Generating functions for scattering amplitudes in N=4 SYM and N=8 supergravity

Henriette Elvang
MIT

Based on:

- arXiv:0805.0757 with M. Bianchi and D. Z. Freedman.
- arXiv:0710.1270 with D. Z. Freedman

Gravitational Thermodynamics and the Quantum Nature of Space Time
Edinburgh, June 18, 2008

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Perturbative

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Gravity

Gravity difficult to quantize as field theory:
Non-renormalizable

Supersymmetry improves divergent behavior

↓

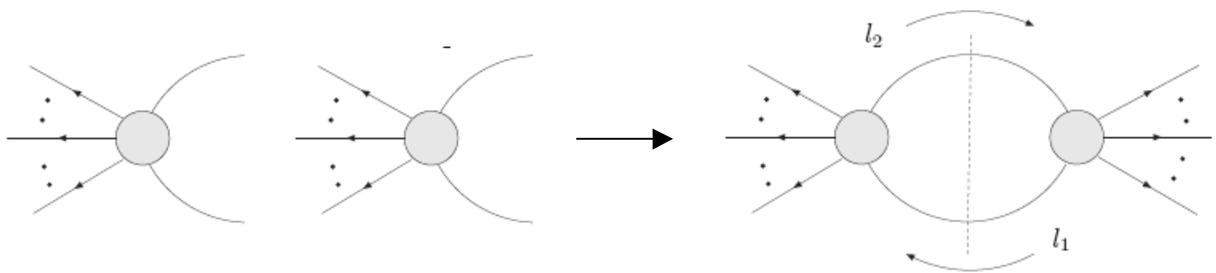
Recent proposal: [Bern, Dixon, Roiban (2007a)]

Is N=8 maximal supergravity in 3+1d perturbatively finite?

- N=8 supergravity known to be 1- and 2-loop finite.
- 4-pt graviton amplitude found to be finite at 3-loops [Bern, Dixon, Roiban (2007b)]

Tree amplitudes

Tree amplitudes → loop amplitudes → finiteness?

