5D Stringy Black Holes

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hep-th/0702072
hep-th/0703087
0705.1847
0801.1863
} Castro, Davis, P.K., Larsen

0708.1001, P.K., Larsen, Shah

Outline

- Logic of standard black hole entropy computations. What is (not) being tested.
- 5D gives rich set of examples:

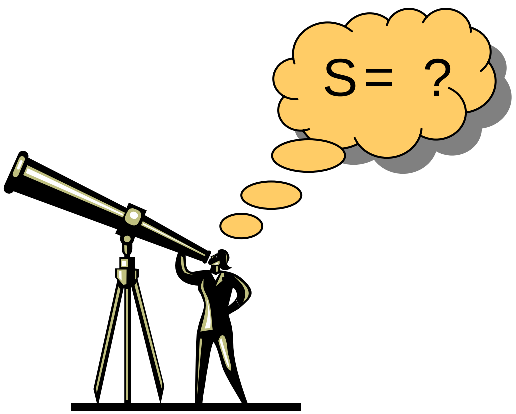
  holes, strings, and rings

- 4D BHs can be obtained via dim. red. from 5D
- Study these solutions in the presence of quantum/string corrections.
- Gives new quantitative and qualitative results:

  entropy corrections, singularity resolution, fundamental string solutions, 4D/5D connection, …

Why do statistical and gravitational entropies agree?

- Usual story: Count BPS states. Degeneracies don't change between weak and strong coupling.
- But…
    - •sometimes get agreement for non-BPS and non-extremal black holes
    - •not obvious a priori that gravitational entropy counts the total number of states. Maybe only those "at the horizon"; entanglement entropy …
- Stat. and grav. computations look very different. Sensitive test of string theory?
- Can we instead show they are the same computation?



weak coupling: brane

- Cardy formula: $S = 2\pi\sqrt{\frac{c_L}{6}h_L} + 2\pi\sqrt{\frac{c_R}{6}h_R}$

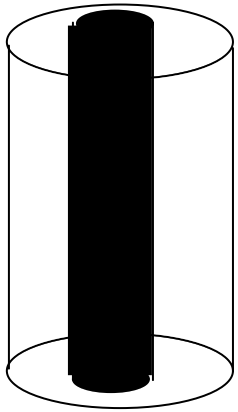
- (0,2) susy: $J(z)J(0) \sim \frac{k}{z^2} + \cdots ; \quad c = 6k$

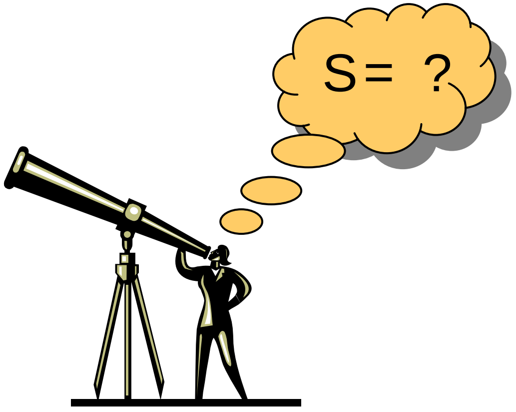
- chiral anomaly: $\partial J^\mu \sim k\,\epsilon^{\mu\nu} F_{\mu\nu}$

- inflow: $S_{bulk} \sim k \int A \wedge dA$

$c_L - c_R$ determined from grav. CS term via diff. anomaly

- Summary: entropy can be determined by asymptotic observer





strong coupling: AdS

- Cardy formula: $S = 2\pi\sqrt{\frac{c_L}{6}h_L} + 2\pi\sqrt{\frac{c_R}{6}h_R}$

- (0,2) susy: $J(z)J(0) \sim \frac{k}{z^2} + \cdots; \quad c = 6k$

- chiral anomaly: $\partial J^\mu \sim k\epsilon^{\mu\nu}F_{\mu\nu}$

- inflow: $S_{bulk} \sim k\int A \wedge dA$

$c_L - c_R$ determined from grav. CS term via diff. anomaly

- Summary: Same computation as before

Comments

- Agreement follows, regardless of what is actually counted by S_{grav}
- All higher deriv. corrections appearing though c taken into account.
- Argument says nothing about other corrections subleading in $h_{L,R}$
- Explains all “solid” entropy comparisons.
- Highlights importance of understanding other cases.

