{1/4})$  vertices, there exists a source-wise distance preserver with  $O(N)$  edges. On the negative side, for any sufficiently large  $N$  and  $s$  that satisfy  $\omega(N^{1/4}) = s = o(N^{9/16})$  there exists an  $N$ -vertex graph  $G$  and a subset  $S$  of  $s$  vertices such that any source-wise preserver of  $G$  with respect to  $S$  contains  $\omega(N + s^2)$  edges. The range of parameters  $\Omega(N^{9/16}) = s = o(N)$  constitutes an open problem. In particular, it is not known whether for any graph  $G$  and a subset  $S$  of  $N^{2/3}$  vertices, there exists a source-wise preserver of  $G$  with respect to  $S$  with  $O(N^{4/3})$  edges. This problem is open even for unweighted graphs. If one allows the subgraph to approximate the distances within an additive error of 2, then an upper bound of  $O(N^{4/3})$  on the size the subgraph with these properties with respect to  $S$  of size  $N^{2/3}$  is known.

---

\*Department of Computer Science, Ben-Gurion University of the Negev, Beer-Sheva, Israel, [elkinm@cs.bgu.ac.il](mailto:elkinm@cs.bgu.ac.il)