Gauss' Law, Observables, and Black Hole Information
or
Information Conservation in Quantum Gravity

Don Marolf 6/19/08
UCSB

What is there to do?

- D-branes, AdS/CFT indicate unitarity.
- AdS/CFT as example of holography

But how does it work?

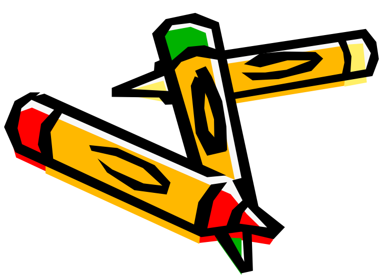
Mechanism for unitarity?
Bulk gravity explanation?
What's wrong with the usual
perturbative treatment?

Warning: Rehash of old chestnut.

Claims to strike gold
in much-searched terrain.

Claims:

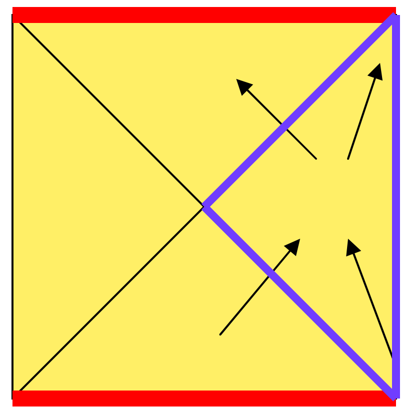
- Despite black holes & causality, unitary evolution should be expected in any theory of quantum gravity.
- *Just* GR & QM! No need for stringy effects or other new non-localities.
- Little to do w/ BH evaporation or resolving singularities. Argument is "more holographic." Info is *always* available at infinity.
- Applies to both full theory and perturbation theory about a black hole background.



- Key point: Proper treatment of observables and gauge invariance.

Unitarity & Black Holes (AdS)

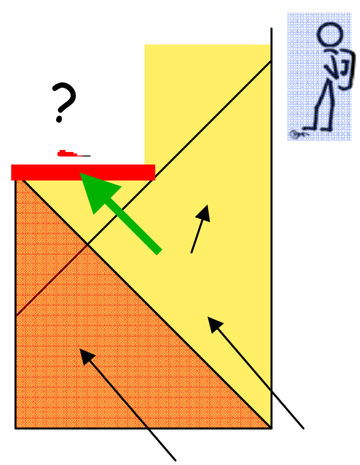
QFT on a fixed black hole background



"Non-unitary evolution"

though system conserves information, so long as one includes all degrees of freedom

Evaporating black hole?



What happens to the green information?

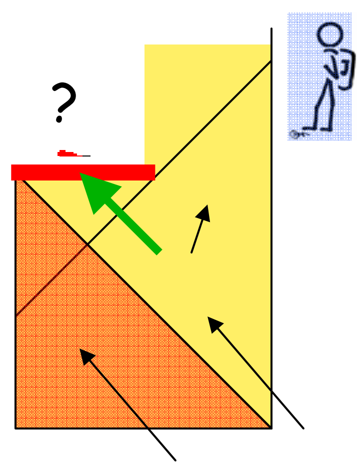
Is it accessible to a (somewhat) late-time external observer (blue)?

Or, do other states carry it away? (for a long time)

Unitarity & Black Holes (AdS)

Def of Unitarity in AdS: Info encoded in Ψ by using boundary observables at t_1 can be recovered using boundary observables at t_2 (for reasonable separations in time).

Evaporating black hole?



What happens to the green information?

Is it accessible to a (somewhat) late-time external observer (blue)?

Or, do other states carry it away? (for a long time)

Combine BPS + BMR?

Consider a theory with:

1. $A_{\text{bndy obs}}(t)$ = algebra of boundary observables at time t
2. Self-adjoint $H = H(t) \in A_{\text{bndy obs}}(t)$

   Conservation of E, Bndy Term

3. H generates time translations for Observables

Assumes unitarity on *some* Hilbert space.

Note: H is a boundary term in classical GR!

In fact, "separate Hamiltonians" for each boundary component.

Above are also true in perturbative Quantum Gravity!

…and in AdS/CFT.

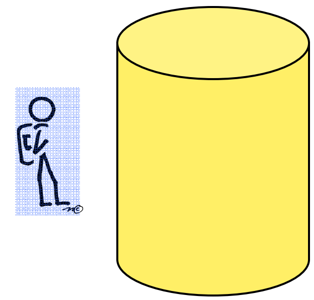
Work Backward

Q: At time t_1, what info cannot be read out of Ψ using $A_{\text{bndy obs}}(t_1)$?

A: Info associated with $\mathcal{O}$ such that

$$[\mathcal{O}, A_{\text{bndy obs}}(t_1)]=0$$

$$\rightarrow [\mathcal{O}, H]=0.$$



Q: What happens at time t_2?