4D higher derivative corrections

- Much work on 4D BPS solutions with R^2 corrections (Cardoso, de Wit, Mohaupt, …)

- Gives agreement with BPS micro. entropy, but wrong answer for non-BPS (Sen)

- Agreement also for BPS small black holes (Dabholkar)
  - Surprising (at first), since no small parameter suppressing R^4 terms

5D higher derivative corrections

- Situation clearer in 5D:

  Just need to check symmetries, anomalies, and finite size AdS_3

- Only higher derivative anomalous term:

$$c_{2I} A^{I} \, \mathrm{Tr} R \wedge R$$

coefficient known

- Would like to find solutions in presence of susy completion of this term

5D R^2 supergravity

- Fully off-shell $N = 2$ action constructed by
  Hanaki, Ohashi, Tachikawa

$$e_\mu^a;\ \psi_\mu;\ A_\mu^I;\ M^I;\ \Omega_I;\ \underbrace{v^{ab};\ D;\ \chi}_{\text{auxiliary fields}}$$

- Two deriv. action specified by

$$c_{IJK}\, A^I \wedge F^J \wedge F^K$$

CY intersection numbers

- 4-deriv action believed to be uniquely fixed by susy

5D 4-deriv terms

$$\frac{c_{2I}}{24}\Big(\frac{1}{16}\epsilon_{abcde}A^{Ia}C^{bcfg}C^{de}{}_{fg}+\frac{1}{8}M^{I}C^{abcd}C_{abcd}+\frac{1}{12}M^{I}D^{2}+\frac{1}{6}F^{Iab}v_{ab}D$$

$$+\frac{1}{3}M^{I}C_{abcd}v^{ab}v^{cd}+\frac{1}{2}F^{Iab}C_{abcd}v^{cd}+\frac{8}{3}M^{I}v_{ab}\hat{D}^{b}\hat{D}_{c}v^{ac}$$

$$+\frac{4}{3}M^{I}\hat{D}^{a}v^{bc}\hat{D}_{a}v_{bc}+\frac{4}{3}M^{I}\hat{D}^{a}v^{bc}\hat{D}_{b}v_{ca}-\frac{2}{3}M^{I}\epsilon_{abcde}v^{ab}v^{cd}\hat{D}_{f}v^{ef}$$

$$+\frac{2}{3}F^{Iab}\epsilon_{abcde}v^{cf}\hat{D}_{f}v^{de}+F^{Iab}\epsilon_{abcde}v^{c}{}_{f}\hat{D}^{d}v^{ef}-\frac{4}{3}F^{Iab}v_{ac}v^{cd}v_{db}$$

$$-\frac{1}{3}F^{Iab}v_{ab}v^{2}+4M^{I}v_{ab}v^{bc}v_{cd}v^{da}-M^{I}(v^{2})^{2}\Big)$$

•susy variations:

$$\delta\psi_{\mu}=\left(D_{\mu}+\frac{1}{2}v^{ab}\gamma_{\mu ab}-\frac{1}{3}\gamma_{\mu}\gamma\cdot v\right)\epsilon\,;$$

$$\delta\Omega^{I}=\left(-\frac{1}{4}\gamma\cdot F^{I}-\frac{1}{2}\gamma^{a}\partial_{a}M^{I}-\frac{1}{3}M^{I}\gamma\cdot v\right)\epsilon\,;$$

$$\delta\chi=\left(D-2\gamma^{c}\gamma^{ab}D_{a}v_{bc}-2\gamma^{a}\epsilon_{abcde}v^{bc}v^{de}+\frac{4}{3}(\gamma\cdot v)^{2}\right)\epsilon$$

Two-derivative solutions (Gauntlett et. al.)

$$ds^2 = e^{4U}(dt + \omega)^2 - e^{-2U} ds_B^2$$

hyperkahler base

1-form on base

$$F^I = d[M^I e^{2U}(dt + \omega)] + \Theta^I$$

closed anti-self-dual 2-form on base

• BPS equations:

$$\nabla^2 (M_I e^{-2U}) = \frac{1}{2} c_{IJK} \Theta^J \cdot \Theta^K$$

$$d\omega - \star_4 d\omega = - e^{-2U} M_I \Theta^I$$

$$\frac{1}{6} c_{IJK} M^I M^J M^K = 1$$

$$M_I \equiv \tfrac{1}{2} c_{IJK} M^J M^K$$

• Simplest solution: $\omega = \Theta^I = 0\,; \quad ds_B^2 = ds_{R^4}^2$

$$M_I e^{-2U} = H_I = 1 + \frac{q_I}{r^2}$$

Gives 5D static BH with near horizon $AdS_2 \times S^3$

Two-derivative solutions cont.