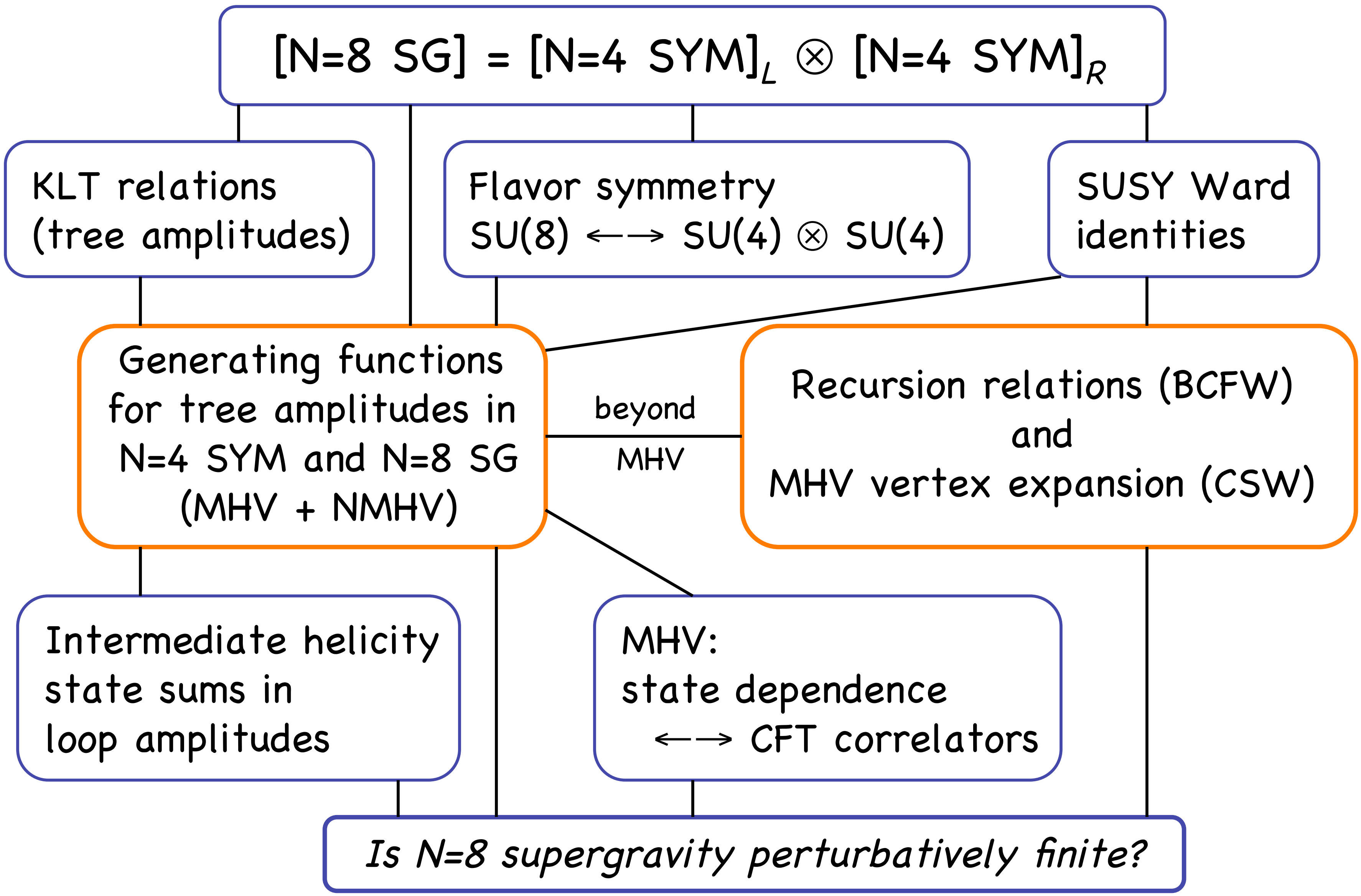


Generalized unitarity

[Bern, Dixon, Kosower,...]

Our work:
focuses on **tree amplitudes** of *N=8 supergravity* and their relationship with tree amplitudes in *N=4 super Yang-Mills theory*.

The bigger picture



N=4 Super Yang-Mills Theory

2^4 = 16 massless states

	gluon	gluino	self-dual scalar	gluino	gluon
	$B_+(p)\,,$	$F^a_+(p)\,,$	$B^{ab}(p) = \frac{1}{2}\,\alpha_4\,\epsilon^{abcd}B_{cd}(p)\,,$	$F^-_a(p)\,,$	$B^-(p)$
Helicity:	+1	+1/2	0	-1/2	-1
#states:	1	4	6	4	1

a,*b*,*c*,*d*=1,2,3,4 are SU(4) flavor symmetry indices

N=8 supergravity

2^8 = 256 massless states

1	8	28	56
graviton	gravitino	gravi-photon	gravi-photino
$b_+(p)\,,$	$f_+^A(p)\,,$	$b_+^{AB}(p)\,,$	$f_+^{ABC}(p)\,,$

$$b^{ABCD}(p) \;=\; \frac{1}{4!}\,\alpha_8\,\epsilon^{ABCDEFGH}\,b_{EFGH}(p)\,,$$ ← 70 self-dual scalars

$f^-_{ABC}(p)\,,$	$b^-_{AB}(p)\,,$	$f^-_A(p)\,,$	$b^-(p)\,.$
gravi-photino	gravi-photon	gravitino	graviton
56	28	8	1

A,B,C,... = 1,...,8 are SU(8) flavor symmetry indices.

