Quantum propagation across cosmological singularities

Ben Craps

Vrije Universiteit Brussel & The International Solvay Institutes

work with T. Hertog and N. Turok

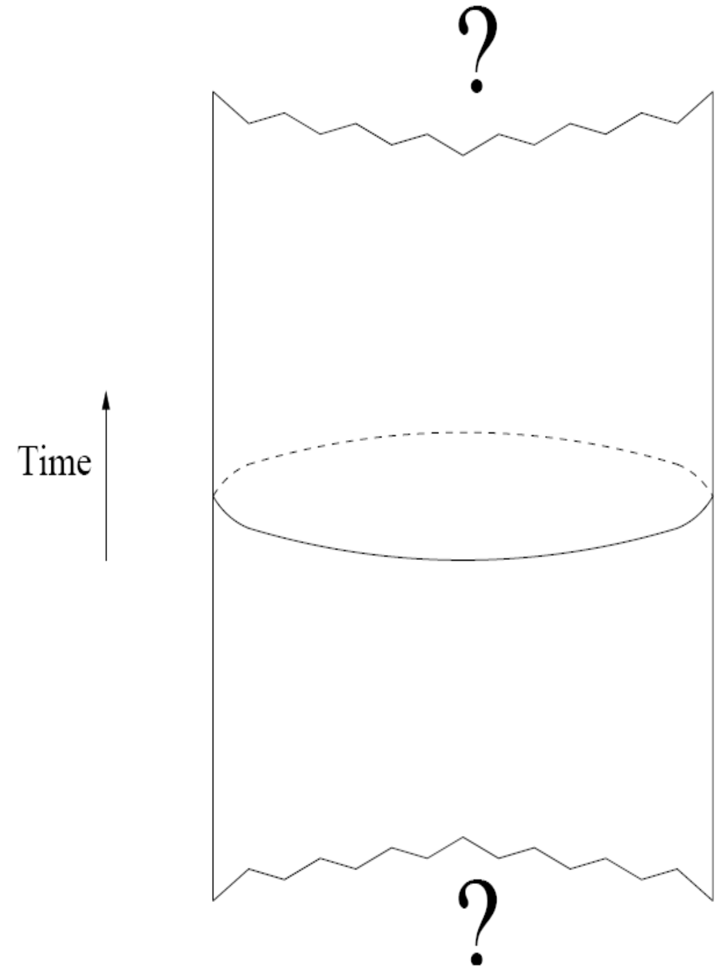
ICMS workshop on Gravitational Thermodynamics and the Quantum Nature of Space Time

Edinburgh, June 17, 2008

AdS cosmologies: basic idea

Starting point: supergravity solutions in which smooth, asymptotically AdS initial data evolve to a big crunch singularity.

Can a dual gauge theory be used to study the singularity in quantum gravity?



AdS cosmologies: basic idea

- AdS: boundary conditions required

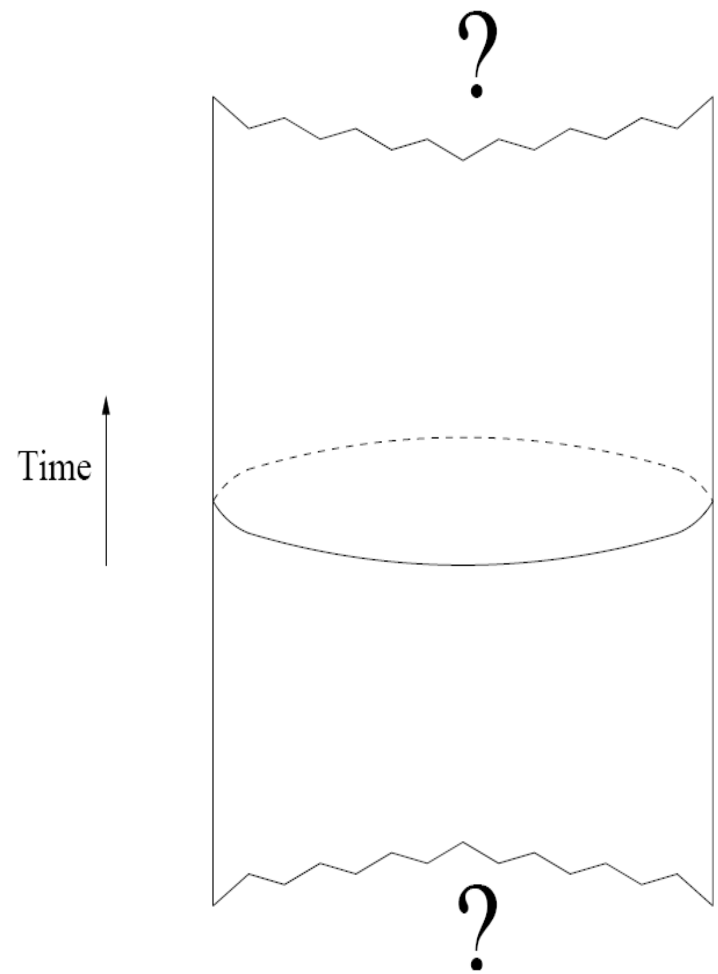
- Usual supersymmetric boundary conditions: stable

