A black hole solution of scalar field theory and the RHIC fireball

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SUMMARY

1. Introduction

2. "Black hole" solution of toy scalar field theory: "pionless hole"

3. Heisenberg's model for Froissart saturation and its AdS-CFT dual

4. A simple model for RHIC collisions and its dual

1. Introduction

- Hawking $\longrightarrow$ proof of black hole radiation led to the black hole information paradox: quantum information "swallowed by horizon," only mixed state out

- **Paradox** (not merely a problem): Info "loss" $\longrightarrow$ only horizon needed, but horizon doesn't have transplanckian curvatures

  $\longrightarrow$ should be able to use QFT in curved spacetime (?)

- Usual answer: info is retrieved (mostly) from the late stages of decay, by a quantum gravitational process

  $\longrightarrow$ many problems with it.

- Also, known (toy model) solutions seem to suggest this picture, but with caveats:

•Maldacena-Strominger (hep-th/9609026): Stringy **unitary** decay of near-extremal black holes. $\longrightarrow$ info gets out in greybody factors

But! Is quantum gravity (string theory) essential to this, or rather the *scattering formalism*? (black hole decay = scattering). Also $\longrightarrow$ info leaks out from the begining, when we still are in the semiclassical region.

•Fuzzball picture $\longrightarrow$ also in quantum gravity (string theory) context. $\longrightarrow$ in as sense, ignore quantum mechanics: have a (semi)classical picture $\longrightarrow$ avoid paradox, concentrate on info retrieval.

Is the quantum gravity aspect essential?

•But perhaps instead:

•**It is a failure of formalisms for finite temperature in QFT**.

There is no good formalism to deal with the formation of $T \neq 0$ objects (mixed state) out of the collision of $T = 0$ quantum objects (pure state)

$\rightarrow$ only formalisms $\rightarrow T =$ const. in $t, \vec{x}$.

•Yet, it is the same process that happens *experimentally* at RHIC!

•Two $T = 0$ quantum objects (Au nuclei) collide and form a $T \neq 0$ plasma (sQGP) that decays.

•Not merely a classical "thermalization" of the $\sim$ 200 nucleons (i.e., collide systems of classical balls at rest $\Rightarrow$ thermalize)

$\longrightarrow$ tens of thousands of emittted *pions*

$\longrightarrow$ info is lost on what kind of hadrons (n,p,etc.) collided! (same kind of info lost inside a black hole)

•And we can't "thermalize quarks" (they have no free classical state)

$\Rightarrow$ We have "**aparent** info loss" at RHIC

- But QCD is unitary $\Rightarrow$ the finite T formalism is flawed $\rightarrow$ need better one.

- We should be able to find a field theory analog of the black hole $\rightarrow$ find toy model.

- See that it is also a toy model for RHIC.

2. Black hole solution of toy scalar field theory = "pionless hole".

•Already exists nonrelativistic example of "black hole-like" solution:

Unruh's "dumb holes" (sound absorbing)

•Nonrelativistic irrotational ($\nabla \times \vec{v} = 0$) fluid flow moving at ultrasonic speeds: "Dumb holes".

•Surface of v=c acts as the thermal horizon of a black hole

•Parameters: $\vec{v} = \vec{\nabla}\Phi$, ρ, sound speed $c^2 = dp/d\rho$.

•Fluctuation equation for the potential, $\phi = \delta\Phi$, describing the fluid is

$$\partial_0 \frac{\rho/c^2}{1 - v^2/c^2} \partial_0 \phi + \partial_i \rho (\frac{v^i v^j}{c^2} - \delta^{ij}) \partial_j \phi = 0$$

•Matches the fluctuation equation of scalar field in

$$ds^2 = \frac{\rho}{c}[(c^2 - v^2) d\tau^2 - (\delta^{ij} + \frac{v^i v^j}{c^2 - v^2}) dx^i dx^j]$$

•Following Hawking's calculation, the temperature of the thermal horizon at v=c is

$$T = \frac{k}{2\pi} = \frac{1}{4\pi} \frac{1}{\rho} \partial_r [\rho c (1 - v^2/c^2)]|_{v=c}$$

$\longrightarrow$ very useful description, especially since now there are several proposals for gravity duals to (relativistic, turbulent) fluid flow $\longrightarrow$ there should be a relation.