The Map

$$\begin{pmatrix} \text{256 states} \\ \text{of} \\ \text{N=8 supergravity} \end{pmatrix} = \begin{pmatrix} \text{16 states} \\ \text{of} \\ \text{N=4 super Yang-Mills} \end{pmatrix} \otimes \begin{pmatrix} \text{16 states} \\ \text{of} \\ \text{N=4 super Yang-Mills} \end{pmatrix}$$

graviton = gluon $\otimes$ *gluon′*

$$b_+ = B_+ \tilde{B}_+ \qquad b^- = B^- \tilde{B}^-$$

Note: Split $SU(8) \to SU(4)_L \otimes SU(4)_R$ as $a,b,..=1,2,3,4$ and $r,s,..=5,6,7,8$.

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graviton = gluon ⊗ gluon'

$$b_+ = B_+ \tilde{B}_+ \qquad b^- = B^- \tilde{B}^-$$

gravitino = gluon ⊗ gluino'

$$f_a^- = F_a^- \tilde{B}^-$$
$$f_r^- = B^- \tilde{F}_r^-$$

Note: Split $SU(8) \rightarrow SU(4)_L \otimes SU(4)_R$ as *a,b,..*=1,2,3,4 and *r,s,..*=5,6,7,8.

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graviton = *gluon* ⊗ *gluon′*

$$b_+ = B_+ \tilde{B}_+ \qquad b^- = B^- \tilde{B}^-$$

gravitino = *gluon* ⊗ *gluino′*

$$f_a^- = F_a^- \tilde{B}^-$$
$$f_r^- = B^- \tilde{F}_r^-$$

spin-1 *gravi-photon* = { spin-1 *gluon* ⊗ spin-0 *scalar*; spin-1/2 *gluino* ⊗ spin-1/2 *gluino* }

$$b_{ab}^- = B_{ab} \tilde{B}^-$$
$$b_{ar}^- = -F_a^- \tilde{F}_r^-$$
$$b_{rs}^- = B^- \tilde{B}_{rs}$$

etc

Note: Split SU(8) → $SU(4)_L \otimes SU(4)_R$ as *a,b,..*=1,2,3,4 and *r,s,..*=5,6,7,8.

The Map

b_+	$=$	$B_+\,\tilde{B}_+$	b^-	$=$	$B^-\,\tilde{B}^-$
f_+^a	$=$	$F_+^a\,\tilde{B}_+$	f_a^-	$=$	$F_a^-\,\tilde{B}^-$
f_+^r	$=$	$B_+\,\tilde{F}_+^r$	f_r^-	$=$	$B^-\,\tilde{F}_r^-$
b_+^{ab}	$=$	$B_+^{ab}\,\tilde{B}_+$	b_{ab}^-	$=$	$B_{ab}^-\,\tilde{B}^-$
b_+^{ar}	$=$	$F_+^a\,\tilde{F}_+^r$	b_{ar}^-	$=$	$-F_a^-\,\tilde{F}_r^-$
b_+^{rs}	$=$	$B_+\,\tilde{B}_+^{rs}$	b_{rs}^-	$=$	$B^-\,\tilde{B}_{rs}^-$
f_+^{abc}	$=$	$\alpha_4\,\epsilon^{abcd}\,F_d^-\,\tilde{B}_+$	f_{abc}^-	$=$	$-\alpha_4\,\epsilon_{abcd}\,F_+^d\,\tilde{B}^-$
f_+^{abr}	$=$	$B_+^{ab}\,\tilde{F}_+^r$	f_{abr}^-	$=$	$B_{ab}^-\,\tilde{F}_r^-$
f_+^{ars}	$=$	$F_+^a\,\tilde{B}_+^{rs}$	f_{ars}^-	$=$	$F_a^-\,\tilde{B}_{rs}^-$
f_+^{rst}	$=$	$\tilde{\alpha}_4\,\epsilon^{rstu}\,B_+\,\tilde{F}_u^-$	f_{rst}^-	$=$	$-\tilde{\alpha}_4\,\epsilon_{rstu}\,B^-\,\tilde{F}_+^u$
b^{abcd}	$=$	$\alpha_4\,\epsilon^{abcd}\,B^-\,\tilde{B}_+$	b_{abcd}	$=$	$\alpha_4\,\epsilon_{abcd}\,B_+\,\tilde{B}^-$
b^{abcr}	$=$	$\alpha_4\,\epsilon^{abcd}\,F_d^-\,\tilde{F}_+^r$	b_{abcr}	$=$	$\alpha_4\,\epsilon_{abcd}\,F_+^d\,\tilde{F}_r^-$
b^{abrs}	$=$	$B_+^{ab}\,\tilde{B}_+^{rs}$	b_{abrs}	$=$	$B_{ab}^-\,\tilde{B}_{rs}^-$
b^{arst}	$=$	$\tilde{\alpha}_4\,\epsilon^{rstu}\,F_+^a\,\tilde{F}_u^-$	b_{arst}	$=$	$\tilde{\alpha}_4\,\epsilon_{rstu}\,F_a^-\,\tilde{F}_+^u$
b^{rstu}	$=$	$\tilde{\alpha}_4\,\epsilon^{rstu}\,B_+\,\tilde{B}^-$	b_{rstu}	$=$	$\tilde{\alpha}_4\,\epsilon_{rstu}\,B^-\,\tilde{B}_+$