- Modified boundary conditions: smooth initial data that evolve into big crunch (which extends to the boundary of AdS in finite time)

- AdS/CFT relates quantum gravity in AdS to field theory on its conformal boundary

- Modified boundary conditions → potential unbounded from below in boundary field theory; scalar field reaches infinity in finite time

- Goal: learn something about cosmological singularities (in the bulk theory) by studying unbounded potentials (in the boundary theory)



Plan

- Bulk theory: strings on $AdS_5 \times S^5$ with modified boundary conditions
- Boundary theory: N=4 SYM with unstable double trace deformation
- Beyond the singularity: self-adjoint extensions
- Quantum evolution of the homogeneous component
- Particle creation: does the universe bounce?
- Summary and outlook

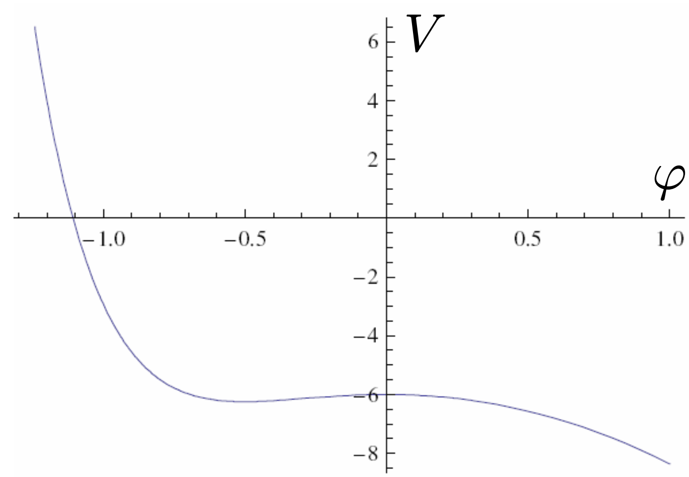
AdS cosmologies: the bulk theory

Compactify 10d type IIB sugra on S^5 and truncate (consistently) to

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{4R_{AdS}^2}\left(15e^{2\gamma\varphi} + 10e^{-4\gamma\varphi} - e^{-10\gamma\varphi}\right)\right]$$

Freedman, Gubser, Pilch, Warner

with $\gamma = \sqrt{2/15}$



This describes a scalar whose mass $m^2 = -4/R_{AdS}^2$ saturates the BF bound.

Breitenlohner, Freedman

In all solutions asymptotic to the AdS_5 metric

$$ds^2 = R_{AdS}^2\left(-(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_3^2\right)$$

the scalar field decays at large radius as $\varphi(r) \sim \frac{\alpha(t,\Omega)\log r}{r^2} + \frac{\beta(t,\Omega)}{r^2}$

Consider boundary conditions $\alpha = f\beta$

AdS cosmologies: bulk solution

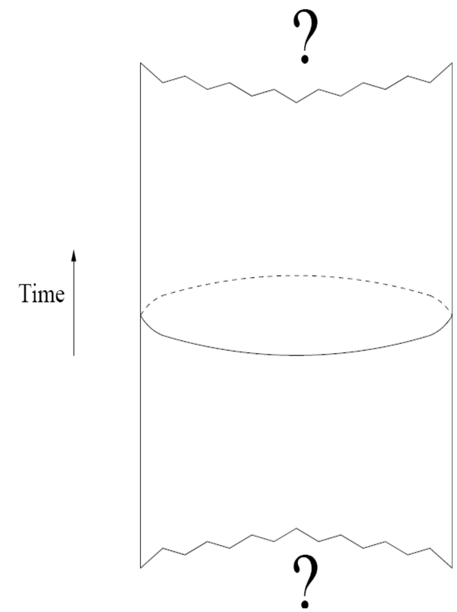
$$\varphi(r) \sim \frac{\alpha \log r}{r^2} + \frac{\beta}{r^2} \qquad\qquad \alpha = f\beta$$

Standard supersymmetric boundary conditions: $f = 0$. Preserves AdS symmetry group.
Pure AdS solution is stable ground state.

Gibbons, Hull, Warner; Townsend

For $f > 0$, there exist smooth asymptotically AdS initial data that evolve to a singularity that (plausibly) reaches the boundary of AdS in finite global time.

cf. Hertog, Horowitz



Plan

- Bulk theory: strings on $AdS_5 \times S^5$ with modified boundary conditions
- Boundary theory: N=4 SYM with unstable double trace deformation
- Beyond the singularity: self-adjoint extensions
- Quantum evolution of the homogeneous component
- Particle creation: does the universe bounce?
- Summary and outlook

The AdS/CFT correspondence

Type IIB string theory on $AdS_5 \times S^5$ with string coupling g_s and radius of curvature R	dual $\longleftrightarrow$	$\mathcal{N}$=4 SU(N) super-Yang-Mills theory with Yang-Mills coupling g_{YM} on the 3+1 dimensional boundary of AdS_5
g_s	$\longleftrightarrow$	$(g_{YM})^2$
$(R/l_s)^4$	$\longleftrightarrow$	$(g_{YM})^2\, N$

Maldacena

The AdS/CFT correspondence gives a non-perturbative definition of string theory in (asymptotically) anti-de Sitter space.