•Look for a solution to a relativistic Lagrangean of type

$$\mathcal{L}(\phi, (\partial_\mu \phi)^2) = \mathcal{L}(\phi, -\dot{\phi}^2 + (\vec{\nabla}\phi)^2)$$

with a spherically symmetric solution $\phi(r)$.

•The fluctuation equation in this background is of type

$$-A\delta\ddot{\phi} + \nabla_i A_{ij} \nabla_j \delta\phi - M^2 \delta\phi = 0$$

•Existence of horizon $\Rightarrow$

$\longrightarrow |\phi'|/|\phi| \to \infty \Rightarrow \mathcal{L}_{horizon} \sim [(\partial_\mu \phi)^2]^n$.

$\longrightarrow$ velocity of propagation of perturbations $\to$ 0 at horizon: $A_{ij}/A \to 0 \Rightarrow$ n=1/2.

•Then $\mathcal{L}_{horizon} \simeq \sqrt{(\partial_\mu\phi)^2}$.

•Identification map to Unruh's dumb holes (so that the fluctuation equation for ϕ matches)

$$\rho = 2\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi)^2}; \quad c = \frac{1}{\sqrt{1+2(\nabla\phi)^2(\frac{\delta^2\mathcal{L}}{\delta((\partial_\mu\phi)^2)^2}/\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi)^2})}}$$

$$v^i = \nabla^i\Phi\frac{\sqrt{-2(\frac{\delta^2\mathcal{L}}{\delta((\partial_\mu\phi)^2)^2}/\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi)^2})}}{\sqrt{1+2(\nabla\phi)^2(\frac{\delta^2\mathcal{L}}{\delta((\partial_\mu\phi)^2)^2}/\frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi)^2})}}$$

•Temperature of horizon is $\neq 0, \infty$

$$\Rightarrow \mathcal{L} = \sqrt{(\partial_\mu\phi)^2 + 1} + \phi g(r)$$

$\rightarrow$ DBI action with a source term $g(r)$! Will find constraints on $g(r)$ later.

Scalar DBI action (with δ function source)

$$S = \beta^{-2} \int d^4x [\sqrt{1 + \beta^2 (\partial_\mu \phi)^2} - 1] + \int d^4x \phi (\bar{C} \delta(r))$$

has "catenoid" solutions (analog of BIon solution of electro-magnetic BI action)

$$\phi(r) = \bar{C} \int_r^\infty \frac{dx}{\sqrt{x^4 - \beta^2 \bar{C}^2}}$$

$\rightarrow$ aparent singularity at $r_0 = \sqrt{\beta \bar{C}}$: $\phi' \rightarrow \infty, \phi_0$=finite.

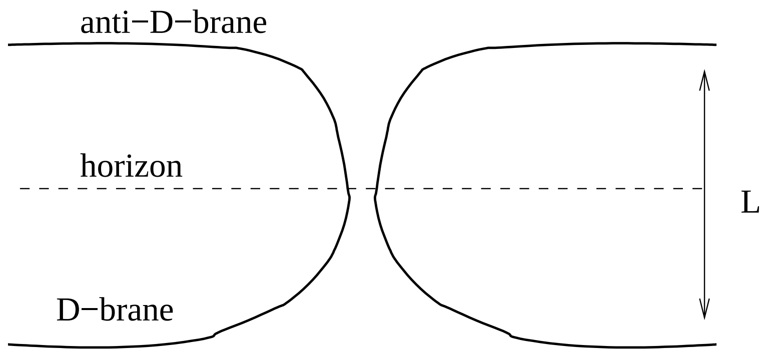
•$\bar{C}$ = scalar field charge.