KLT relations

Kawai, Lewellen, Tye (1986)

Tree level scattering amplitudes:

$s_{12} = -(p_1 + p_2)^2$

$$M_4(h_1\ h_2\ h_3\ h_4) = s_{12}\, A_4(g_1\ g_2\ g_3\ g_4)\, A_4(g'_1\ g'_2\ g'_4\ g'_3)$$

where $h = g \otimes g'$, *e.g.* a graviton from two gluons.

(n-pt relations more complicated)

Natural in **string theory**:
Closed string = (open string) $\otimes$ (open string)

But VERY surprising in **field theory**:

- Yang-Mills theory $\quad \partial A^3, \quad A^4$
- Gravity $\quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \longrightarrow \partial^2 h^n \quad$ for any n

Field theory derivation of KLT?

Steps in this direction: Bern & Grant (1998); Freedman & HE (2007); Bern et al (2008).

Appetizer: susy Ward identities and the map

$$\underset{\text{spin-1}}{\text{gravi-photon}} = \begin{cases} \overset{\text{spin-1}}{gluon} \otimes \overset{\text{spin-0}}{scalar} \\ \underset{\text{spin-1/2}}{gluino} \otimes \underset{\text{spin-1/2}}{gluino} \end{cases}$$

$$M_4 = s_{12}\, A_4\, A_4' \quad \rightarrow \quad \begin{aligned} &A_4(\text{all bosons})\, A_4(\text{all bosons})' \\ &= A_4(\text{fermions})\, A_4(\text{fermions})' \end{aligned}$$

How does it work?

⟶ Generating functions

Generating functions

for

MHV

and

NMHV

amplitudes

MHV amplitudes

Gluon (or graviton) amplitude:

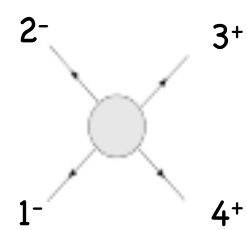
No helicity violation!

$$\langle ++++\ldots+\rangle = 0$$
$$\langle -+++\ldots+\rangle = 0$$

Maximally Helicity Violating amplitudes: $\langle --++\ldots+\rangle$

e.g.

$$\langle B^-(1)\, B^-(2)\, B_+(3)\, B_+(4)\rangle \quad =$$



MHV *n*-gluon amplitude: Parke-Taylor formula [Parke, Taylor (1986)]

$$A_n(1^+,\ldots,a^-,\ldots,b^-,\ldots,n^+) = \frac{\langle ab\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\ldots\langle n1\rangle}$$