- More generally, choose Gibbons-Hawking base space:

$$ds_B^2 = \frac{1}{H^0}(dx^5 + \chi)^2 + H^0 d\vec{x}^2$$

$$\vec{\nabla} \times \vec{\chi} = -\vec{\nabla} H^0$$

- System admits solution in terms of harmonic functions on R^3

$$H^0;\ H^I;\ H_0;\ H_I$$

- Various choices give 5D black holes, 5D black strings, 5D black rings, and 4D black holes

Relation to 4D BPS equations

Change of variables:

$$Y^0 = -\frac{1}{2}(e^{2\phi+2U}\omega_5 - i)H^0$$

$$Y^I = -e^{2\phi-2U}M^I + \frac{1}{2}(e^{2\phi+2U}\omega_5 - i)H^I$$

$$\omega = \omega_5(dx^5 + \chi) + \hat{\omega}$$

$$e^{-4\phi} = \frac{e^{-2U}}{H^0} - e^{4U}\omega_5^2$$

$$e^{2g} = \frac{e^{2U+2\phi}}{H^0}$$

5D BPS equations become:

$$Y^A - \overline{Y}^A = iH^A$$

$$F_A - \overline{F}_A = iH_A$$

$$i(\overline{Y}^A F_A - \overline{F}_A Y^A) = \frac{1}{2}e^{-2g}$$

$$H^A\vec{\nabla}H_A - H_A\vec{\nabla}H^A = \frac{1}{2}\vec{\nabla}\times\hat{\omega}$$

$$F = -\frac{1}{6}\frac{c_{IJK}Y^IY^JY^K}{Y^0}$$

Higher deriv BPS equations

- Maxwell equation

$$\nabla^2 (M_I e^{-2U}) = \tfrac{1}{2} c_{IJK}\, \Theta^J \cdot \Theta^K$$

becomes:

$$\vec\nabla^2 \left\{ M_I e^{-2U} - \frac{c_{2I}}{24}\left[3(\vec\nabla U)^2 - \frac{1}{4} e^{6U} (d\omega^+)^2 - \frac{1}{12} e^{6U} (d\omega^-)^2 \right] \right\}$$

$$= \frac{1}{2} c_{IJK}\, \Theta^J \cdot \Theta^K + \frac{c_{2I}}{24} \left\{ \frac{1}{8} R^{\hat i \hat j \hat k \hat l} d\omega^-_{\hat i \hat j} d\omega^-_{\hat k \hat l} + \frac{1}{32} (R^{\hat i \hat j \hat k \hat l})^2 \right\}$$

curvature of base space

Higher deriv BPS equations

•v equation of motion:

$$d\omega - \star_4 d\omega + e^{-2U} M_I \Sigma^I = 0$$

becomes:

$$(d\omega - \star_4 d\omega)_{ij} + e^{-2U} M_I \Sigma^I_{ij} = -\frac{c_{2I}}{24}\Big[-\frac{1}{6}\nabla^2 \big[M^I e^{2U}(3d\omega^+_{ij} - d\omega^-_{ij})\big] - \frac{1}{12} e^{-6U}\nabla^2 \big(e^{6U}\Sigma^I_{ij}\big)$$
$$- 2\partial_k \partial_{[i}\big(M^I e^{2U} d\omega^+_{j]k}\big) + \frac{1}{12} e^{6U}\Sigma^I_{ij}\big(3(d\omega^+)^2 + (d\omega^-)^2\big)$$
$$+ 2e^{2U}\big(\partial_k M^I \partial_{[i} U - \partial_k U \partial_{[i} M^I\big)\big(d\omega^+_{i]k} - 3d\omega^-_{i]k}\big)\Big]$$

Higher deriv BPS equations

- Special geometry constraint:

$$\frac{1}{6}c_{IJK}M^I M^J M^K = 1$$

becomes:

$$\frac{1}{6}c_{IJK}M^I M^J M^K = 1 - \frac{c_{2I}}{24}e^{2U}\Big\{\Big(M^I\Big[\vec\nabla^2 U - 4(\vec\nabla U)^2\Big] + \vec\nabla M^I\cdot\vec\nabla U\Big) + \frac{1}{4}e^{6U}M^I\Big((d\omega^+)^2 + \frac{1}{3}(d\omega^-)^2\Big) - \frac{1}{12}e^{4U}\Theta^I\cdot d\omega^-\Big\}$$