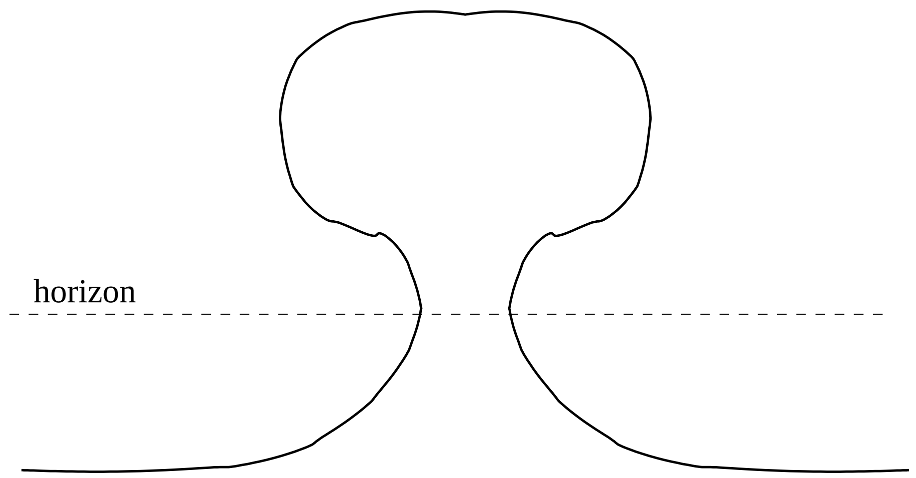
•δ function behind horizon $\rightarrow$ sometimes we drop it.

•Unlike BH, behind horizon - no continuation to singularity, only to different asymptotic space $\longrightarrow$ Einstein-Rosen bridge (which is a different 3d foliation of the 4d Schwarzschild black hole)

- brane-antibrane

- "bubble"


anti−D−brane
horizon
L
D−brane


horizon

•But like black hole, this solution can be formed by e.g., collapse of a spherical shell of ϕ field, or in a high energy collision.

•In $\phi(r)$= position of a brane (radion), horizon: brane creates a tube $\rightarrow$ clearly nonsingular!

•$r = r_0$ is a thermal horizon. Map to fluid parameters

$$c^2 = 1 + (\nabla\phi)^2; \quad \rho = \frac{1}{\sqrt{1 + (\nabla\phi)^2}}; \quad \vec{v} = \vec{\nabla}\phi$$

•Unfortunately,

$$T_{horizon} = \frac{k}{2\pi} \sim \partial_r g_{tt} \rightarrow \frac{dv}{dr}|_{v=c} \rightarrow \infty \quad \text{now!}$$

•How about info?

•Time it takes for $\delta\phi$ to propagate to horizon: from

$$c_{ph}^2 = \frac{\omega^2}{k^2} \rightarrow \frac{6}{kr_0} \text{ as r} \rightarrow \text{r}_0 \quad c_{gr} = \frac{d\omega}{dk} \rightarrow \frac{1}{2}\sqrt{\frac{6}{kr_0}} \text{ as r} \rightarrow \text{r}_0$$

•In the effective metric

$$\int^{horizon} dt = \int^{horizon} \frac{dr}{c(1 - v^2/c^2)} \rightarrow \text{ finite}$$

•So we are close to black hole solution properties, but not quite (we need finite T, $c_{phase} \rightarrow 0$ and $c_{group} \rightarrow 0$ at horizon for all k, and $\int dt$ infinite at horizon).

Pionless hole

- $g(r)$= nucleon ($\bar{N}N$) distribution.

- Toy model: $g(r) = \alpha/r^2$. Then

$$\phi(r) = \int_r^\infty dx \frac{\bar{C} + \alpha x}{\sqrt{x^4 - (\bar{C} + \alpha x)^2}}$$

- For $\bar{C} = -\alpha^2/4$, horizon at $r_0 = \alpha/2$. There

$$\phi(r) \simeq \frac{\alpha}{2\sqrt{2}} \ln(r - r_0) \to \infty \text{ as } r \to r_0$$

- It has finite temperature, $T = \sqrt{2}/(\pi\alpha) = 1/(\sqrt{2}\pi r_0)$!