Spinor helicity formalism

Up to complex phase:

$$\langle 12 \rangle \ \sim \ [12] \ \sim \ \sqrt{s_{12}}$$

$$\langle ji \rangle = -\langle ij \rangle \,, \qquad [ji] = -[ij]$$

SUSY Ward identities

SUSY Ward identities relate n-pt amplitudes with different external states

- "MHV sector" = amplitudes proportional to $\langle --++\ldots+\rangle$ via SUSY Ward identities. In N=1 SYM:

$$\langle B^-(1)\, B^-(2)\, B_+(3)\cdots B_+(n)\rangle$$

$$\langle B^-(1)\, F^-(2)\, F_+(3)\, B_+(4)\cdots B_+(n)\rangle \;=\; \frac{\langle 13\rangle}{\langle 12\rangle}\langle B^-(1)\, B^-(2)\, B_+(3)\cdots B_+(n)\rangle$$

$$\langle F^-(1)\, F^-(2)\, F_+(3)\, F_+(4)\, B_+(5)\cdots B_+(n)\rangle \;=\; -\frac{\langle 34\rangle}{\langle 12\rangle}\langle B^-(1)\, B^-(2)\, B_+(3)\cdots B_+(n)\rangle$$

- MHV $\langle --++\ldots+\rangle$ simple!
- Next-to-MHV $\langle ---+\ldots+\rangle$ not-so-simple

etc

SUSY Ward identities

Extended SUSY: similar!

Generators Q_a, $a = 1,\ldots,N$ for N-extended SUSY.

How many different amplitudes?

Theory:	*MHV sector:*	*Next-to-MHV sector:*
N=4 SYM	15 amplitudes	34 amplitudes
N=8 S.G.	186 amplitudes	919 amplitudes

SUSY Ward identities

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Counting ⇔ # partitions of integers!!

From generating functions…

Generating functions Z --- idea

Each particle state X $\longleftrightarrow$ differential operator D_X

$\downarrow$ $\downarrow$

Amplitude $A_n(X_1\, X_2 \ldots X_n) = D_{X1}\, D_{X2} \ldots D_{Xn}\, Z$

Generating functions

Original work by Nair (1988),
Extended by Georgiou, Glover, Khoze (2004),
Bianchi, Freedman, HE (2008)

Properties:

1) Bookkeeping for state dependence.
2) Practical tool for calculating MHV and next-to-MHV amplitudes.
3) Clean connection between N=8 amplitudes and N=4 amplitudes.

Generating function for N=4 SYM MHV amplitudes

Define gen func:

Gluon MHV amplitude

Grassman

$$Z_n = \underbrace{\frac{A_n(1^-, 2^-, 3^+, \ldots, n^+)}{\langle 12 \rangle^4}}_{\text{cyclic symmetric}} 2^{-4} \prod_{a=1}^{4} \sum_{i,j=1}^{n} \langle ij \rangle \, \eta_{i\,a} \, \eta_{j\,a}$$

cyclic symmetric

SU(4) flavor index

momentum labels

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Associate with each state a differential operator:

$$B_+(i) \leftrightarrow 1, \quad F_+^a(i) \leftrightarrow D_i^a = \frac{\partial}{\partial \eta_{ia}}, \quad B^{ab}(i) \leftrightarrow D_i^{ab} = \frac{\partial^2}{\partial \eta_{ia} \partial \eta_{ib}},$$

$$\ldots \qquad B^-(i) \leftrightarrow D_i = \frac{1}{24} \epsilon_{abcd} \frac{\partial^4}{\partial \eta_{ia} \partial \eta_{ib} \partial \eta_{ic} \partial \eta_{id}}$$

To calculate amplitude, apply operators to gen func.