We shall work at large N (planar limit in field theory).

Our field theory computations will be trustworthy for small 't Hooft coupling, corresponding to a stringy bulk. Not clear yet to what extent the results carry over to large 't Hooft coupling. The unbounded potential does survive at large 't Hooft coupling and the bulk solution (and D3-brane picture) suggests that the effective 't Hooft coupling decreases near the singularity.

AdS cosmologies: dual field theory

$$\varphi(r) \sim \frac{\alpha \log r}{r^2} + \frac{\beta}{r^2} \qquad\qquad \alpha = f\beta$$

- For $f = 0$ (supersymmetric) boundary conditions: dual field theory is N=4 SYM on R x S^3.

- Scalar field φ couples to operator $\mathcal{O} = \frac{1}{N}\mathrm{Tr}\left[\Phi_1^2 - \frac{1}{5}\sum_{i=2}^{6}\Phi_i^2\right]$

- Boundary conditions with $f > 0$ correspond to deforming the CFT by a double trace operator:

$$S \to S + \frac{f}{2}\int \mathcal{O}^2$$

Aharony, Berkooz, Silverstein;
Witten; Berkooz, Sever, Shomer

This corresponds to a potential that is unbounded from below, and $\langle \mathcal{O} \rangle$ becomes infinite in finite time.

cf. Hertog, Horowitz

- Does this extend to the full quantum theory?

cf. Elitzur, Giveon, Porrati, Rabinovici

Quantum corrections do not turn potential around

$$V = -\frac{f}{2}\mathcal{O}^2$$

- The coupling f is asymptotically free → quantum corrections to potential under excellent perturbative control for large $\mathcal{O}$ (close to singularity)

Witten

- One-loop (Coleman-Weinberg) effective potential:

$$V = -\frac{f_{\mathcal{O}}}{2}\mathcal{O}^2 \qquad \text{with} \qquad f_{\mathcal{O}} = \frac{2}{\log(\mathcal{O}/M^2)}$$

Banados, Schwimmer, Theisen

- Coupling is replaced by arbitrary mass scale M: dimensional transmutation

- If $f_{\mathcal{O}}$ is small for certain value of $\mathcal{O}$, then it is even smaller for larger values of $\mathcal{O}$: quantum corrections do not turn the potential around!

- Caveat: we still have to assume small 't Hooft coupling to have complete control over the field theory, so strictly speaking the bulk is in a stringy regime.

- Beta-function for f is one-loop exact in large N limit, so running of f does extend to large 't Hooft coupling.

Witten

Strategy

- Have seen: AdS/CFT maps gravitational bulk theory with big crunch singularity to dual field theory with unbounded potential

- Thus questions about cosmological singularities can be studied in a (non-gravitational) field theory

- The cosmological singularity in the bulk reaches the boundary when $\mathcal{O}$ reaches infinity. Can time evolution be continued beyond that point?

- Shall attempt to define quantum evolution beyond the singularity and check for consistency.

So what happens when $\mathcal{O}$ reaches infinity?

Plan

- Bulk theory: strings on $AdS_5 \times S^5$ with modified boundary conditions
- Boundary theory: N=4 SYM with unstable double trace deformation
- Beyond the singularity: self-adjoint extensions
- Quantum evolution of the homogeneous component
- Particle creation: does the universe bounce?
- Summary and outlook

Quantum mechanics with unbounded potentials

Consider $\hat{H} = -\frac{d^2}{dx^2} + V(x)$ with $V(x) = -a^2 x^p$ for $x > 0$ and $p > 2$. For such potentials, classical trajectories can reach infinity in finite time. So do quantum mechanical wavepackets, which would seem to lead to loss of probability/unitarity.

Unitarity can be restored by restricting the domain of allowed wavefunctions such that the Hamiltonian is self-adjoint ("self-adjoint extension"):

$$(\hat{H}\phi_1, \phi_2) = (\phi_1, \hat{H}\phi_2) \qquad \leftrightarrow \qquad \left[\frac{d\phi_1^*}{dx}\phi_2 - \phi_1^*\frac{d\phi_2}{dx}\right]_{x=\infty} = 0$$

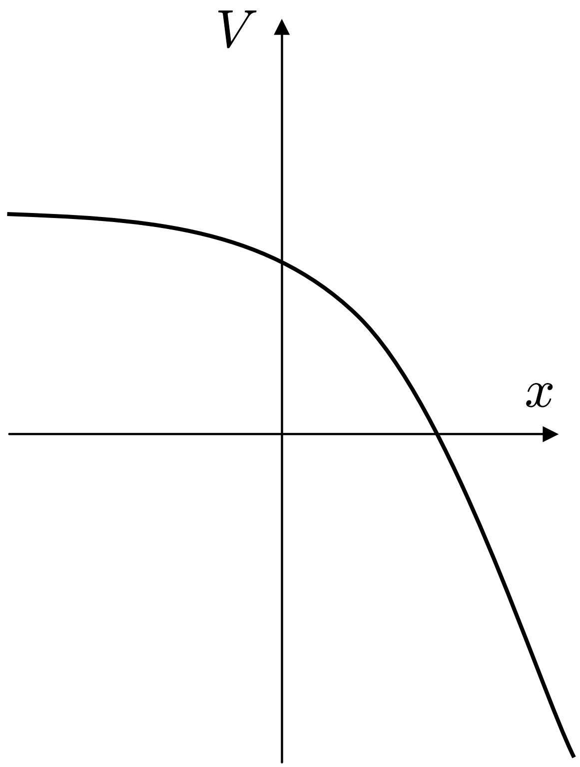
The WKB energy eigenfunctions $[2(E + a^2 x^p)]^{-1/4} \exp\left(\pm i \int_0^x \sqrt{2(E + a^2 y^p)} dy\right)$

