New phases of black holes in higher dimensions

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- Review w/ H. Reall: **0801.3471 [hep-th]**
- M.Caldarelli, RE, M.J.Rodríguez: **0806.1954 [hep-th]**
- RE, T.Harmark, V.Niarchos, N.Obers **to appear**

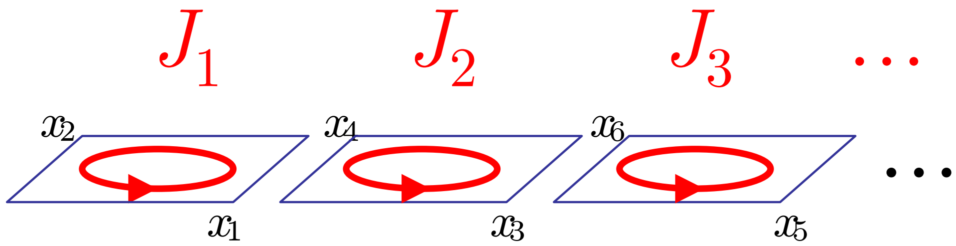
- Recently (~7 yrs ago) we've fully realized how *little* we know about black holes (even classical ones) and their dynamics in D>4
- Progress will have impact on:
  - String / M theory
  - Large Extra Dimensions & TeV gravity
  - AdS/CFT
  - Mathematics of Lorentzian geometry
  - **General understanding of spacetime at its most extreme**
- Activity launched initially by:
  - **GL-instability**, its endpoint, and inhomogeneous phases
  - **Black rings**, non-uniqueness, non-spherical topologies
- This talk:
  - First: vacuum, $R_{\mu\nu} = 0$, asymptotically flat solutions
  - Then, AdS etc

FAQ's

- Why is D>4 richer?

  – More degrees of freedom

  – Rotation:



  - more rotation planes

  - gravitational attraction ⇔ centrifugal repulsion

$$-\frac{GM}{r^{D-3}} + \frac{J^2}{M^2 r^2}$$

  – ∃ extended black objects: black p-branes

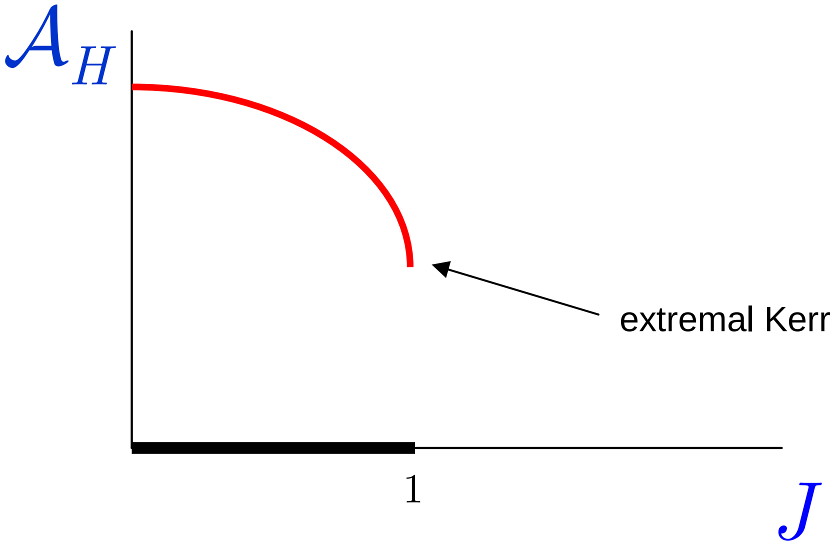
- Why is D>4 harder?

  – More degrees of freedom

  – Axial symmetries: U(1)'s at asymptotic infinity appear only every 2 more dimensions -- not enough to reduce to 2D σ-model if D>5

Phases of 4D black holes

- Just the Kerr black hole: Uniqueness thm



fixed Mass

- **End of the story!**

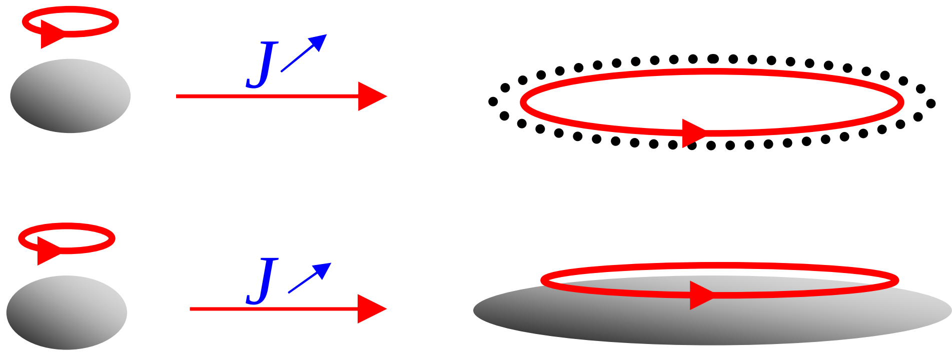
Multi-bhs not rigorously ruled out, but physically unlikely to be stationary (eg multi-Kerr can't be balanced)

Myers-Perry black holes in D dimensions

- They all have spherical topology S^{D-2}
- Consider a **single spin**:

$D = 5$

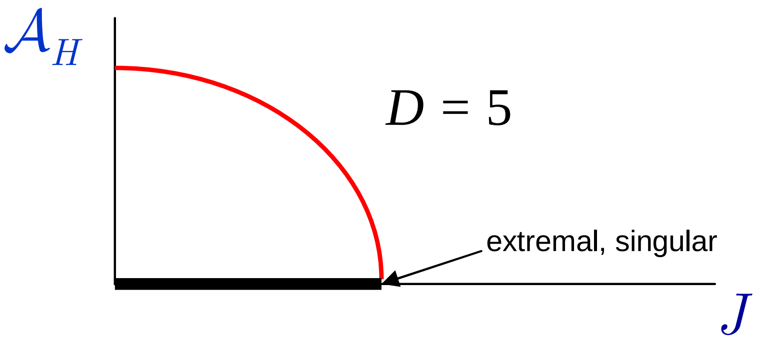
$D \geqslant 6$

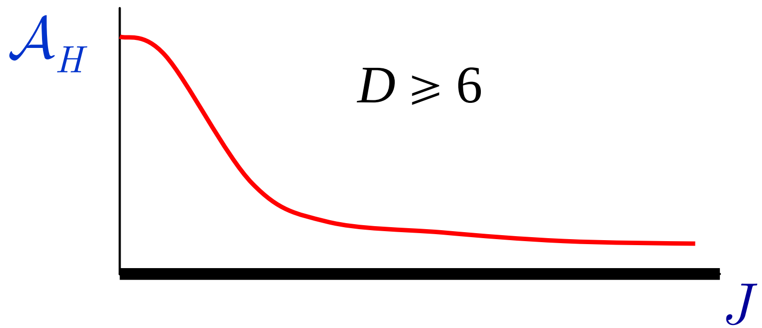


upper (extremal) limit on J is singular

no upper limit on J for fixed M

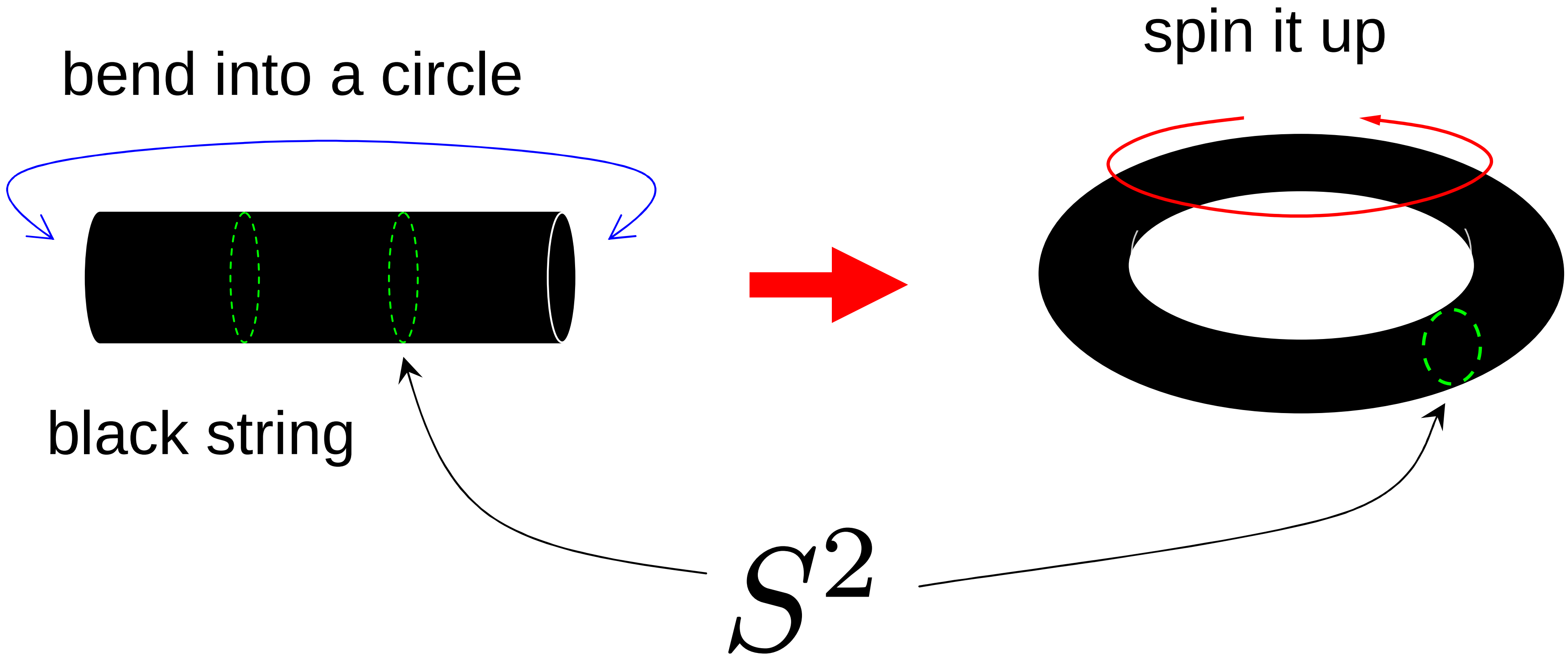
ultraspinning bh → black membrane





But there's much more…

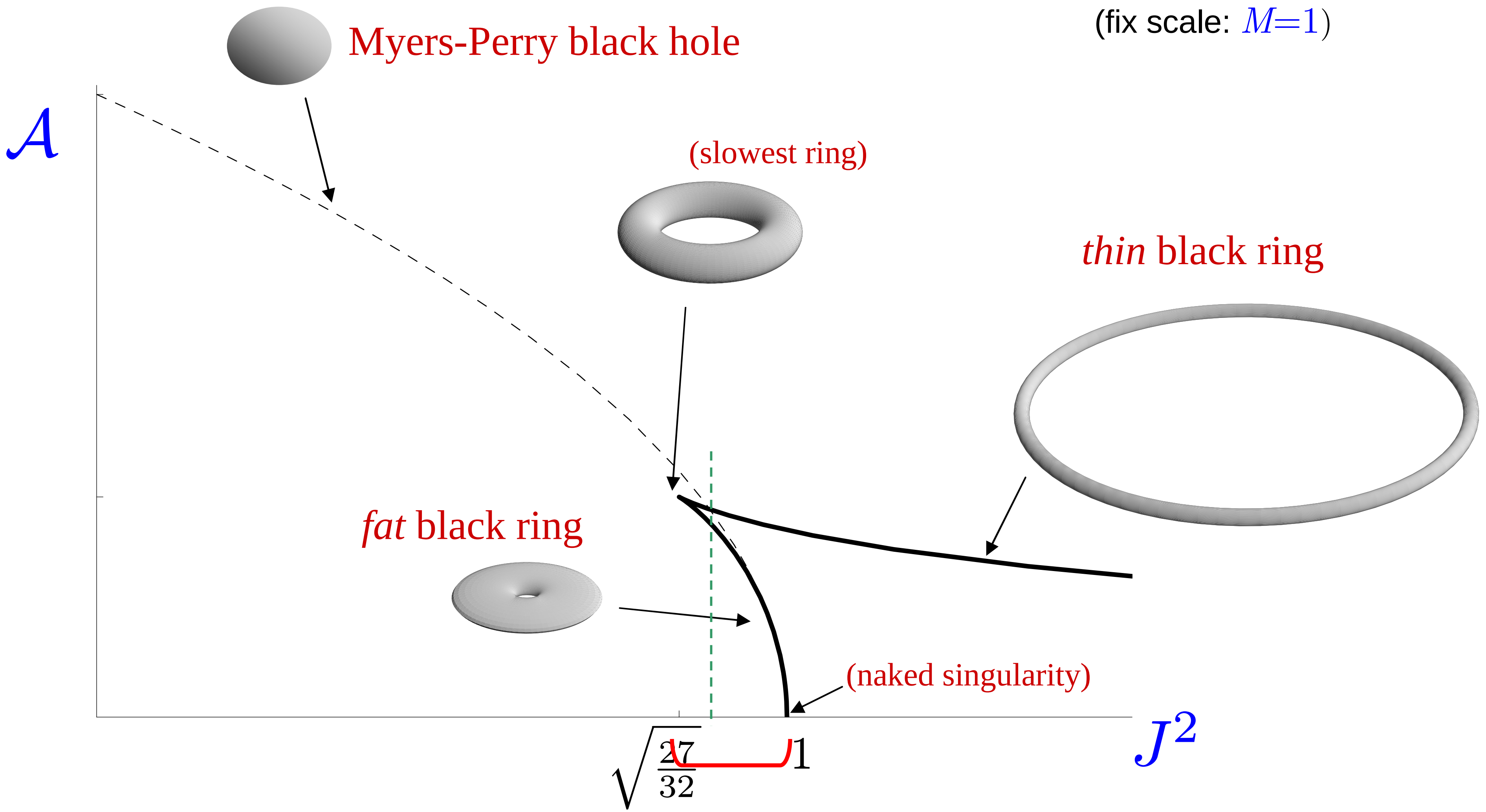
The forging of the ring (in $D\!=\!5$)