Generating function for N=4 SYM MHV amplitudes

$$Z_n = \frac{A_n(1^-, 2^-, 3^+, \ldots, n^+)}{\langle 12\rangle^4} \, 2^{-4} \prod_{a=1}^{4} \sum_{i,j=1}^{n} \langle ij\rangle \, \eta_{i\,a} \, \eta_{j\,a}$$

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Example:

$$\langle B^-(1)\, B^-(2)\, B_+(3)\, B_+(4)\rangle \;=\; \underbrace{D_1\, D_2}\, Z_n \;=\; A_4(1^-, 2^-, 3^+, 4^+)$$

→ produces $\langle 12\rangle^4$

Pure gluon amplitude correctly reproduced!

What about all the other MHV amplitudes?

Generating function for N=4 SYM MHV amplitudes

Introduce SUSY generators: $\tilde{Q}_a = \sum_{i=1}^{n} |i\rangle \, \eta_{ia}$

Fact 1: $[\tilde{Q}_a$, diff ope] = <εp> diff ope

in precise agreement with N=4 SUSY algebra.

Fact 2: $\tilde{Q}_a Z_n = 0$

Fact 3: $[\, \tilde{Q}_a, D^{(9)}\,]\, Z_n = 0$ ← SUSY Ward identity!

9th order diff ops ⇔ string of annihilation ops

$$0 = \langle [\tilde{Q}, B^-(1) B^-(2) F_+(3)\, B_+(4)] \rangle$$

4th 4th 1st 0th order derivatives

Generating function for N=4 SYM MHV amplitudes

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9th order diff ops ⇔ string of annihilation ops

MHV Ward identities have *unique* solutions, hence the gen func correctly reproduces *all* MHV amplitudes.

Application I: Counting MHV amplitudes

- The generating function is 8th order in η's.
- Any 8th order derivative operator $D^{(8)}$ produces an MHV amplitude.
- States $\Leftrightarrow$ operators order 0,1,...,4.

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- States ⇔ operators order 0,1,…,4.

Conclusion:

\# MHV amplitudes in N=4 SYM

=

\# integer partitions of 8 with n_{max}=4.

8=4+4	← --++..+ pure gluon amplitude	Total number is 15
8=4+3+1	← two gluinos	
…		
8=1+….+1	← 8 gluinos	

Generating function for N=8 S.G. MHV amplitudes

Define gen func:

Graviton MHV amplitude

Grassman

$$\Omega_n = \underbrace{\frac{M_n(1^-, 2^-, 3^+, \ldots, n^+)}{\langle 12 \rangle^8}} \, 2^{-8} \prod_{A=1}^{8} \sum_{i,j=1}^{n} \langle ij \rangle \, \eta_{i\,A} \, \eta_{j\,A}$$

fully bose symmetric

SU(8) flavor index

momentum labels

Associate with each state a differential operator:

$$b_+(i) \leftrightarrow 1 \,, \qquad f_+^A(i) \leftrightarrow \frac{\partial}{\partial \eta_{iA}} \qquad \ldots$$

$$\ldots \qquad b^-(i) \leftrightarrow \quad \mathcal{D}_i = \frac{1}{8!} \epsilon_{ABCDEFGH} \frac{\partial^8}{\partial \eta_{iA} \cdots \partial \eta_{iH}}$$

To calculate amplitude, apply operators to gen func.

Generating function for N=8 S.G. MHV amplitudes

As in N=4 SYM, the generating function satisfies the SUSY Ward ids.

So gen funct gives correct MHV amplitudes. AND:

MHV amplitudes in N=8 supergravity
=
integer partitions of 16 with n_{max}=8.

16=8+8 ← --++..+ pure graviton amplitude
16=8+7+1 ← two gravitinos
…
16=1+….+1 ← 16 gravitinos

Total number is 186

Application II: N=8 and $(N=4)^2$

N=4 SYM $$Z_n = \frac{A_n(1^-,2^-,3^+,\ldots,n^+)}{\langle 12\rangle^4}\, 2^{-4} \prod_{a=1}^{4} \sum_{i,j=1}^{n} \langle ij\rangle\, \eta_{i\,a}\, \eta_{j\,a}$$

N=8 S.G. $$\Omega_n = \frac{M_n(1^-,2^-,3^+,\ldots,n^+)}{\langle 12\rangle^8}\, 2^{-8} \prod_{A=1}^{8} \sum_{i,j=1}^{n} \langle ij\rangle\, \eta_{i\,A}\, \eta_{j\,A}$$

So $$\Omega_n \propto (Z_n)_L (Z_n)_R$$

Moreover, differential operators of N=8 S.G. factorize into $(\text{N=4 SYM})_L \otimes (\text{N=4 SYM})_R$ precisely following the map!