are an increasingly good approximation for large x. Unitarity can be achieved by only allowing the linear combination that for large x behaves as

$$\psi_E^\alpha(x) \sim (2a^2 x^p)^{-1/4} \cos\left(\frac{\sqrt{2} a x^{p/2+1}}{p/2+1} + \underset{\substack{\uparrow \\ \text{arbitrary phase}}}{\alpha}\right)$$

Reed, Simon

Interpretation of the self-adjoint extensions



Rightmoving wavepacket disappearing at infinity is always accompanied by leftmoving wavepacket appearing at infinity (think of brick wall at infinity)

Carreau, Farhi, Gutmann, Mende

Energy spectrum consists of bound states (energy levels depend on phase α and are unbounded from below) as well as scattering states (if potential is bounded from above)

Self-adjoint extensions in quantum field theory

$$V(\mathcal{O}) = -\frac{\mathcal{O}^2}{\ln(\mathcal{O}^2/\tilde{M}^2)} \qquad \mathcal{O} = \frac{1}{N}\mathrm{Tr}[\Phi_1^2 - \frac{1}{5}\sum_{i=2}^{6}\Phi_i^2]$$

Focus on steepest unstable direction:

$$\Phi_1(x) = \phi(x)U$$

canonically normalized scalar field

constant Hermitean matrix,

$$\mathrm{Tr}U^2 = 1$$

$$V(\phi) = -\frac{\lambda_\phi}{4}\phi^4 \qquad \text{with} \qquad \lambda_\phi = \frac{1}{N^2 \ln\left(\frac{\phi}{NM}\right)}$$

For now, ignore running of coupling.

Self-adjoint extensions in quantum field theory

$$V = -\frac{\lambda}{4}\,\phi^4$$

Equation of motion: $$\partial^2\phi = -\lambda\phi^3 + \cancel{\frac{1}{6}R(S^3)\phi}$$

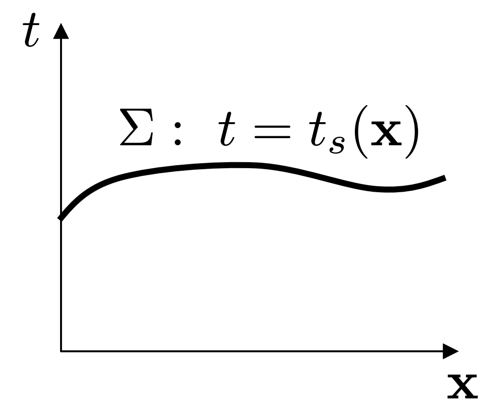
Ricci scalar; ignore for large ϕ

Homogeneous background solution: $\phi = \sqrt{(2/\lambda)}\,t^{-1}$. Define $\chi = (2/\lambda)^{1/2}\phi^{-1}$.

Can construct generic, spatially inhomogeneous solution to e.o.m. in expansion around space-like singular surface $\Sigma:\ t = t_s(\mathbf{x})$ where ϕ is infinite:

$$\chi(t,\mathbf{x}) = -t + \underbrace{t_s(\mathbf{x})}_{\text{time delay}} + \frac{1}{6}t^2\nabla^2 t_s - \frac{1}{24}t^4(\nabla^4 t_s) + \ldots$$
$$-\frac{\lambda\rho(\mathbf{x})}{10}\,t^5 + \cdots + \quad \text{(non-linear in } t_s,\rho\text{)}$$

energy perturbation



Main observation: spatial gradients become unimportant near the singularity

→ evolution becomes ultralocal

Thus different spatial points decouple, and we can define a self-adjoint extension point by point!

How unique is the self-adjoint extension?

A priori ambiguity: one phase for every point of S^3

- It is natural to choose the theory (e.g. Lagrangian) to be symmetric
- It is unnatural to choose a state that is very symmetric

Choice of self-adjoint extension is part of definition of theory (not a choice of state)

→ Natural to choose same phase at every point

Plan

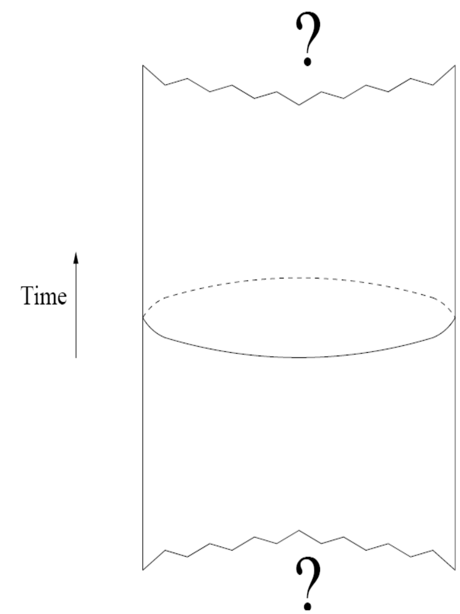
- Bulk theory: strings on AdS_5 x S^5 with modified boundary conditions
- Boundary theory: N=4 SYM with unstable double trace deformation
- Beyond the singularity: self-adjoint extensions
- Quantum evolution of the homogeneous component
- Particle creation: does the universe bounce?
- Summary and outlook