•Now: Time it takes a perturbation to reach a point r near r_0

$$t \sim \frac{\alpha}{2\sqrt{2}} \ln(r - r_0) \to \infty!$$

•Phase and group velocities of scalar pert. near r_0

$$c_{ph} = \frac{\omega}{k} \propto r - r_0 \to 0$$
$$c_{gr} = \frac{d\omega}{dk} \propto \sqrt{r - r_0} \to 0$$

•"Pionless" hole (absorbs pions) $\rightarrow$ traps scalar info at horizon and emits thermal radiation.

•Uniqueness:

$$\phi(r) = \int_r^\infty \frac{\bar{C} + (\int x^2 g(x) dx)}{\sqrt{x^4 - (\bar{C} + (\int x^2 g(x) dx)^2)}}$$

•Finite temperature at $r = r_0 \Rightarrow x = r_0$ is a minimum of

$$f(x) \equiv x^4 - (\bar{C} + (\int x^2 g(x) dx))^2 \quad \text{at} \quad \text{f} = 0$$

•$\Rightarrow g(x)$ decays faster than $1/x$ at ∞ (faster than a free field!)

Hawking's derivation: sketch and comments

•Only kinematics of gravity (curved space) used, not dynamics (Einstein's equation)

•$\longrightarrow$ The effective black hole metric

$$ds^2 = \frac{\rho}{c}[(c^2 - v^2)d\tau^2 - (\delta^{ij} + \frac{v^i v^j}{c^2 - v^2})dx^i dx^j]$$

is defined by the *scalar* field equation (not Einstein's equation!).

•Hawking $\rightarrow$ study propagation of scalar field in black hole background

$$\partial_\mu \sqrt{g} g^{\mu\nu} \partial_\nu \delta\phi = 0$$

•Then, Bogoliubov transformation: On $\mathcal{I}^-$ we have

$$\phi = \sum_i (f_i a_i + \bar{f}_i a_i^+)$$

whereas on $\mathcal{I}^+$ we have

$$\phi = \sum_i (p_i b_i + \bar{p}_i b_i^+ + q_i c_i + \bar{q}_i c_i^+)$$

and the relation between the two is

$$p_i = \sum_i (\alpha_{ij} f_j + \beta_{ij} \bar{f}_j)$$

The f_i ($f_{\omega lm}$) and p_i ($p_{\omega lm}$) have the expansions

$$f_{\omega lm} \sim \frac{1}{r\sqrt{\omega}} F_\omega(r) e^{i\omega v} Y_{lm}(\theta, \phi)$$

$$p_{\omega lm} \sim \frac{1}{r\sqrt{\omega}} P_\omega(r) e^{i\omega u} Y_{lm}(\theta, \phi)$$

•Here the null coordinates are

$$v = t + r_* = t + r + 2M \log |\frac{r}{2M} - 1|$$
$$u = t - r_* = t - r - 2M \log |\frac{r}{2M} - 1|$$

•In our case, that is the only difference now:

$$v = t + r_* \sim t + r + \frac{\alpha}{2\sqrt{2}} \ln(r - r_0)$$
$$u = t - r_* \sim t - r - \frac{\alpha}{2\sqrt{2}} \ln(r - r_0)$$

•Then α_{ij} and β_{ij} ($\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$) are calculated by continuing

$$e^{i\omega u} \to \exp(-i\frac{\omega}{k} \log \left(\frac{v - v_0}{CD}\right))$$

where the affine parameter on the horizon is written in terms of the surface gravity at the horizon as

$$\lambda = -Ce^{-\kappa u}$$

•**Assumptions:**

-∃ scalar field propagating in *effective* metric (no dynamics needed for the background!)