5D static black hole

- Built out of harmonic functions:

$$H_I = H_I^{\infty} + \frac{q_I}{r^2}$$

- Moduli determined by:

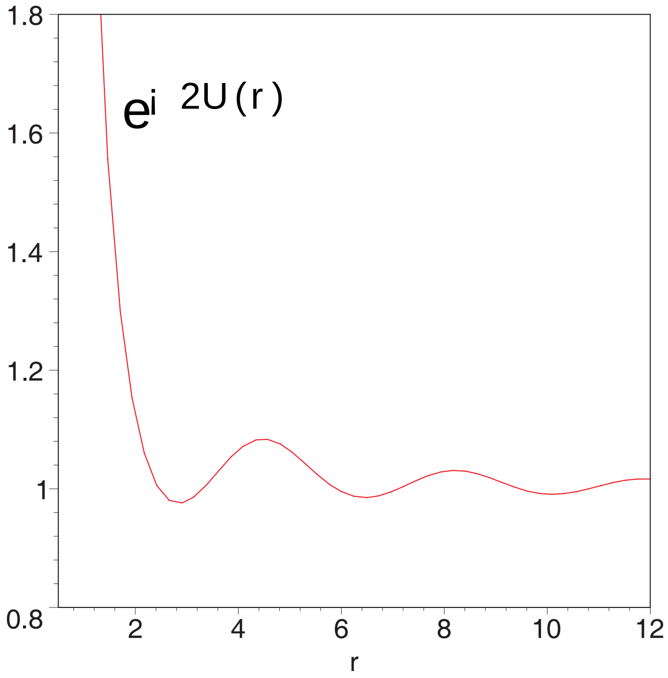
$$\frac{1}{2} c_{IJK} M^J M^K = e^{2U} \left(H_I + \frac{c_{2I}}{8} (\nabla U)^2 \right)$$

- Special geometry constraint yields ODE:

$$\frac{1}{6} c_{IJK} M^I M^J M^K = 1 + \frac{c_{2I}}{24} \left((\nabla^2 U - 4 (\nabla U)^2) M^I + \nabla U \cdot \nabla M^I \right)$$

- Admits solutions with near horizon $AdS_2 \times S^3$ with corrected radii and moduli

Numerical solution



•Oscillatory behavior similar to 4D case (Sen), (Hubeny, Maloney, Rangamani)

Due to existence of spurious solutions of linearized HD theory

Entropy

- Compute entropy from Wald formula.

- Just find simple charge shift:

$$S(q_I) \Rightarrow S(q_I + \tfrac{1}{8} c_{2I})$$

- Geometrically:

$$S = \underbrace{\tfrac{1}{6} c_{IJK} M^I M^J M^K}_{=1 \text{ in 2-deriv theory}} \cdot \frac{A}{4G}$$

- No micro formula in general, but agrees with available results (Vafa), (Huang, Klemm, Marino, Tavanfar)

Rotating black holes

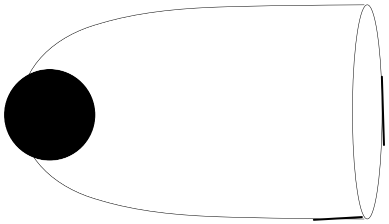
- In 2-deriv theory include spin via $d\omega^+$

  Doesn't feed into rest of solution.
- With HD, spin does affect rest of solution (attractor moduli, etc.)
- Entropy formula for $K3 \times T^2$

$$S = 2\pi \sqrt{\frac{1}{2} c^{ij} q_i q_j (q_1 + 3) - \frac{(q_1 - 1)(q_1 + 3)}{(q_1 + 1)^2} J^2}$$

q_1 = M2-brane on T^2

Black holes on Taub-NUT



asymptotically $R^3 \times S^1$

- Moduli independence of entropy leads to 2-deriv formula:

$$S_{5D}(q_I; J) = S_{4D}(q_I; q_0 = J; p^0 = 1)$$ (Gaiotto, Strominger, Yin)

- Modified in HD theory. For static hole:

$$S_{5D}(q_I) = S_{4D}(q_I - \frac{c_{2I}}{24}; p^0 = 1)$$

delocalized charge from $c_{2I} A^I \wedge \mathrm{Tr} R \wedge R$ term

Spinning black hole on $K3 \times T^2$:

$$S_{5D}(q_1; q_i; J) = S_{4D}(q_1 - 1; q_i; q_0 = \frac{q_1 + 3}{q_1 + 1} J; p^0 = 1)$$

