Extremal black holes

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Extremality

- Killing horizon: null hypersurface whose normal is a Killing vector field V
- Event horizon of stationary BH is Killing horizon Hawking 72
- $V^2=0$ on horizon
- Horizon is extremal iff $d(V^2)=0$ too
- Implies vanishing surface gravity/temperature

Motivation

- BPS BH must be extremal
- Recent progress in statistical calculation of entropy of non-BPS extremal BHs via attractor mechanism
- Extremal BHs excluded from usual 4d uniqueness theorems

Motivation

- What are the stationary BH solutions of d>4 gravity?
- Myers-Perry, black rings, what else?
- Topology? Rotational symmetries?
- Supersymmetry or extremality simplifies problem, may be a guide to properties of general solutions

Strategy

- A near-horizon limit can be taken of any extremal black hole solution
- Near-horizon solution simpler than full BH - “radial” dependence determined
- Aim: determine all NH solutions of given theory
- NH solution reveals geometry (hence topology, symmetries) of horizon of full BH solution
- Try to use NH classification to obtain full classification of BPS/extremal BHs

Near-horizon geometry

- Introduce Gaussian null coordinates near extremal horizon (at r=0):

$$ds^2 = r^2 F(r,x) dv^2 + 2dvdr + 2r h_a(r,x) dv dx^a + g_{ab}(r,x) dx^a dx^b$$

- Take *near-horizon limit* HSR 02

$$r \to \epsilon r, \qquad v \to v/\epsilon, \qquad \epsilon \to 0$$

- r-dependence fixed:

$$ds^2 = r^2 F(x) dv^2 + 2dvdr + 2r h_a(x) dv dx^a + g_{ab}(x) dx^a dx^b$$

N=2, d=4 supergravity

- Minimal: Einstein-Maxwell theory
- General BPS solution: Israel-Wilson-Perjes family Tod 83
- Conjecture: all BPS BHs belong to Majumdar-Papapetrou subfamily Hartle & Hawking 72
- BPS NH solutions are $AdS_2 x S^2$ and $R^{1,1} x T^2$, latter excluded by topological censorship
- This is starting point for proof of conjecture (assuming V timelike outside horizon) Chrusciel, HSR & Tod 05

d=5 supergravity

- Minimal: Einstein-Maxwell Chern-Simons
- Canonical form for BPS solutions known Gauntlett et al 02 but not obvious how to construct BHs
- Classify BPS NH geometries: NH BMPV, $AdS_3 x S^2$, flat HSR 02
- Starting point for proof that BMPV is unique BPS spherical topology BH (assuming V timelike outside horizon)

BPS AdS_5 black holes

- First example discovered by determining NH solution Gutowski & HSR 04
- Most general known BPS black hole in AdS_5 has 4 parameters Kunduri, Lucietti & HSR 06
- BPS states in dual CFT have 5 independent charges
- Are there more general BHs? BPS black rings?

d=5 gauged supergravity

- Classify BPS NH solutions with 2 rotational symmetries (true for all known 5d BH solutions) Kunduri, Lucietti & HSR 06
- Unique regular solution is NH solution of known BPS BHs
- There is a black ring NH geometry but it has conical singularity
- Can eliminate singularity in d=10 by changing topology Gauntlett et al 06 - new asymp $AdS_5 x S^5$ BPS BHs with no 5d interpretation?

Static vacuum BHs

- Schwarzschild is unique
- Uniqueness theorem valid for disconnected horizons *assuming no extremal components* Bunting & Masood 86, Gibbons, Ida & Shiromizu 02
- d=4: unique static vacuum NH geometry is $R^{1,1}xT^2$, excluded by topological censorship - eliminates assumption Chrusciel, HSR & Tod 05

The attractor mechanism

- Extremal black holes obey attractor mechanism *assuming* NH geometry involves AdS_2 factor Sen et al 05-06
- E.g. NH extremal Kerr Bardeen & Horowitz 99

$$ds^2 = f(\theta)\left[-(1+Y^2)dT^2 + \frac{dY^2}{1+Y^2}\right] + g(\theta)d\theta^2 + h(\theta)\left(d\phi - cY\,dT\right)^2$$

- Isometry group SO(2,1) x U(1)
- But general NH geometry has only 2d non-abelian isometry group
- Will higher-derivatives spoil AdS_2 symmetry?

Near-horizon symmetries

Kunduri, Lucietti & HSR 07

- Static NH geometry: AdS_2 automatic
- Stationary NH geometry: *assume* existence of d-3 rotational symmetries d=4,5
- AdS_2 emerges from Einstein eqs for gravity coupled to abelian vectors & scalars with non-positive potential
- AdS_2 persists when higher-derivative terms included
- d>5 generalization (for vacuum) Figueras et al 08

Extremal vacuum BHs

- Can we determine most general NH geometry in vacuum?
- Reduces to solving $R_{ab} = \frac{1}{2} h_a h_b - \nabla_{(a} h_{b)} + \Lambda g_{ab}$ on compact d-2 manifold Lewandowski & Pawlowski 02, Chrusciel, HSR & Tod 05
- d=4 with axisymmetry: NH Kerr-AdS L&P 02, Kunduri & Lucietti 08

Extremal vacuum BHs

Kunduri & Lucietti 08

- d=5, two rotational symmetries, $\Lambda=0$
- 3 non-trivial solutions:
  - S^3: NH Myers-Perry/ergo KK BH
  - S^3: NH ergo-free KK BH
  - $S^1 x S^2$: NH black ring/Kerr string
- $\Lambda \neq 0$: reduces to 6th order ODE

Future directions

- Extend BPS NH classification to other theories
- *Prove* existence of rotational symmetry where we had to assume it *or* find examples with less symmetry: easier than for full BH solution
- Warm-up: d=4 vacuum