Thermodynamics of Spacetime

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TJ PRL '95, **gr-qc/9504004**
Chris Eling, Raf Guedens & TJ PRL '06, **gr-qc/0602001**

Three mystifying things:

1) The 1st law of black hole mechanics, $\delta M = \frac{\hbar\kappa}{2\pi}\frac{\delta A}{4\hbar G_N}$
has a vivid thermodynamic meaning, connected via the Hawking effect to QFT, but is encoded in the classical Einstein equation, as is the 2nd law.

2) BH entropy, which (via entanglement) should depend on the nature and number of matter fields, is nevertheless universally equal to $A/4\hbar G_N$

3) BH thermodynamics applies also to de Sitter and Rindler horizons, which are (more) observer dependent and can exist anywhere and everywhere in a spacetime.

Idea:

> The physical origin of all this behavior is "vacuum thermodynamics".

> The Einstein equation is an equation of state, with G_N determined by surface entropy of the vacuum.

Ideal gas analogy

Equation of state follows from few assumptions:

- molecules are tiny
- $S = k \log(\text{number of states})$
- $dS = dQ/T$ (Clausius relation)
- Energy conservation

Entropy and volume

The molecules are so small that they occupy a negligible fraction of the available volume. Therefore…

NUMBER OF STATES: $V^N f(E)$

Entropy = *k* log(*number of states*) $S/k = N \log V + \log f(E)$

$$\frac{\partial S}{\partial V} = \frac{Nk}{V}$$

Equation of State

Clausius relation

$$dS = \frac{\delta Q}{T}$$

Energy conservation

$$\delta Q = dE + pdV$$

Math

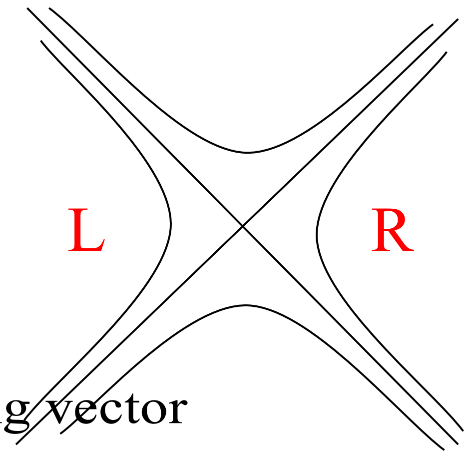
$$\frac{\partial S}{\partial V} = \frac{p}{T} \qquad = \frac{Nk}{V}$$

Counting

$$pV = N\ kT$$

Vacuum as thermal equilibrium

Poincare invariance and energy positivity imply the Minkowsi vacuum for any field restricted to the Rindler wedge is a thermal state:



$$\rho_R = Tr_L |0\rangle\langle 0| = Z^{-1} \exp\left(-H_{Boost}/T\right)$$

$$T = \hbar / 2\pi$$

$$H_{Boost} = \int T_{ab}\, \chi^a\, d\Sigma^b, \quad \chi^a = \text{ boost Killing vector}$$

An observer on a Killing orbit a distance l from the horizon sees the Unruh temperature, $T_{local} = \hbar a / 2\pi = \hbar / 2\pi\, l$.

(Bisognano-Wichmann 1975, Unruh 1976)

Horizon entanglement entropy: $S = -Tr\ \rho_R \ln\ \rho_R$

Horizon entanglement entropy

The total entanglement entropy of the vacuum
$S = -Tr\left(\rho_R \ln \rho_R\right)$ is

$$S \approx N \int dA \; dl \; T^3_{local} \propto N\, A \int dl \; l^{-3} \propto N\, A / l^2_{cutoff}$$

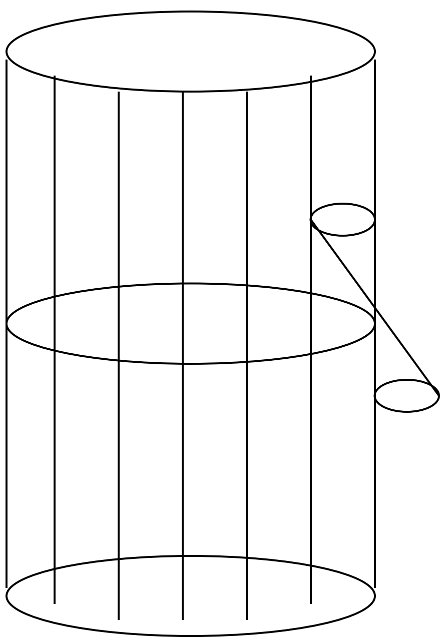
It depends on the number of species, and MUST be somehow cut off…