There's explicit solution, and fairly simple

RE + Reall 2001

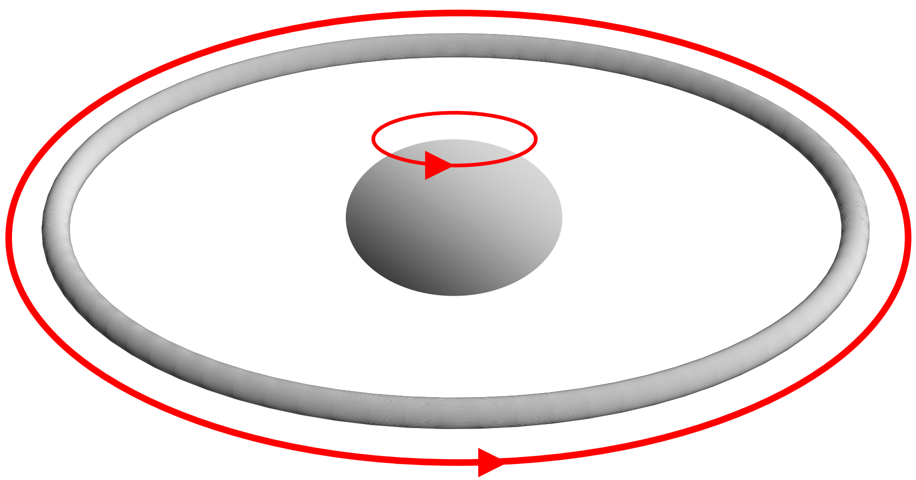
5D: one-black hole phases



3 different black holes with the same value of M,J

Multi-black holes

- *Black Saturn:*



- Exact solutions available *Elvang+Figueras*
- Co- & counter-rotating, rotational dragging…

- Up to iterations of multi-rings, and addition of second angular momentum (not discussed today), this *might* be all…
- …if two rotational symmetries are required:

$\mathbb{R}_t \times U(1)_{\phi_1} \times U(1)_{\phi_2}$ ➔ complete integrability

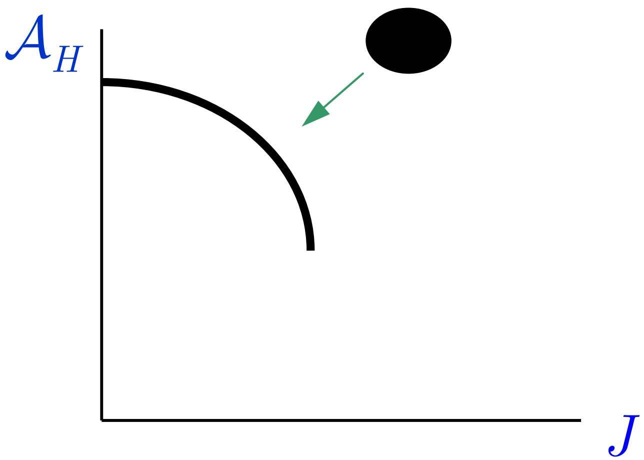
- But: stationarity → one axial $U(1)$, but not (yet?) necessarily two *Hollands et al*

  (new solutions with broken $U(1)$? where/how?)

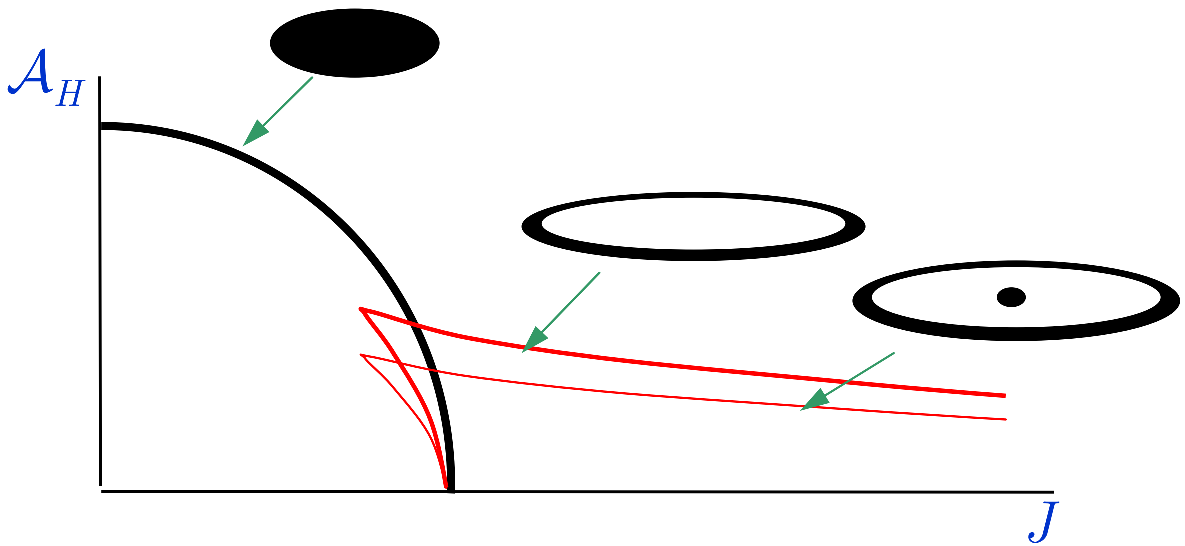
- This remains the main open issue in the classification of 5D black holes (vacuum)

The plot thickens…

$D=4$



$D=5$



Road blocks in $D \geqslant 6$

No solution-construction techniques

- Newman-Penrose formalism: unwieldy in $D > 4$
- Integrability of Weyl class w/ $\mathbb{R}_t \times U(1)^{D\text{-}3}$ symm:

  only helps in $D = 4,\ 5$:

  AF black holes have at most

  $\mathbb{R}_t \times O(D\text{-}1) \supset \mathbb{R}_t \times U(1)^{\lfloor (D\text{-}1)/2 \rfloor}$

  $$\left\lfloor \frac{D-1}{2} \right\rfloor = D-3$$

  $\Rightarrow D = 4, 5$

- Kerr-Schild class:

  MP black holes are K-S, but black rings are not

$$g_{\mu\nu} = \eta_{\mu\nu} + 2H(x) k_\mu k_\nu$$

Horizon topology

- Hawking's 4D theorem relies on Gauss-Bonnet thm:

$$\int_H R^{(2)} > 0 \Rightarrow H = S^2$$

- $D=5$: Galloway+Schoen: S^3, S^1x S^2 (& quotients): +ve Yamabe
- $D=6$: Helfgott et al: S^4, S^2x S^2, S^1x S^3, Σ_gx S^2

  so far: S^4 exactly (MP, but possibly others too)

  S^1x S^3 approximately
- $D>6$: essentially unknown (in progress)

Blackfolds

RE+Harmark+Niarchos+Obers

- **Black** branes w/ worldvolume = curved submani**fold** of spacetime

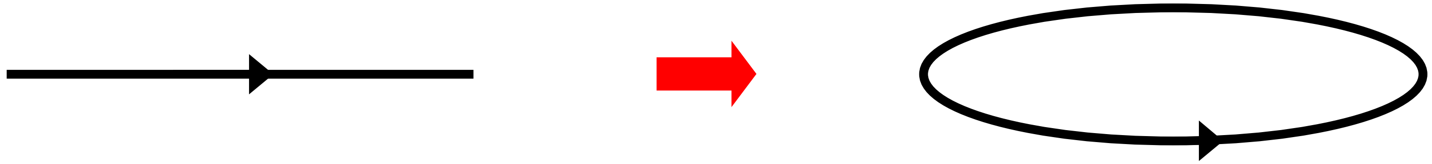
- Analogy: D-branes w/ non-trivial worldvolume geometry arise as solutions of DBI action

- Can this be extended to black branes?