The factorization of amplitudes then follows automatically!

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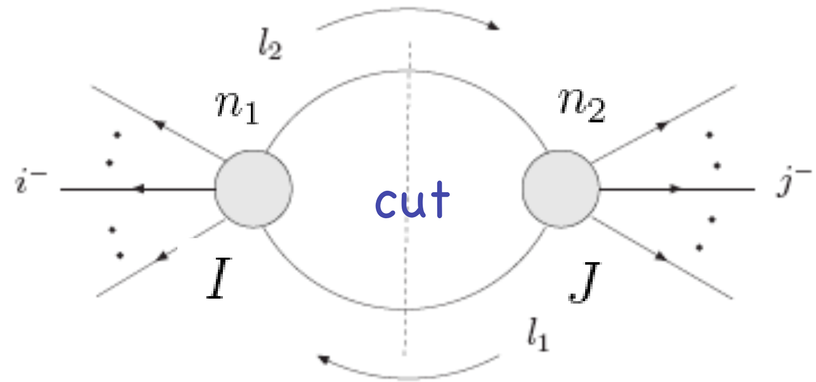
Application III: Intermediate state sum

1-loop pure gluon
MHV amplitude
in N=4 SYM.



$n = n_1 + n_2$

Need to compute

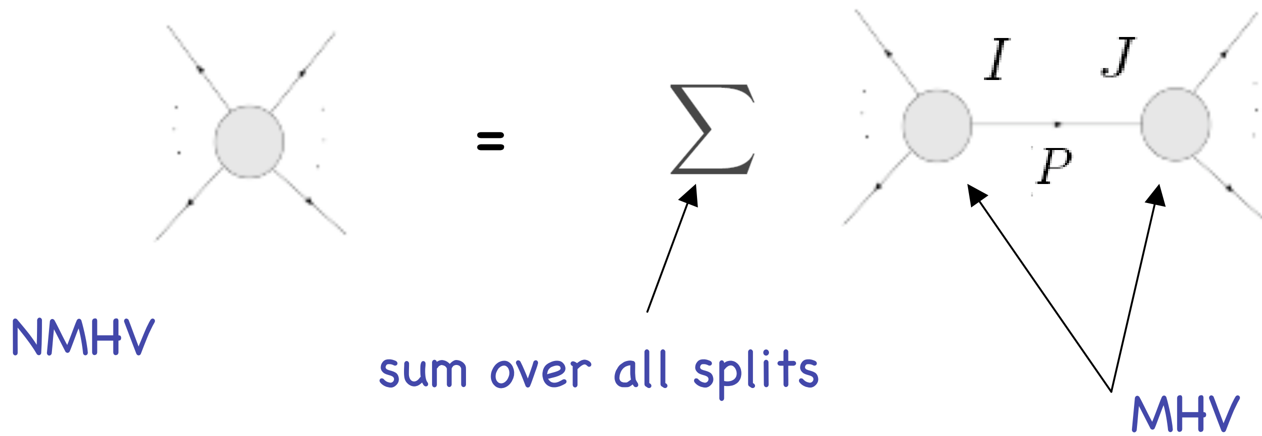
$$D_{l_1}\, D_{l_2}\, [D_i(Z_{n_1})_I][D_j(Z_{n_2})_J] \;=\; \langle i\, j\rangle^4 \langle l_1\, l_2\rangle^4$$

4th order

The point: $D_{l_1}\, D_{l_2}$ automatically sums over all intermediate states!