The homogeneous mode is a quantum mechanical variable

Boundary field theory lives on $R \times S^3$

time

finite volume space



Decompose $$\phi(t, \mathbf{x}) = \bar{\phi}(t) + \delta\phi(t, \mathbf{x})$$

First ignore inhomogeneous modes $\delta\phi(t, \mathbf{x})$,
which start out in ground state.

Kinetic term for homogeneous mode: $$V_3 \int dt \, \frac{1}{2} \dot{\bar{\phi}}^2$$

finite “mass”

Wave function will undergo quantum spreading. This will give rise to UV cutoff on creation of inhomogeneous modes.

Complex classical solutions and quantum mechanics

- Semiclassical expansion for QM particle: $\Psi(x_f, t_f) \sim A(x_f, t_f) e^{iS(x_f, t_f)/\hbar}$
- Solved to leading order in $\hbar$ by $S = S_{Cl}(x_f, t_f)$: classical action with

1) initial condition at $t = t_i$: $$x + 2i\frac{pL^2}{\hbar} = x_c + 2i\frac{p_c L^2}{\hbar}$$

i.e. Gaussian wavepacket centered around (x_c, p_c) with spread L in x

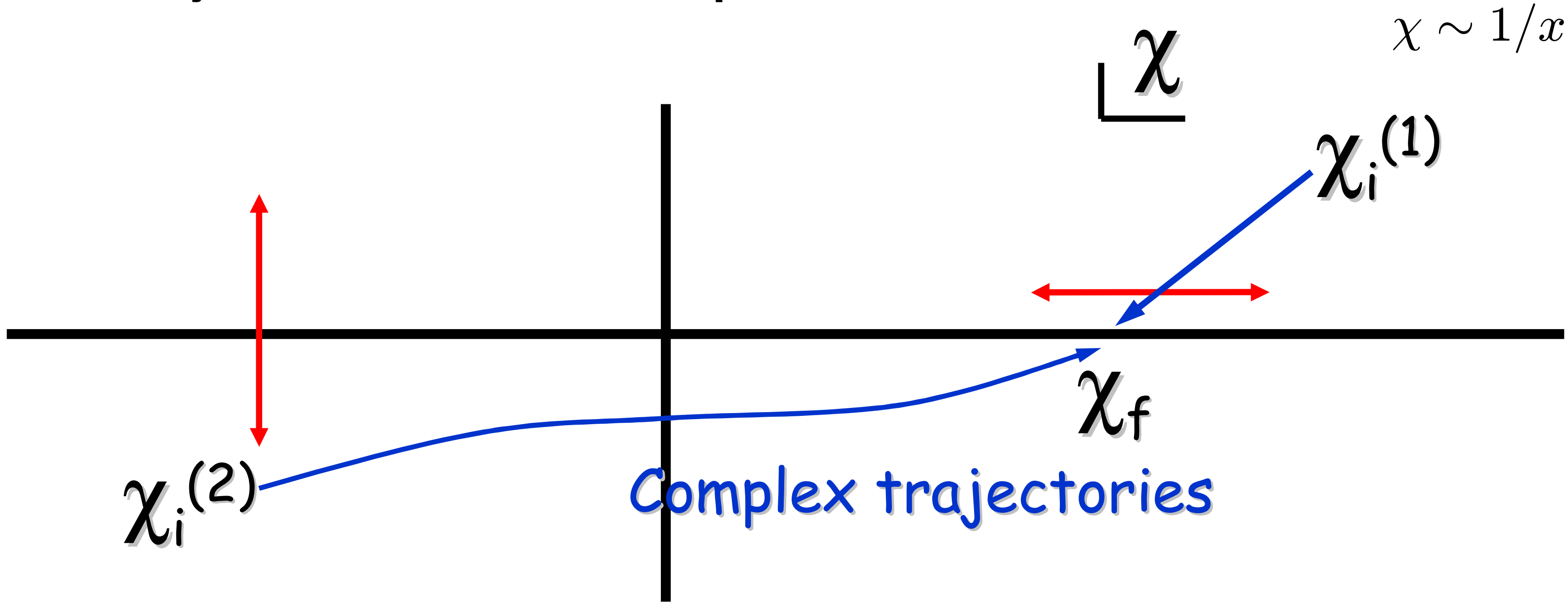
2) final condition at $t = t_f$: $x = x_f$

- Classical solution with these boundary conditions is complex for nonzero spread L (unless x_f lies on classical trajectory)
- Apply to self-adjoint extension: add "mirror" wavefunction corresponding to

initial condition at $t = t_i$: $$x + 2i\frac{pL^2}{\hbar} = -\left(x_c + 2i\frac{p_c L^2}{\hbar}\right)$$