- Yes: we can have a *general theory of classical brane dynamics*, including ***thin black branes*** w/ regular horizons

- Simplest blackfold:

  thin black ring = circular (boosted) black string



- Take a black p-brane, and bend it onto a p-diml submanifold: when is this possible?
- Approx:
  - thin brane: thickness << curvature of submanifold

    → perturbative solution via matched asymptotic expansion
  - equivalent to flat black brane, up to local Lorentz transform (boost & possibly rotation)

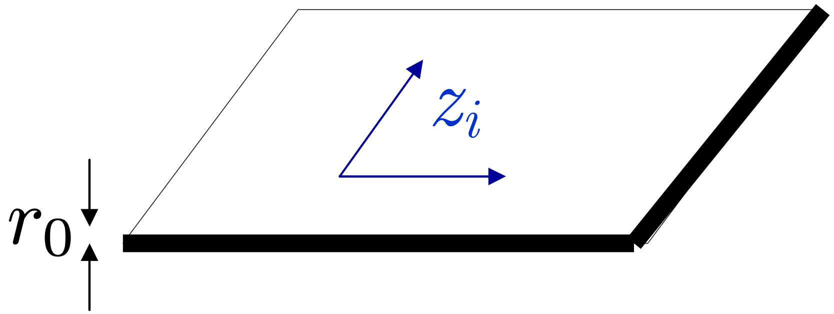
    → determines local stress tensor
  - **blackness condn**: temp and angular velocity uniform over blackfold

    → constraint on eqns to yield black objects

- Black p-brane

$$ds^2 = -\left(1 - \frac{r_0^n}{r^n}\right) dt^2 + \sum_{i=1}^{p} dz_i^2 + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1}^2$$

$$\begin{aligned} T_{tt} &= r_0^n(n+1) \\ T_{ii} &= -r_0^n \end{aligned}$$

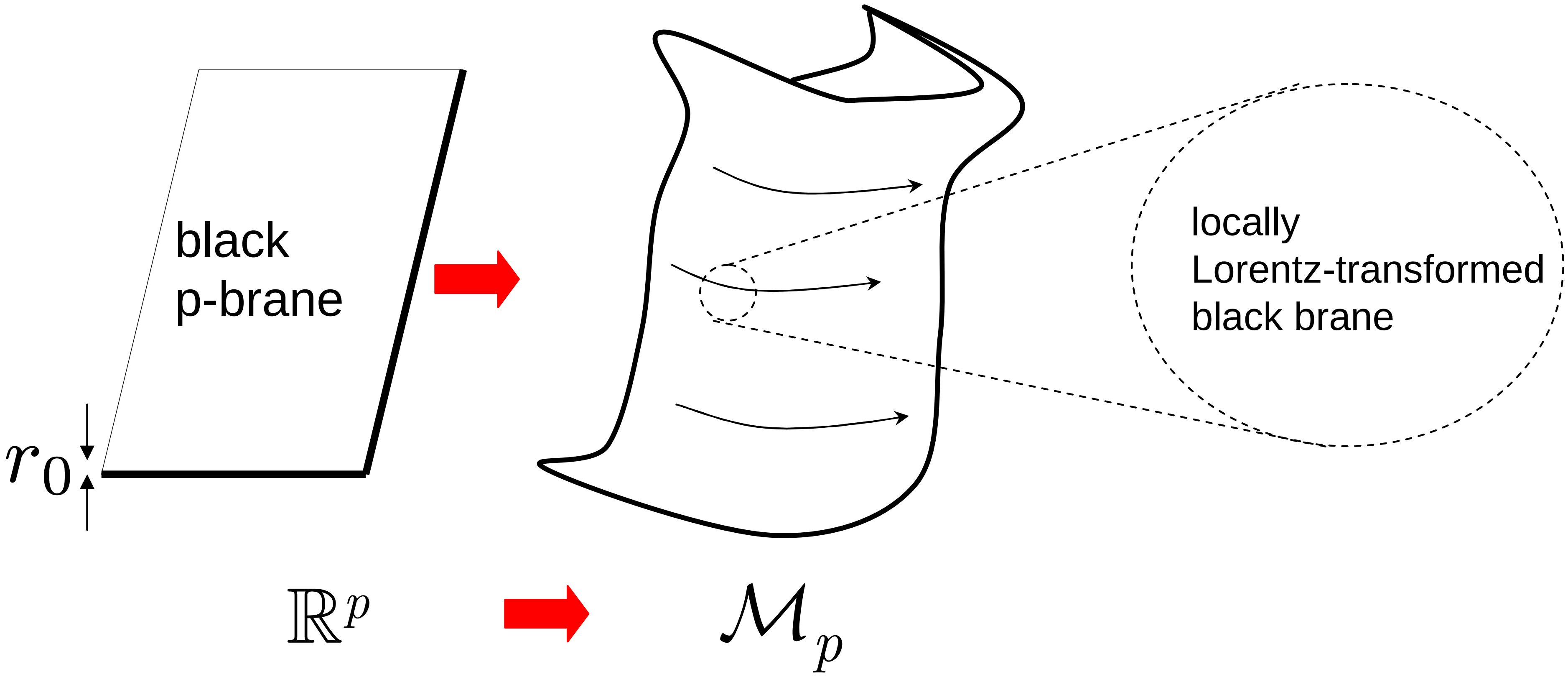


Lorentz-transform:

$$(t, z_i) = \sigma^\mu \,, \quad \sigma^\mu \to a^\mu_\nu \sigma^\nu \,, \quad a^\mu_\nu \in O(1,p)$$

$$T_{\mu\nu} \to T_{\mu\nu} = r_0^n \left[(n+1) a^t_\mu a^t_\nu - \sum_{i=1}^p a^i_\mu a^i_\nu\right]$$

E.g., black string → boosted black string



Brane in $\mathcal{M}_p$ is (extrinsically) curved, and has tension
→ generically not in mechanical equilibrium

- Classical black brane dynamics

$$\nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad K_{\mu\nu}{}^{\rho} T^{\mu\nu} = 0$$

Carter

extrinsic curvature

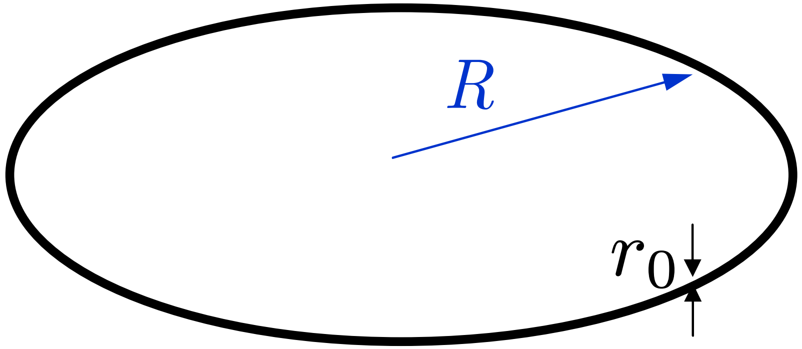
or, with external force: $F^\rho = K^{\rho}_{\mu\nu} T^{\mu\nu}$

- Newton's force law: $F = ma$
- Nambu-Goto-Dirac eqns: $T_{\mu\nu} = T\, g_{\mu\nu}$ → $K^\rho = 0$: minimal surface
- Generic eom's for *given* $T_{\mu\nu}$