5D strings

- Branch of solutions with null $2\circ\,{}^{1}\ 2$

$$ds^2 = e^{2U}(dt^2 \text{ ¡ } dx_4^2) \text{ ¡ } e^{i\ 4U}(dr^2 + r^2 d\text{-}^2)$$

$$F^I = \text{ ¡ } \tfrac{1}{2} p^I\ {}^2{}_{S^2}$$

- Now have:

$$e^{i\ 6U} = \tfrac{1}{6} c_{IJK} H^I H^J H^K + \tfrac{c_{2I}}{24}(r\ H^I\ \text{¢}r\ U + 2H^I r\ {}^2 U)$$

$$H^I = H_1^I + \frac{p^I}{r}$$

- Find solution interpolating between flat space and near horizon $AdS_3 \text{ £ } S^2$

$$\grave{}_s = \tfrac{1}{2}\grave{}_A = \tfrac{1}{2}(\tfrac{1}{6} c_{IJK} p^I p^J p^K + \tfrac{1}{12} c_{2I} p^I)^{1=3}$$

Central charges

- AdS_3 factor implies symmetry algebra

$$Vir_L \times Vir_R$$ (Brown, Henneaux)

- In general $c_{L,R}$ determined by c-extremization (P.K., Larsen) but given (0,4) susy just need CS terms:

$$c_{IJK} A^I \wedge F^J \wedge F^K + c_{2I} A^I \wedge \mathrm{Tr} R \wedge R$$

•Gives:

$$c_L = c_{IJK} p^I p^J p^K + c_{2I} p^I ; \quad c_R = c_{IJK} p^I p^J p^K + \tfrac{1}{2} c_{2I} p^I$$

agrees with MSW

•Unlike other results, these are not corrected by $R^4, \ldots$ terms.

Heterotic F-strings

- For $K3 \times T^2$ with N M5-branes on K3 find

$$c_L = 24N\,; \quad c_R = 12N$$

  - indeed, het. – IIA duality maps this to N heterotic F-strings with string scale $AdS_3 \times S^2$ and $g_s \sim \frac{1}{\sqrt{N}}$

- But now expect (0,8) susy rather than (0,4), so central charges not necessarily exact (or even correct.)

Heterotic F-string cont.

- Anomaly analysis yields:

$$c_L - c_R = 12N$$

$$k_R^{SU(2)} = 2N$$

- For N=4 algebra have $c_R = 6k_R^{SU(2)} = 12N$, but now need to know which N=8 algebra is realized.
- Superisometry group of 5D solution is

$$OSp(4^*|4) \quad \text{(Lapan, Simons, Strominger)}$$

- Natural guess: $\widehat{OSp}(4^*|4)$ nonlinear superconformal algebra (Lapan, Simons, Strominger), (P.K., Larsen, Shah)

Nonlinear superconformal algebras

- OPEs have quadratic nonlinearities:

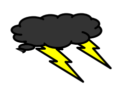
$$GG \sim 1 + T + J + \partial J + JJ$$

- Arises naturally in AdS_3 CS supergravity (Henneaux, Maoz, Schwimmer)
- Jacobi identities imply nonlinear central charge formulas:

level of R-current algebra

$$c(k) = a_0 k + a_1 + \frac{a_2}{k} + \cdots$$

- Since $k \sim N \sim \frac{1}{g_s^2}$ these are string loop effects. So quantum gravity effects are determined algebraically in this theory.
- For $\widehat{OSp}(4^*|4)$ find:



$$c_R = -(12N + 18)$$ non-unitary

Comments on F-string holography

- Standard type reasoning leads to decoupling limit:

$$g_s \to 0\,; \quad N \to \infty\,; \quad g_s^2 N \text{ fixed}$$

(recall solution had $g_s \sim \frac{1}{\sqrt{N}}$)

- Leads to duality between full solution and theory with explicit source strings.
- Unlike D-brane case, low energy limit leads to free CFT.

string scale AdS_3 dual to free CFT_2 ?

many puzzles. (see Dabholkar, Murthy)

Loose ends

- Black rings
- Higher dimensions
- Worldsheet CFT for F-string solutions
- Make sense of non-unitarity of nonlinear algebra, or find actual symmetry algebra.

THE END