The self-adjoint extension via complex classical solutions

$\chi \sim 1/x$



$$\Psi(x_f, t_f) = A_1(x_f, t_f) e^{iS_1/\hbar} + A_2(x_f, t_f) e^{i\theta} e^{iS_2/\hbar}$$

$\chi_i^{(1)} \to \chi_f$ $\qquad$ $\chi_i^{(2)} \to \chi_f$

Can check: belongs to domain of self-adjoint extension

Late times (after bounce): mirror wavepacket will dominate

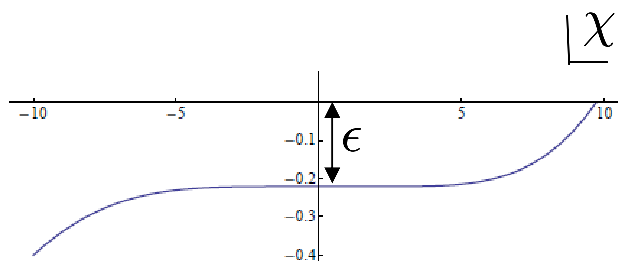
The spread of the wavepacket

Initial Gaussian wave packet with nearly zero energy, centered on ϕ_c

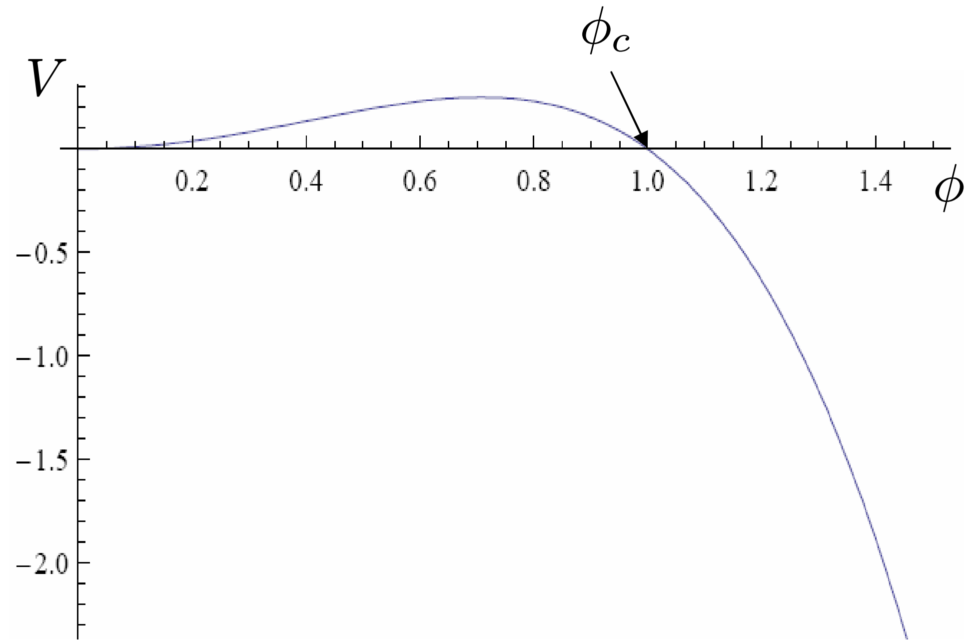
Time to roll to infinity and back is of order R_{AdS}

Introduce $\chi = \left(\frac{2}{\lambda_\phi}\right)^{1/2} \frac{1}{\phi}$

Classical solution: $\chi = |t|$



$\chi_i \approx -R_{AdS}$



Wave packet that minimizes spread over duration of bounce:

$$(\Delta\chi_i)_{min} \approx \lambda_\phi^{1/2} R_{AdS} \ll R_{AdS}$$

For "most" χ_f, away from classical trajectory,

$$\epsilon \sim \lambda_\phi^{1/2} R_{AdS}$$

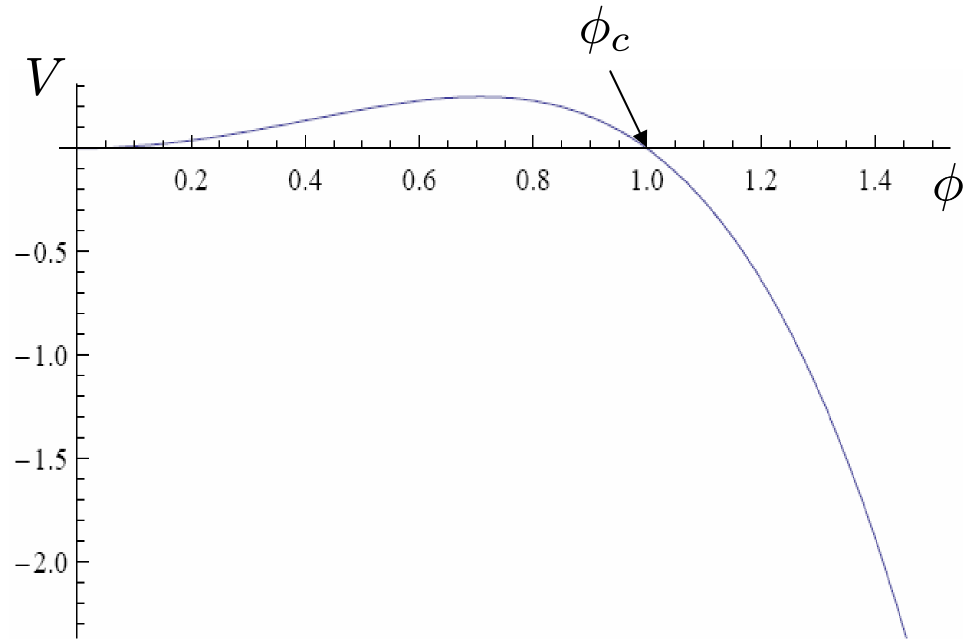
ϵ is smaller for broader/narrower initial wavepackets [corrects earlier version]