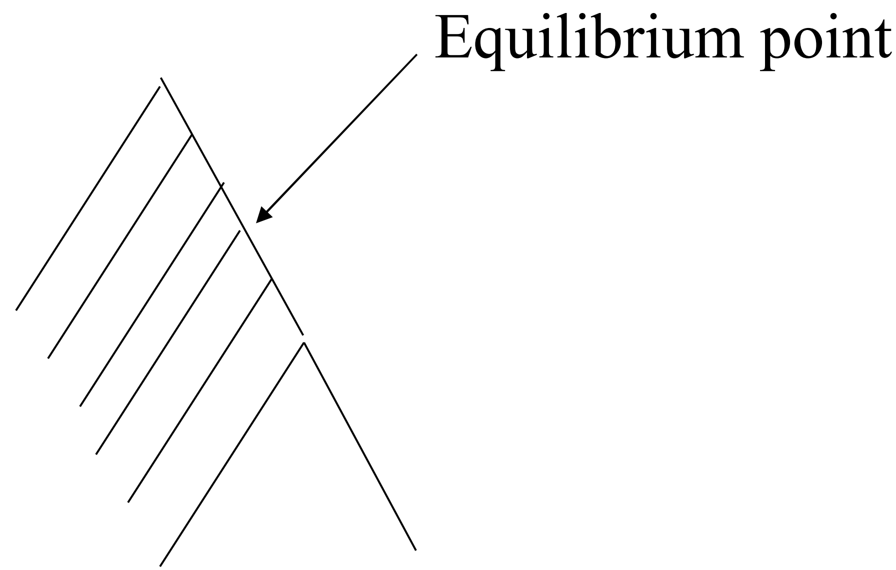
It is uncomputable in current theory (as was Avogadro's number),
so let us simply postulate a universal area entropy density η,

$$S = \eta A$$

Local causal horizons

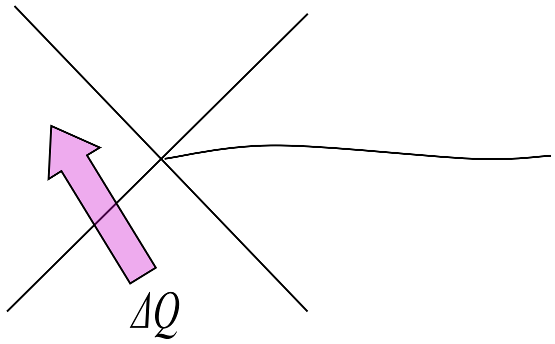
Stationary black hole horizon





Local horizon

Horizon thermodynamics



Postulates

The horizon system is a 'heat bath', with universal entropy area density.

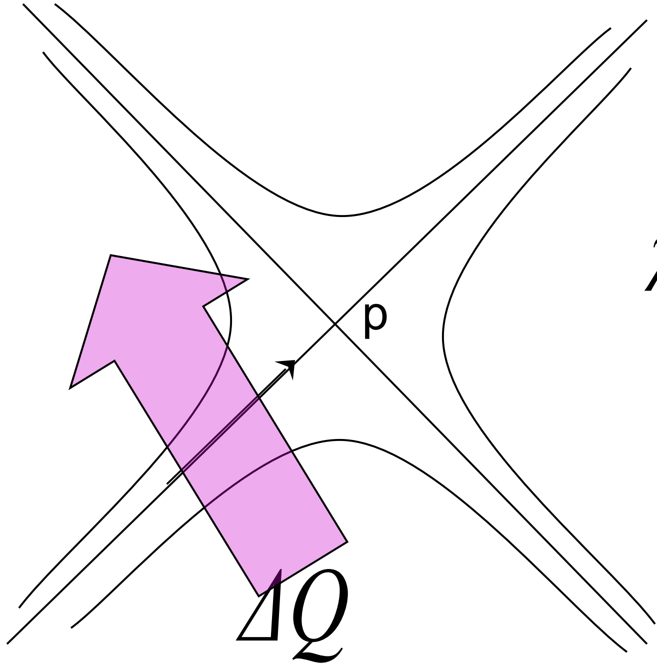
$$S = \eta A$$

Boost energy flux across the horizon is 'thermalized' at the Unruh temperature.