- Blackfolds:
  - $T_{\mu\nu}(\sigma_\mu)$ locally Lorentz equiv to black p-brane: $r_0(\sigma),\ a^\mu{}_\nu(\sigma)$
  - Uniform surface gravity & angular velocities: eliminate *thickness*, and *boost* parameters in favor of geometric parameters

• Simplest example: black rings in $D \geqslant 5$

$$K_{\mu\nu}{}^{\rho} T^{\mu\nu} = 0$$



$$\Rightarrow \quad \frac{T_{11}}{R} = 0$$

$$T_{11} = r_0^{D-3}\left[(D-3)\sinh^2\sigma - 1\right]$$

Tune boost to equilibrium $\Rightarrow \sinh^2\sigma = \dfrac{1}{D-3}$

Horizon S^1 x $s^{D\text{-}3}$

"small" transverse sphere ~ r_0

• p-brane curved into $\mathbf{T}^p$:

$$K_{\mu\nu}{}^{\rho} T^{\mu\nu} = 0$$

$$\Rightarrow \quad \frac{T_{11}}{R_1} = \frac{T_{22}}{R_2} = \cdots = 0$$

$$T_{ii} = r_0^n (n(a_i^t)^2 - 1) \qquad n = D\text{-}p\text{-}3$$

Tune boost to equilibrium $\Rightarrow a_i^t = -\frac{1}{\sqrt{n}}, \quad a_t^t = \sqrt{\frac{n+p}{n}}$

Horizon $\mathbf{T}^p$ x s^{n+1}

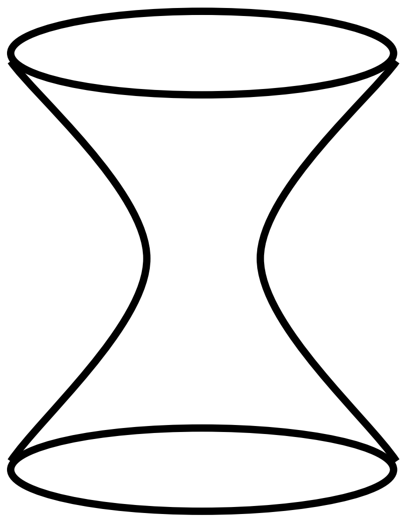
"small" transverse sphere ~ r_0

- Static minimal blackfolds:

If no boost (no local Lorentz transf)

→ $T_{ij} = -Pg_{ij}$ (spatial)

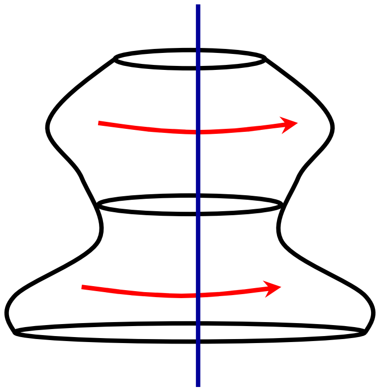
→ $K^{\rho}{}_{ij}=0$: minimal submanifold



E.g.: hyperboloid

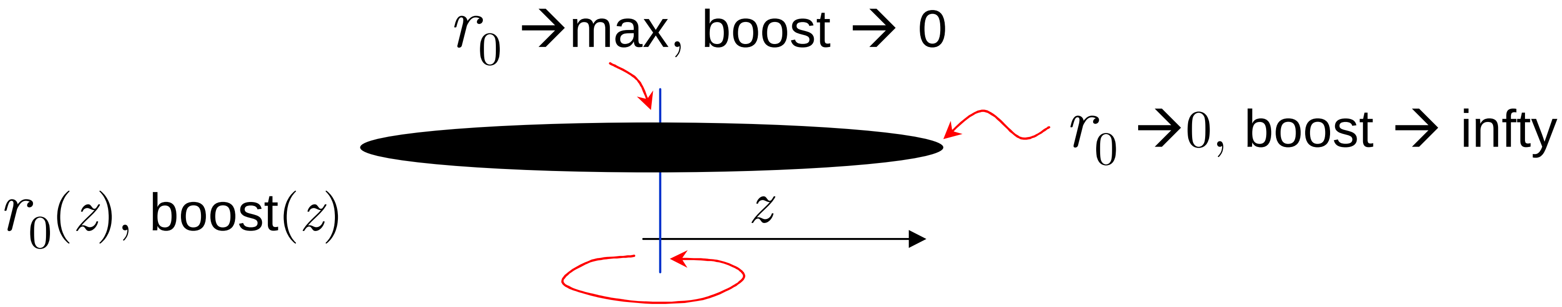
→ non-compact static blackfold

• Axisymmetric blackfolds



(possibly rotations along all axes)

- Simple analytic solutions:
  - odd p : round S^p, with all $(p+1)/2$ rotations equal
  - even p: ultraspinning MP bh, with $p/2$ ultraspins



- In progress: blackfolds with horizon topology

$$\prod_{p_i \in \mathrm{odd}} S^{p_i} \times s^{n+1} \quad ?$$

$$D = \sum_i p_i + n + 3$$

- But: does p-brane horizon remain *regular* after bending?
  - Perturb black p-brane into a slightly curved surface