Plan

- Bulk theory: strings on $AdS_5 \times S^5$ with modified boundary conditions
- Boundary theory: N=4 SYM with unstable double trace deformation
- Beyond the singularity: self-adjoint extensions
- Quantum evolution of the homogeneous component
- Particle creation: does the universe bounce?
- Summary and outlook

Does the universe bounce?

Consider the homogeneous mode $\bar{\phi}(t)$.



Self-adjoint extension would seem to imply that after rolling to infinity, it rolls up the hill again, returning to the original configuration
→ "bounce in spacetime".

However, inhomogeneous modes $\delta\phi(t, \mathbf{x})$ may be created and may drain energy out of the homogeneous mode.

Need to compute the energy in created particles and see how far the homogeneous mode can roll up the hill again.

Note: this is particle creation in the boundary field theory, not in spacetime!

Including the inhomogeneous modes

Have computed the wavefunction $\Psi\left(t_f, \bar{\phi}\right)$, ignoring $\delta\phi$, using complex classical solutions.

Want to compute the full wavefunctional $\Psi\left(t_f, \bar{\phi}, \delta\phi(\mathbf{x})\right)$ at some late time of interest. Fluctuations start out in ground state.

Semiclassical expansion: need complex classical solutions for all modes.

Approximations:

- Work to quadratic order in $\delta\phi$ in the action → linearized field equations around complex homogeneous background.

- Ignore backreaction on homogeneous mode.

Dependence of wavefunctional on Fourier mode $\delta\phi_{\mathbf{k}}$ can be interpreted in terms of number of created particles.

In practice: solve for incoming negative frequency mode $\delta\phi_{\mathbf{k}}$, compute late time mixing with positive frequency mode, read off number of created particles $\langle n \rangle$ from Bogolubov coefficient β.

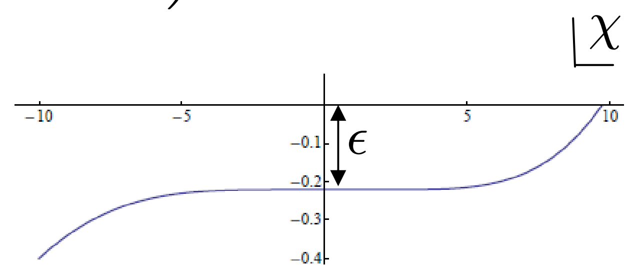
Include other particle species in analysis.

Particle creation using complex classical solutions

$$V(\phi) = -\frac{\lambda_\phi}{4}\phi^4 \qquad \text{with} \qquad \lambda_\phi = \frac{1}{N^2 \ln\left(\frac{\phi}{NM}\right)}; \qquad \text{denote} \quad l \equiv \ln(\phi/NM)$$

Zero energy real scaling solution: $\phi \sim \dfrac{1}{\sqrt{\lambda_\phi}\,|t|}\left(1 + \dfrac{1}{2l} + \dots\right)$

Complex solution (near t=0): replace $t \to t - i\epsilon$



E.o.m. for inhomogeneous perturbations:

$$\delta\ddot{\phi}_{\mathbf{k}} = \left(-k^2 + \frac{6}{(t-i\epsilon)^2}\left(1 + \frac{5}{12}l^{-1} - \frac{2}{3}l^{-2}\dots\right)\right)\delta\phi_{\mathbf{k}}$$

wave number

Aim: solve for incoming negative frequency mode, determine Bogolubov coefficients and $\langle n \rangle$

Particle creation using complex classical solutions

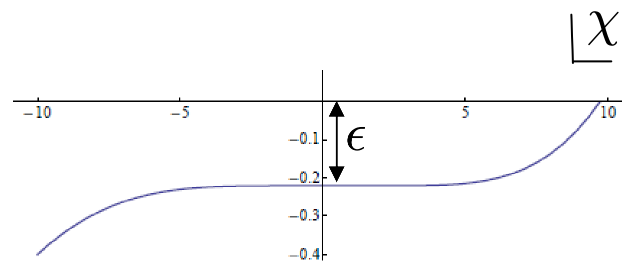
$$l \equiv \ln(\phi/NM)$$

Wave equation for linearized fluctuation with wavenumber k, to leading order in $1/l$:

$$\delta\ddot{\phi}_{\mathbf{k}} = \frac{6}{\tilde{t}^2}\delta\phi_{\mathbf{k}} - k^2\delta\phi_{\mathbf{k}} \qquad (\tilde{t} \approx t - i\epsilon)$$

Incoming negative frequency solution (unambiguously continued through t=0):

$$\delta\phi_{\mathbf{k}} = \frac{e^{ikt}}{(2k)^{1/2}}\left(1 + \frac{3i}{k\tilde{t}} - \frac{3}{(k\tilde{t})^2}\right)$$



Mode oscillates for $|k\tilde{t}| \gg 1$ and is frozen for $|k\tilde{t}| \ll 1$

To leading order in $1/l$, incoming and outgoing positive frequency modes coincide!