A: $$[\mathcal{O}, A_{\text{bndy obs}}(t_2)] = [\mathcal{O}, e^{iH\Delta t} A_{\text{bndy obs}}(t_1)\, e^{-iH\Delta t}]$$
$$= e^{iH\Delta t} [\mathcal{O}, A_{\text{bndy obs}}(t_1)]\, e^{-iH\Delta t}$$
$$= 0$$

$\rightarrow$ There is no time t_2 at which this info cannot be encoded by $A_{\text{bndy obs}}$!!

If an external observer encodes any info in Ψ, it remains available at the boundary at *any* time t.

To Do

<u>Asymptotically Flat Case:</u>

Theorem applies directly.

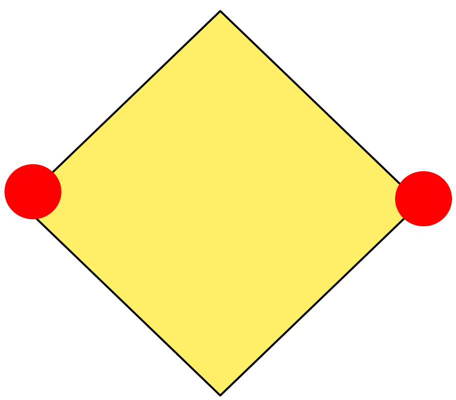
But physics less clear:

What are useful bndy obs?

Relation to “external observers?”

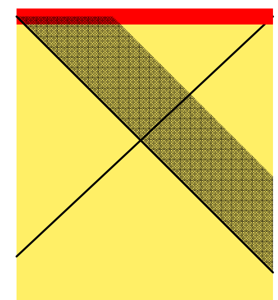
Unitary S-matrix?

How well can cut-offs be controlled?



Perturbation Theory

E.g., collapsing 4d black hole background:



Gravitational Gauss' Law

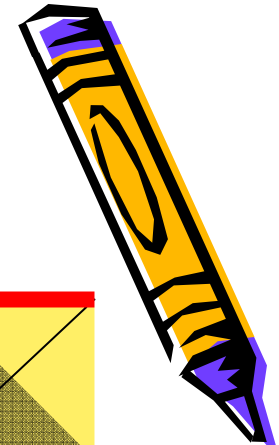
$$0 = \mathcal{H} = \pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2 - {}^3R + \rho.$$

$$g_{ij} = \tilde{g}_{ij} + \ell_p h_{ij} \qquad \ell_p^{-1}\tilde{D}_i v^i = \delta\rho + \delta\rho_{\text{lin grav}} + \delta\rho_{int} \qquad v^i = (\tilde{D}_j h^{ij} - \tilde{D}^i h)$$

$$\begin{aligned} H &= \ell_p^{-2}\int_{\partial\Sigma} r^i \left(\tilde{g}^{jk}\tilde{D}_j g_{ik} - \tilde{g}^{jk}\tilde{D}_i g_{kj}\right) + sources \\ &= \tilde{E} + \ell_p^{-1}\int_{\partial\Sigma} r^i v_i + sources \end{aligned}$$

i.e., $\ell_p \delta H := \int_{\partial\Sigma} r^i v_i + sources = \ell_p H_{lin} + O(\ell_p^2)$

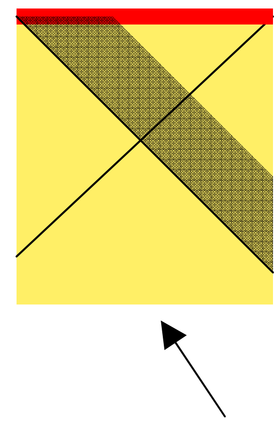
Perturbation Theory



$$\ell_p \delta H := \int_{\partial\Sigma} r^i v_i + sources = \ell_p H_{lin} + O(\ell_p^2)$$

At this order $\mathcal{O} = \mathcal{O}_0 + \ell_p \mathcal{O}_1$ commutes with $A_{\text{bndy obs}}(t)$ only if $[\mathcal{O}, H_{lin}] = O(\ell_p)$.

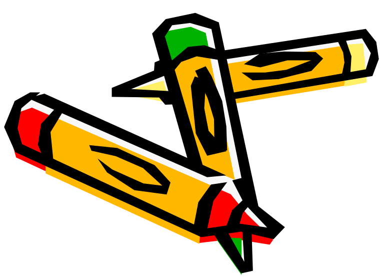
i.e., $[\mathcal{O}_0, H_{lin}] = 0$.



Pure AdS

Also need $[\mathcal{O}_0, A_{\text{lin bndy obs}}(t)] = 0$...
for all t, including pure AdS past.

But there aren't any!



At this order pert. grav. *is* "holographic."

Summary

- Despite black holes & causality, unitary evolution should be expected in quantum gravity.

- *Just* GR & QM! No need for stringy effects or other new non-localities.

- Little to do w/ BH evaporation.
  Argument is "more holographic."
  Info is *always* available at infinty.

- Applies to both full theory and perturbation theory about a black hole background.
  Such pert. theory *is* holographic at 2nd order.

- Key point: Proper treatment of observables in GR.