$$r_0 << R_{\mathrm{curvature}}$$

  - Regularity checked in several cases – hard work, not most general yet

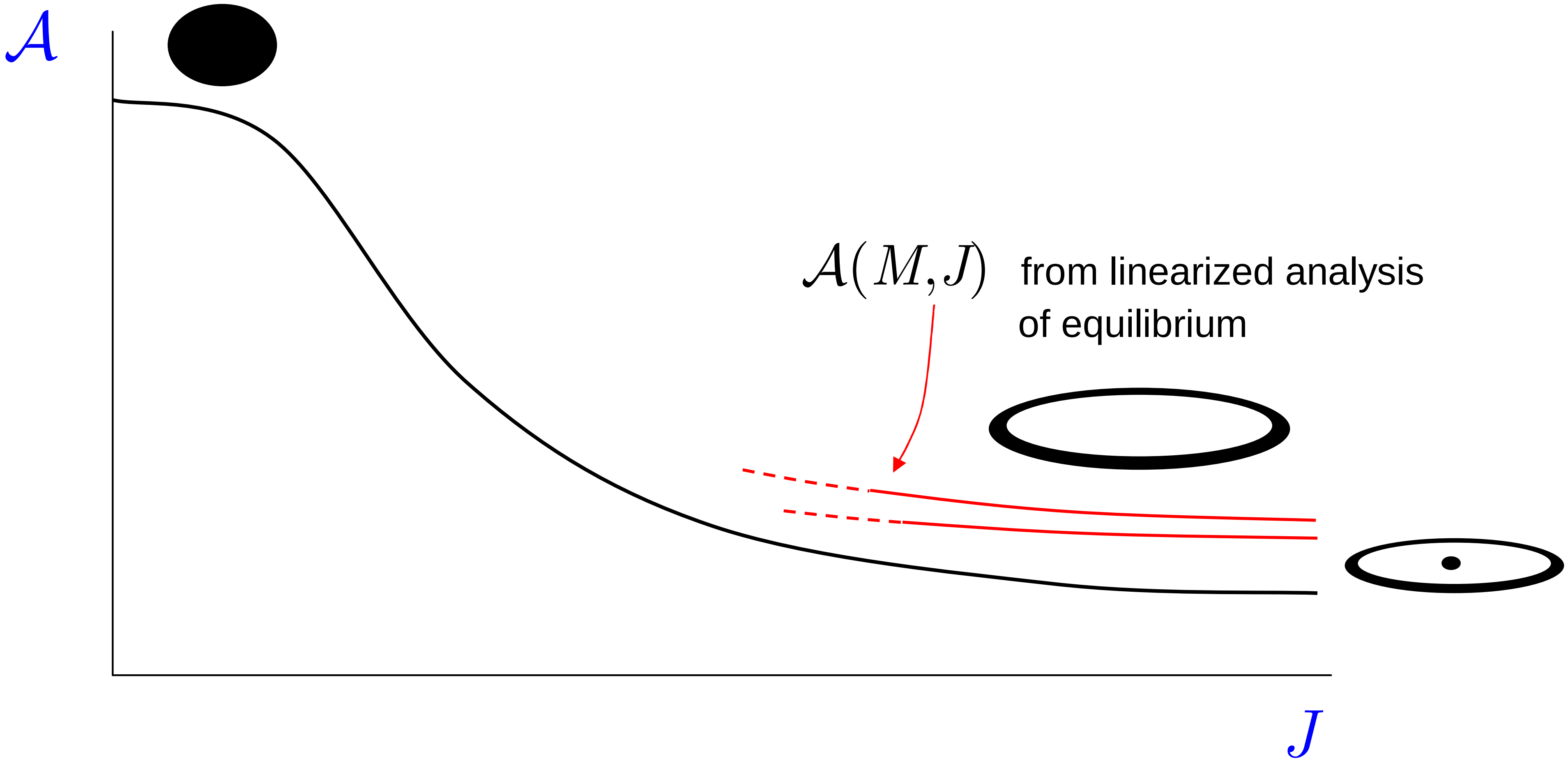
Other blackfold applications

- Curved strings & branes in gravitational potentials: *Caldarelli+RE+Rodriguez*
  - black rings in AdS: ∃ w/ arbitrarily large radius
  - black rings in dS: ∃ static ones
  - black saturns, charged and susy blackfolds…
- Very powerful and general approach to hi-d bh dynamics, extending (& including) DBI approach to D-branes

Caveat emptor: blackfold *may not* remain stationary beyond linear approx -- use with caution

$D \geqslant 6$ phase diagram

(fix M)



➔ Black rings dominate the entropy at large J

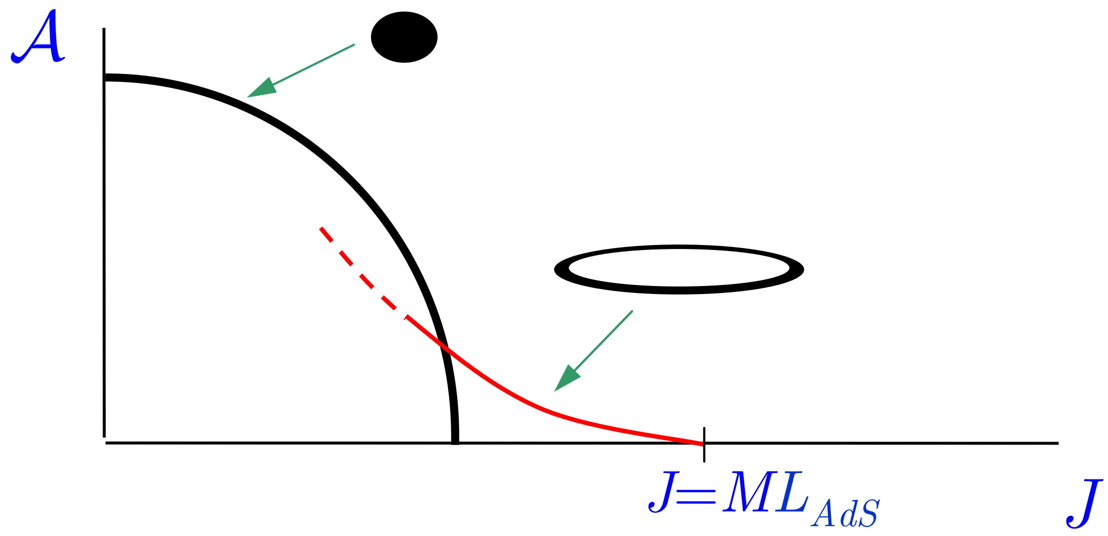
AdS phase diagram

"BPS" bound $J = ML_{AdS}$

Chrusciel et al

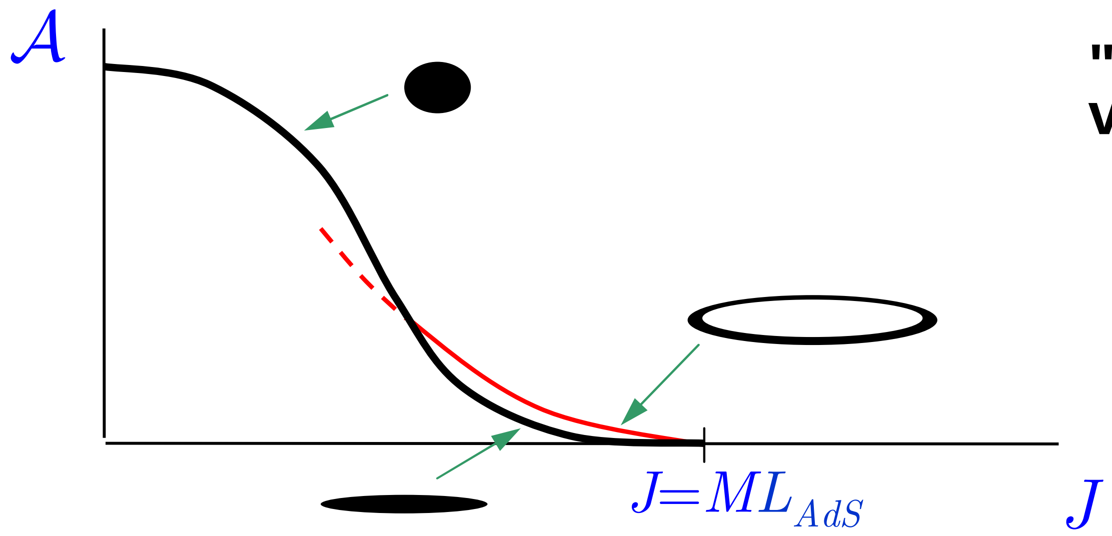
rot AdS bh:
Hawking+Hunter+Taylor
Gibbons et al