$$dS = \delta Q / T$$

Implies focusing of light rays: the causal structure must satisfy Einstein's equation, with $G = 1/4\hbar\eta$

Local Rindler Horizon



χ^a approximate boost Killing field

$$\chi^a = -\lambda\, k^a$$

$$k^a = dx^a / d\lambda \quad \text{affine param. tangent}$$

$$\Delta S = \Delta Q / T$$

$$\Delta S = \eta \; \Delta A = \eta \int \theta \, d\lambda \, dA$$

$$\theta = \frac{d(dA)/d\lambda}{dA}$$

"expansion" of the
horizon generating null geodesics

$$\Delta Q = \int T_{ab} \chi^a k^b \, d\lambda \, dA = \int -\lambda \, T_{ab} k^a k^b \, d\lambda \, dA$$

Heat current is linear in λ,
so the expansion must vanish at p.

Focussing equation

$$d\theta / d\lambda = -\frac{1}{2}\,\theta^2 - \sigma_{ab}\sigma^{ab} - R_{ab}k^a k^b$$

$$\Rightarrow \quad \theta = -\lambda\,\left(\sigma_{ab}\sigma^{ab} + R_{ab}k^a k^b\right)_p + O\left(\lambda^2\right)$$

$$\Delta S = \Delta Q / T$$

$$\Delta S = \eta\ \Delta A = \eta \int \theta\ d\lambda\ dA \approx \eta \int -\lambda\ (\sigma_{ab}\sigma^{ab} + R_{ab} k^a k^b)\, d\lambda\ dA$$

$$\Delta Q = \int\ T_{ab}\, \chi^a k^b\, d\lambda\ dA = \int -\lambda\ T_{ab} k^a k^b\, d\lambda\ dA$$

$T = \hbar / 2\pi =$ Unruh temperature

If the shear does not vanish at p cannot satisfy dS = dQ/T.

If valid for all LRH implies tracefree Einstein equation with

$$G = 1/4\hbar\,\eta$$

100% "induced" gravity

If also $\nabla^a T_{ab} = 0$ then

$$R_{ab} - \frac{1}{2} R g_{ab} - \Lambda g_{ab} = 8\pi\ G\ \ T_{ab}$$

Shear viscosity of the horizon

To allow for shear must account for INTERNAL ENTROPY PRODUCTION out of equilibrium: can satisfy $dS = dQ/T + d_iS$:

$$\frac{d_i S}{dt\ d^2 A} = \frac{1}{T}\ 2\varsigma\ \hat{\sigma}_{ab}\hat{\sigma}^{ab}, \quad \text{where} \quad \varsigma = \eta\ T/2 = \hbar\,\eta/4\pi = 1/16\pi G$$

and $\hat{\sigma}_{ab} = (d\lambda/dt)\ \sigma_{ab}$ is the rate of shear wrt Killing time t.

- Universal ratio shear viscosity/entropy density = $\hbar/4\pi$ same as found for black hole horizons. (Damour, Price & Thorne)
- Viscosity coefficient is phenomenological…as is entropy density.
- Why universal?? Fluctuation-dissipation theorem? Entropy counts the states into which the shear dissipates?
- AdS/CFT analog for black holes. (Policastro, Son & Starinets)

AdS/CFT Test

What theory has enough local Poincare invariance *and* a UV cutoff on entanglement entropy?

The CFT in AdS/CFT with the boundary at large finite AdS radius is locally Poincare invariant and has a UV cutoff … (RS brane)

…and gravity!

Moreover, entanglement entropy scales as $S \sim \left(N^2 / L^2_{cutoff}\right) A$,
and the induced 4D Newton constant on the boundary scales as
$G \sim \left(L^2_{cutoff} / N^2\right)$, where $N = \left(R^4_{AdS} / g l^4_s\right)$ is the number of colors.

(Hawking, Maldacena, and Strominger '01, Emparan '06)

comments and questions

1. The UV entropy finiteness assumption implies gravity, and gravity becomes strong in the UV, providing a potential cutoff mechanism. Does this signal a deep self-consistency?

2. If entropy is UV divergent we seem to infer G=0, but still the vacuum Einstein equation. Is this invalid? Why or why not?

3. The “horizon system” can be identified with the “atmosphere” behind a stretched local Rindler horizon, leading to a picture of “vacuum hydrodynamics”, as discussed in a new paper by Chris Eling (out today).