To next order in $1/l$, particles are created:

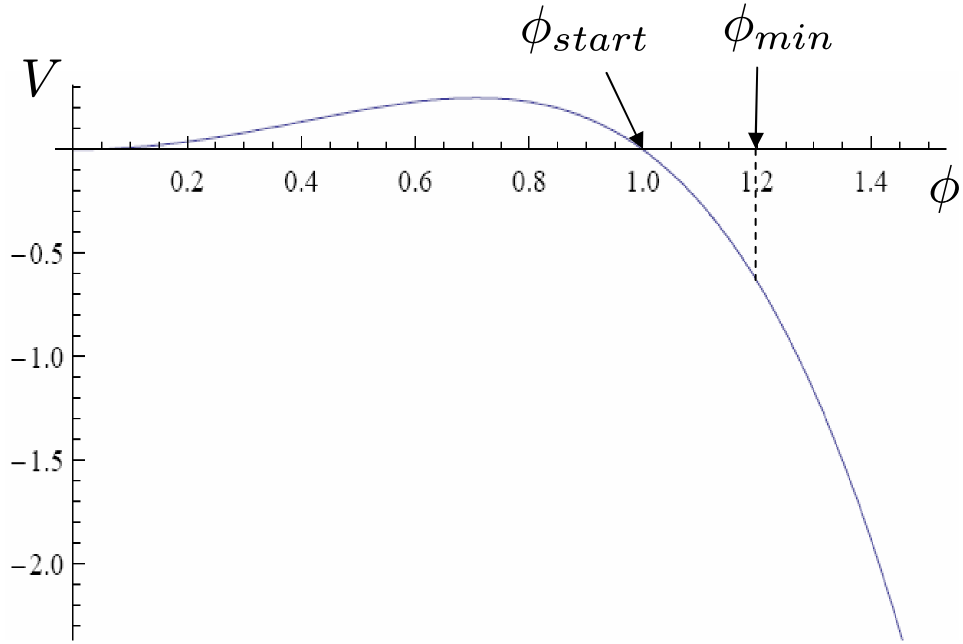
$$\beta_k \approx -\frac{i\pi e^{-2k\epsilon}}{\ln(k/M)} \qquad (k \gg M) \qquad \rightarrow \qquad \langle n \rangle = \frac{\pi^2}{\ln(k/M)^2}e^{-4k\epsilon}$$

Modes with $k\epsilon \gg 1$ never freeze → UV cutoff on particle creation!

Interpretation: quantum spread and “brick wall” keep homogeneous mode away from infinity.

Backreaction of created particles

$$V = \frac{\phi^2}{6R_{AdS}^2} - \frac{\lambda_0 \phi^4}{4\ln(\phi/NM)}$$



Field starts at ϕ_{start}, rolls to infinity and back to ϕ_{min}

"Small backreaction" if

$$\phi_{min} - \phi_{start} \ll \phi_{start}$$

Result:

Satisfied for "most" $\bar{\phi}$ only if $|\ln(MR_{AdS})| \gg N^3$

However, then the unstable double trace deformation would be small compared to terms neglected in planar limit

→ Not possible in controlled regime in this model

→ Despite UV cutoff, too many particles are produced to allow bounce (in controlled regime)

Plan

- Bulk theory: strings on $AdS_5 \times S^5$ with modified boundary conditions
- Boundary theory: N=4 SYM with unstable double trace deformation
- Beyond the singularity: self-adjoint extensions
- Quantum evolution of the homogeneous component
- Particle creation: does the universe bounce?
- Summary and outlook

Summary

- String theory in AdS_5 x S^5 with modified b.c.; smooth initial data evolve into big crunch.
- In dual N=4 SYM: unbounded potential, operator reaches infinity in finite time.
- Potential under excellent control near singularity. Quantum effective potential is really unbounded from below.
- Need small 't Hooft coupling to have field theory completely under control → stringy bulk. At least certain features expected to extend to large 't Hooft coupling.
- QM with unbounded potentials: self-adjoint extensions define unitary quantum evolution.
- QFT: ultralocality → define self-adjoint extension point by point.
- QM spread of the homogeneous mode (due to finite volume of 3-sphere) provides UV cutoff on particle creation for "most" of the wavepacket for the homogeneous mode. Still too much particle creation for bounce in controlled regime.

Outlook

- D-brane interpretation of big crunch singularity
- Numerical tests on UV cutoff on particle creation
- More systematic study of self-adjoint extensions in quantum field theory
- Effect of non-linearities? Comparison with tachyonic preheating
- Related models with zero beta function → less particle creation → bounce possible?