-∃ horizon with finite surface gravity (nothing needed for inside horizon!)

-we can use the geometric optics approximation up to arbitrarily high energies of $\delta\phi$

(sometimes $\rightarrow$ way out: $E > M_{Pl}$ virtual modes give quantum gravity by interacting with gravity background)

•But now: $S_{DBI+source}$ =full quantum effective action

→ as a toy model, can be assumed to be valid for all energies (in a pion theory without gravity), same as

$$S_4 = l^{-4} \int d^4x \sqrt{det(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}$$

is a good D3-brane action for any energy.

•Now, S_{DBI} = radion action:

$$\mathcal{L} = l_s^{-4} \sqrt{\text{det}(\partial_a X^\mu \partial_b X^\nu g_{\mu\nu})} = l_s^{-4} \sqrt{g} \sqrt{1 + l_s^4 (\partial_\mu \phi)^2}$$

→ is trivially valid for any "energy" → geometric D-brane action (volume)! $|\partial_\mu X^a|$ is a coordinate choice on the D-brane!

•Now: better than Hawking (gravity case)

$\longrightarrow$ Both background ϕ_0 and $\delta\phi$ come from the same effective action S

$\longrightarrow$ No need to speculate on the interaction of virtual $E > M_P$ scalar modes with gravity

$\longrightarrow$ Everything well defined.

3. Heisenberg's model for Froissart saturation and its AdS-CFT dual

•Simple model that gives saturation of the Froissart (unitarity) bound for $\sigma_{tot}(s)$ in QCD. As $s \to \infty$,

$$\sigma_{tot}(s) \leq \frac{\pi}{m_\pi^2} \ln^2(s/s_0)$$

•Hadrons scattering Lorentz contract

$\to$ $\exists$ also pion field Lorentz contracting

$\to$ limit: collision of pion field shockwaves

$$\phi = \phi_1(x^i)\delta(x^+) + \phi_2(x^i)\delta(x^i)$$

for $x^\pm \leq 0$. Here $\phi =$ pion field wavefunction.

•Energy loss proportional to

•total energy

•overlap of pion wavefunctions

$$\mathcal{E} = \alpha\sqrt{s}, \alpha = e^{-bm_\pi} \quad \Rightarrow e^{-b_{max}m_\pi}\sqrt{s} =< E_0 > \quad \Rightarrow \sigma_{tot} = \frac{\pi}{m_\pi^2} ln^2 \frac{\sqrt{s}}{< E_0 >}$$