$D=5$



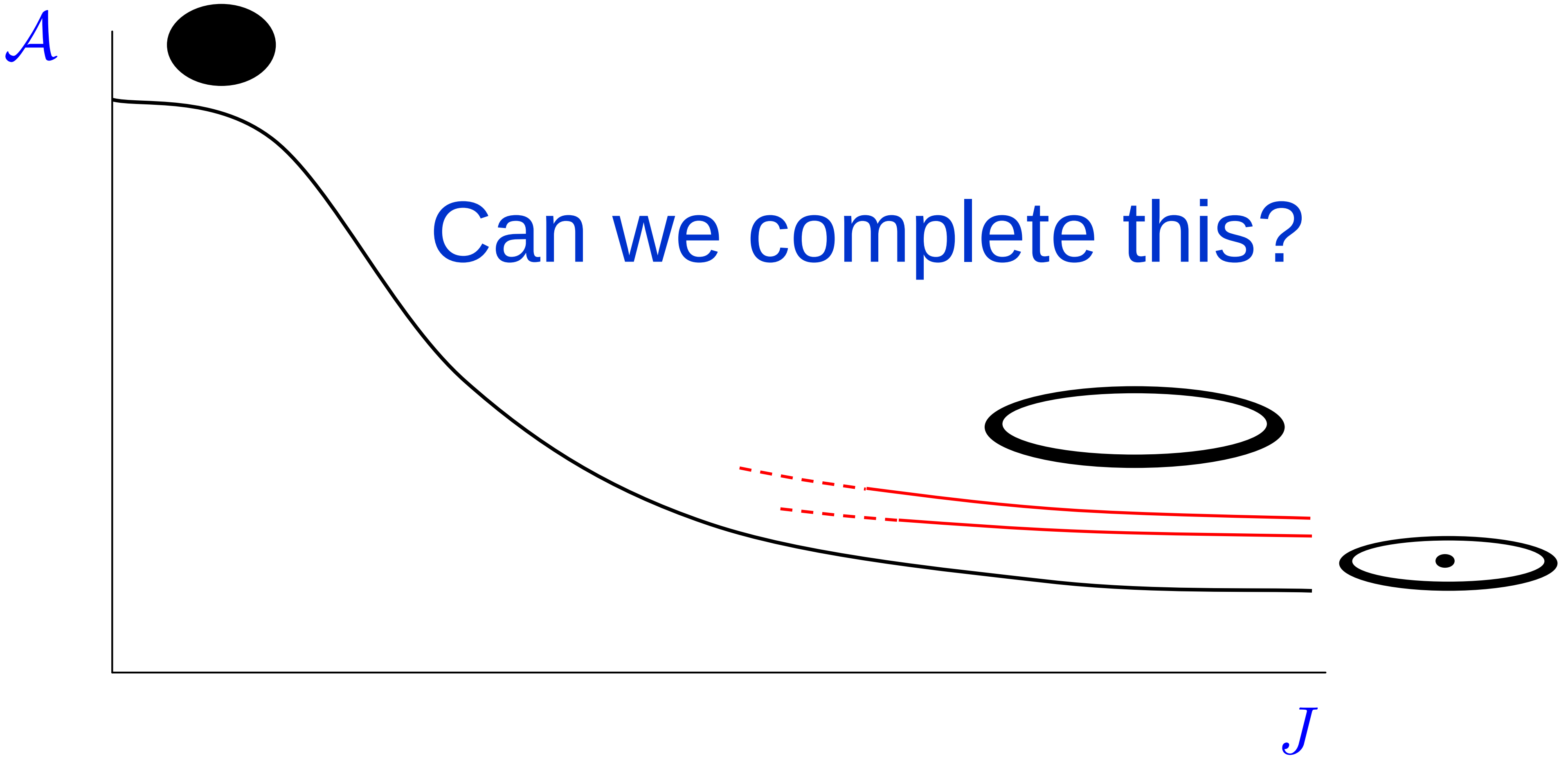
fixed M

$D \geqslant 6$



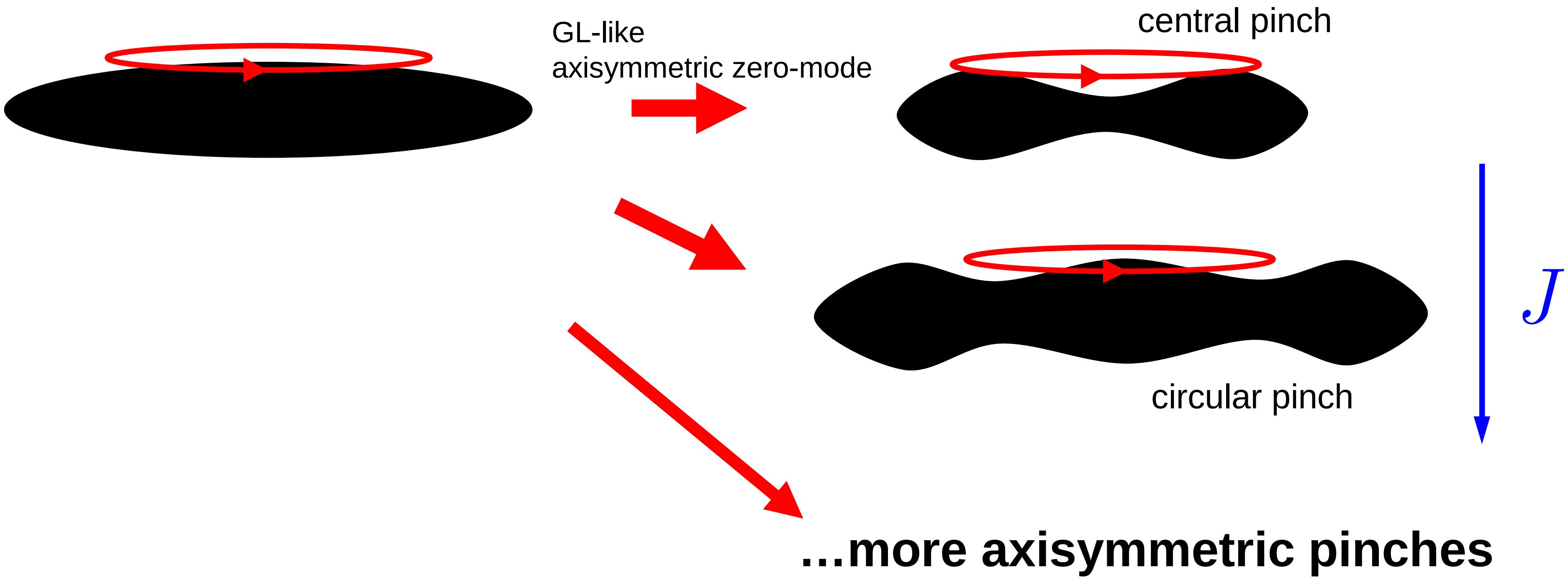
"compressed" versions of $\Lambda = 0$

Back to AF $D \geqslant 6$ phase diagram



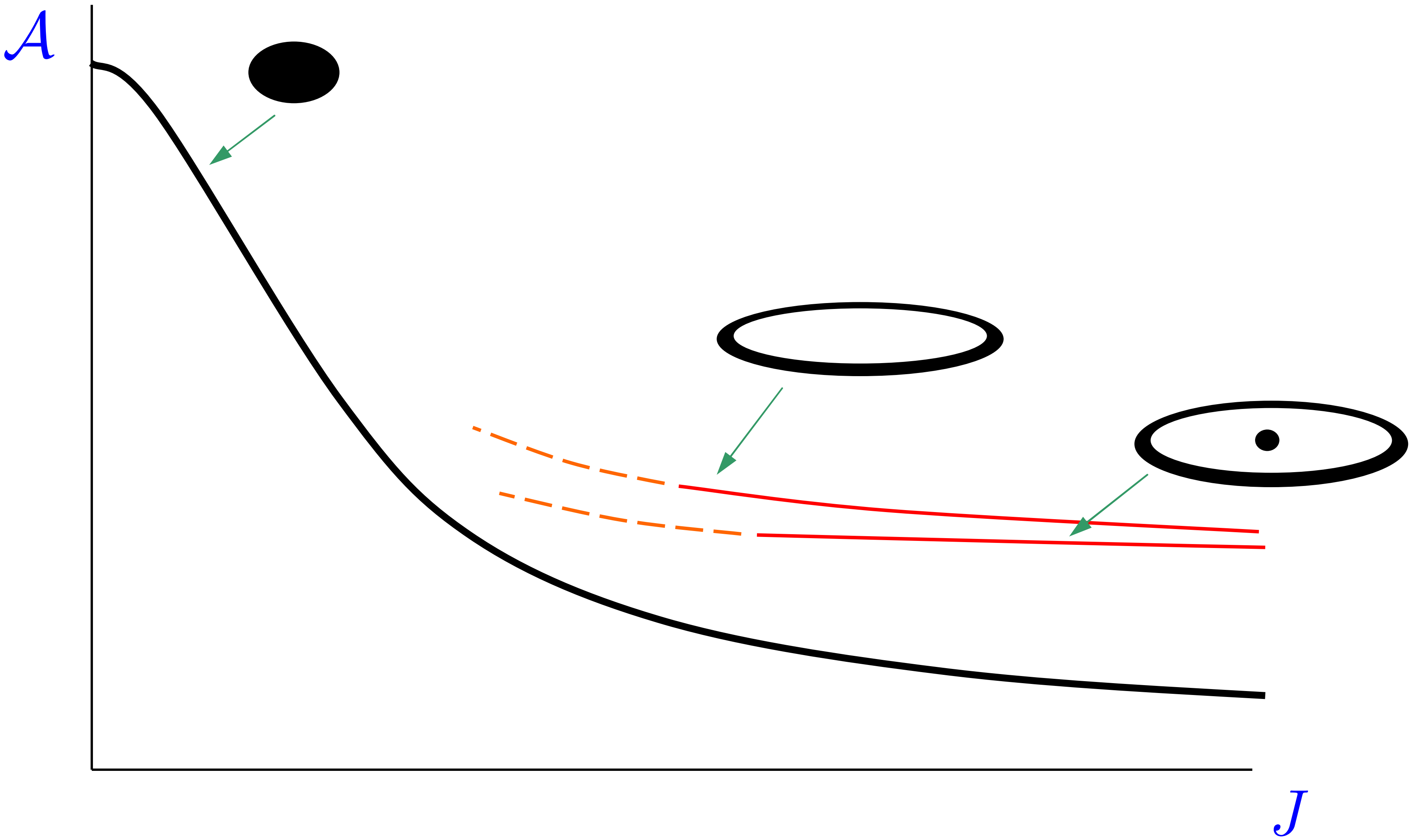
Multiply pinched black holes from axisymmetric zero-modes:

RE+Myers

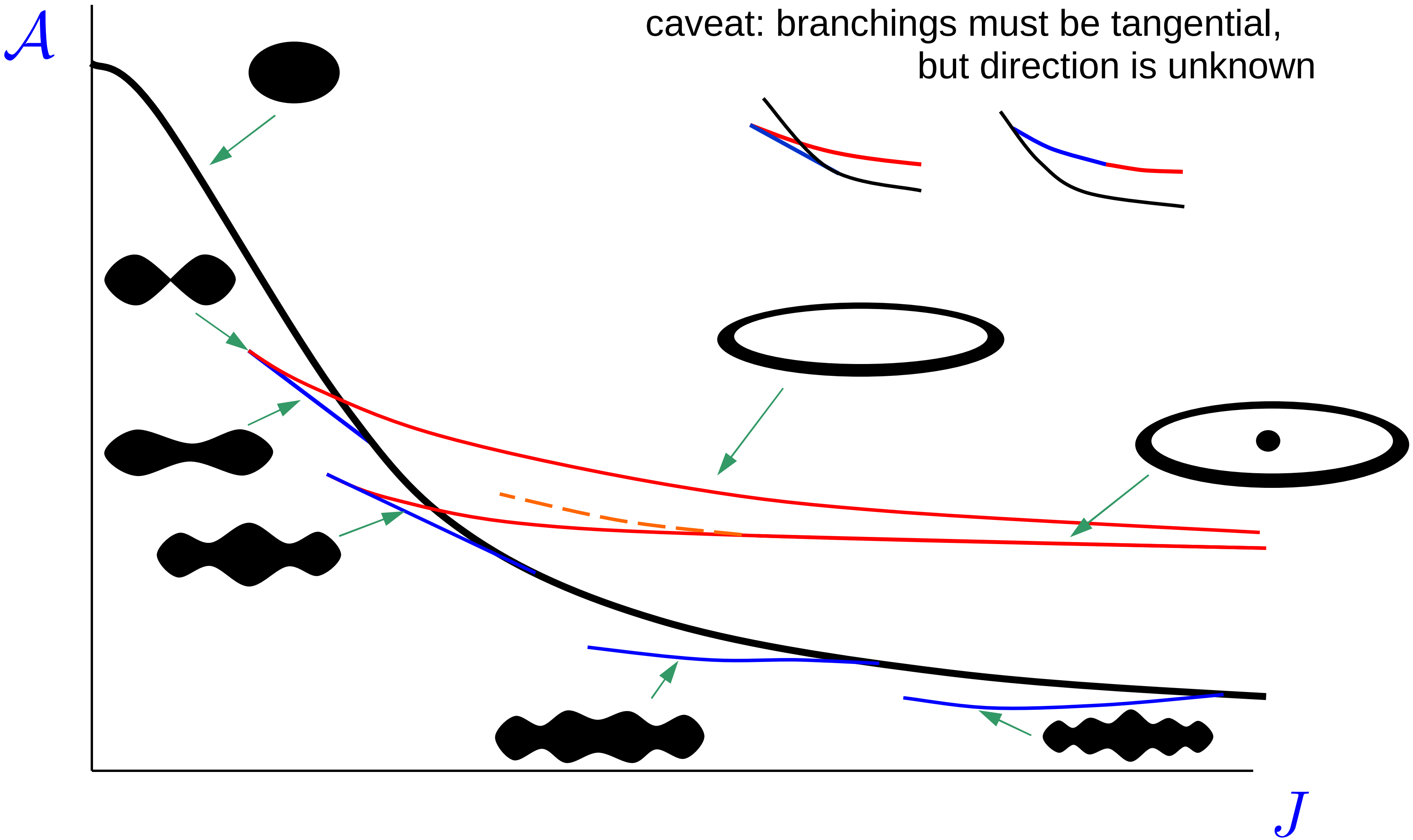


- Not yet found --- presumably numerically or approximately
- *Pinched plasma balls* found by *Lahiri+Minwalla*:
  dual to pinched black holes at IR in AdS w/ confining vacuum

$D \geqslant 6$ phase diagram: a proposal



$D \geqslant 6$ phase diagram: a proposal



Conclusions: *More is different*

Vacuum gravity $R_{\mu\nu} = 0$ in

- $D=3$ has no black holes
  - GM is dimensionless ➔ can't construct a length scale

    (Λ, or h, provide length scale)
- $D=4$ has **one** black hole
  - but no 3D bh ➔ no 4D black strings → no 4D black rings
- $D=5$ has **three** black holes (two topologies); black strings → black rings, infinitely many multi-bhs…
- $D \geqslant 6$ seem to have **infinitely many** ***blackfolds***, ***lumpy black holes***… (& possibly others that lie away from black brane limit)