Generating functions for the Next-to-MHV sector

N=4 SYM NMHV amplitudes can be calculated using the "MHV vertex method" [Cachazo, Svrcek, Witten (2004)] + [Risager (2005)]



Generating function for each diagram:

$$\tilde{Z}_n = \frac{1}{D_I} Z_n \prod_{a=1}^{4} \sum_{k \in I} \langle P k \rangle \, \eta_{k\,a}$$

Diagram prefactor

MHV gen func

SUSY Ward ids OK.

Derive gen func via recursion relations.

Counting NMHV amplitudes

N=4 SYM NMHV amplitudes
=
partitions of 12 with n_{max}=4

That's 34.

In N=8 S.G.: similar set-up, analogue gen func:

N=8 supergravity NMHV amplitudes
=
partitions of 24 with n_{max}=8

That's 919.

BUT:

When is the MHV vertex decomposition valid?

Can be made precise using recursion relations from complex shifts of 3 external momenta: [Risager (2005)]

$$p_i^\mu \quad \rightarrow \quad \hat{p}_i^\mu = p_i^\mu + z\, q_i^\mu \,, \qquad i = 1, 2, 3$$

such that

$$q_1^\mu + q_2^\mu + q_3^\mu = 0 \qquad q_i^2 = 0 \qquad q_i \cdot p_i = 0$$

Momentum conservation ✓ On-shell $\hat{p}_i^2 = 0$ ✓

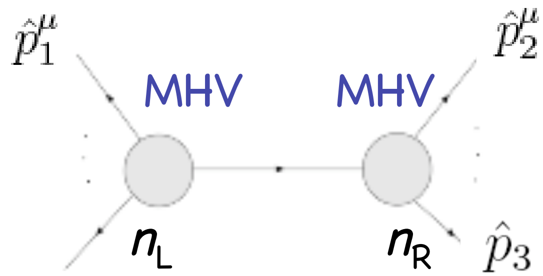
$A_n(z)$ is a rational function of z, with simple poles only (tree level!)

Recursion relations

IF $A_n(z) \to 0$ as $z \to \infty$, then by Cauchy's theorem:

$$\oint dz \frac{A_n(z)}{z} = 0 \quad \to \quad A_n(0) = -\sum_{z \neq 0} \text{Res} \frac{A_n(z)}{z}$$

$$\underset{\text{NMHV}}{A_n(0)} = \sum_{\text{splits}}$$



$n = n_L + n_R - 2$

n-pt NMHV amplitude calculated from *k*-pt MHV amplitudes.

This is precisely the MHV vertex expansion.

Generating functions for the Next-to-MHV sector

Uses MHV vertex expansion, so is valid IF there exists a 3-line shift such that $A_n^{\text{NMHV}}(z) \rightarrow 0$ for $z \rightarrow \infty$.

- Not formally proven for full N=4 SYM theory.
  … no counter examples found.

- In N=8 S.G.: similar set-up, analogue gen func.

BUT we have found *counter examples* to $M_n^{\text{NMHV}}(z) \rightarrow 0$ for $z \rightarrow \infty$.

Ex. $M_6(\phi^{1234}\,\phi^{1358}\,\phi^{1278}\,\phi^{5678}\,\phi^{2467}\,\phi^{3456}) \rightarrow O(1)$ for $z \rightarrow \infty$.

Many others O(1): can be dealt with. But also O(z).

Generating functions for the Next-to-MHV sector

Trouble even for *pure gravity*.

Under the best 3-line shift, we find
(numerical analysis for n=5,…,11)

$$M_n(1^-,2^-,3^-,4^+,\ldots,n^+) \to z^{n-12} \quad \text{for} \quad z \to \infty,$$

So recursion relations can be expected to fail
at n=12.

Must be careful when applying generating
function, e.g. in intermediate state sums.

… Consequence for UV divergencies in loops?

The bigger picture