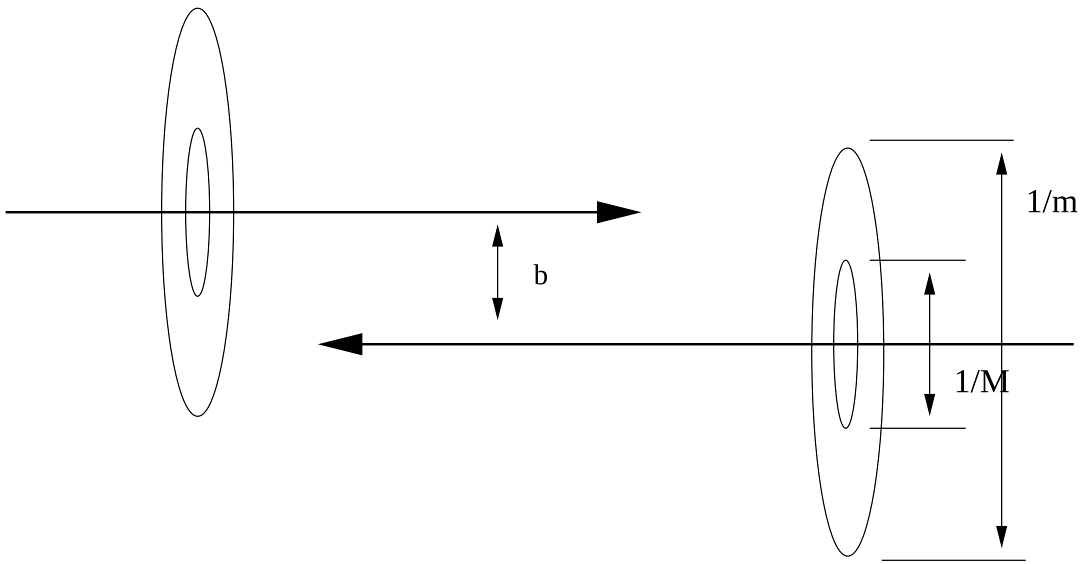
•But $< E_0 > \simeq$ constant only for

$$S = l^{-4} \int d^4x \sqrt{1 + l^4[(\partial_\mu\phi)^2 + m^2\phi^2]}$$

$$< E_0 > = \frac{\mathcal{E}}{n} = m_\pi \frac{\ln\gamma}{1 - 1/\gamma} \simeq ct.$$

•For $\lambda\phi^4$ or free theory,

$$< E_0 > = \gamma m_\pi \frac{1}{\ln\gamma} \propto \sqrt{s} \Rightarrow \sigma = \text{constant.}$$



b
1/m
1/M

•Note that the action needs to be valid at $(\partial_\mu \phi)^2 \gg l^{-4}$ again!

•Thus this is a good high energy ($s \to \infty$) scattering $\to$ should apply to RHIC.

•But this is a radion action

•And mass term $m^2 \to$ nonlinear generalization of radion stabilization. Linear:

-Flux stabilization for Polchinski-Strassler

-Golberger-Wise stabilization for RS

•Simple dual model for QCD with pions = lightest:

AdS with cut-off $\bar{r}_{min} = R^2 \Lambda_{QCD}$ (gravity $\leftrightarrow$ glueballs) and the cut-off promotedf to a (IR) brane: radion $\leftrightarrow$ pion (toy model for behaviour of IR fields).

•High energy scattering in the dual model $\longrightarrow$ infinite s limit found to be:

•Gravitational shockwave scattering on (IR) brane, producing black holes.

•Gravitational (shockwave) wavefunction on IR brane:

$$\Phi = R_s \sqrt{\frac{2\pi R}{r}} C_1 e^{-M_1 r}; \quad M_1 = \frac{j_{1,1}}{R} \sim \frac{3.83}{R}$$

$\longrightarrow$ find again: for M_1 lightest, exactly:

$$\sigma_{dual}(s) = \bar{K} \frac{\pi}{M_1^2} \ln^2 \frac{s}{s_0}$$

$\longrightarrow$ for m_{radion} ($\leftrightarrow m_\pi$) lightest, argument gives

$$\sigma_{dual}(s) \sim \frac{\pi}{m_{radion}^2} \ln^2 \frac{s}{s_0}$$

4. A simple (toy) model for RHIC collisions and its dual

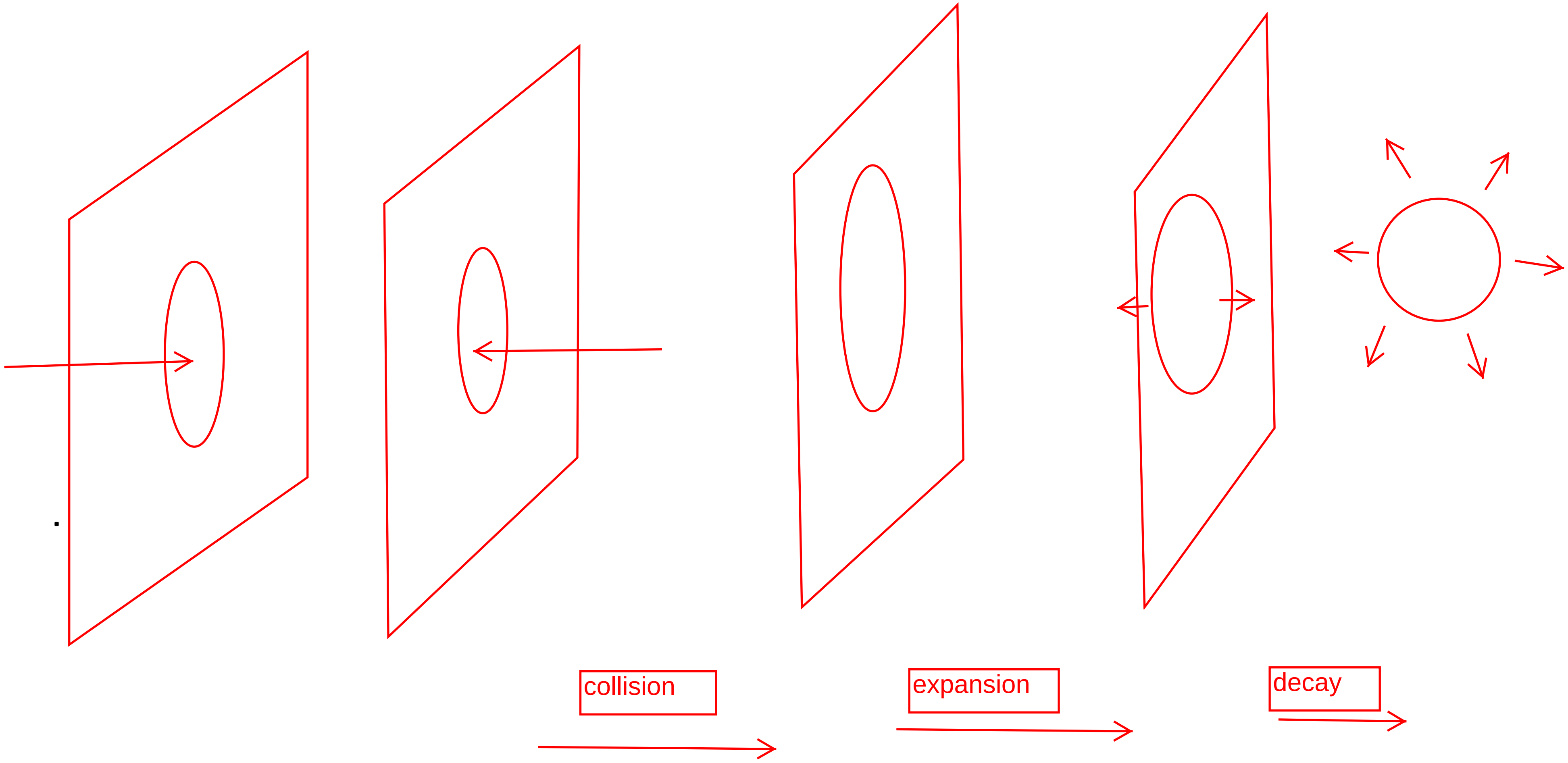
- Heisenberg: As $s \to \infty$, hadrons $\to$ δ function source of pion field.

- But: observe finite T ($\sim 175 MeV$) and aparent info loss $\leftrightarrow$ dual black holes produced.

- Found that we need source $g(r) < 1/r$ at ∞ to obtain that.

- Pion field with DBI action, and $g(r)$ standing in for a nucleon ($\bar{N}N$ condensate) source.

- Shockwaves of $S_{DBI} + S_{source}$: collision $\to$ expansion $\to$ decay.


collision
expansion
decay

•Intermediate stage $\sim$ spherical symmetry $\simeq$ pionless hole.

•In dual: same for radion shockwaves.

•If gravity only (no radion) $\Rightarrow$ gravitational shockwave collision:

trapped surface $\rightarrow$ expansion to black hole (on IR brane) $\rightarrow$ quantum decay.

•Aparent info loss in pionless hole $\leftrightarrow$ aparent info loss in black hole.

•But in both $\rightarrow$ no info loss.

•Need new finite T formalism in QFT and QGr to deal with finite T formation out of collision of $T = 0$ quantum objects.