Harmonic analysis and its interactions: in honour of Tony Carbery

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International Centre for Mathematical Sciences, Edinburgh

Abstracts

Ball, Keith
Rational approximations to zeta
We construct a sequence of rational functions converging locally uniformly to zeta and satisfying a simple recurrence relation.

Beckner, William
On Lie groups and beyond - Kunze-Stein phenomena, $SL(2,R)$, the Heisenberg group, and the role of symmetry
Analysis arises on manifolds motivated by applications to the description of dynamical and physical phenomena. Breaking the classical Euclidean symmetry through growth of balls depending on negative curvature and non-uniform homogeneity, brings in to play two intrinsic Lie groups, $SL(2,R)$ and the Heisenberg group. Applications of symmetry in the study of Riesz potentials and fractional embedding on the group manifold highlights the path to deeper understanding of the mathematical interface between these two groups.

Bez, Neal
On the kinetic transport equation
Despite the simplicity of the linear kinetic transport equation, establishing certain desirable estimates gives rise to some interesting challenges. We will present some results in this direction, including the so-called Strichartz estimates for this equation and some smoothing estimates for the velocity average of the solution. Surprising interactions will emerge, including direct connections with certain extensions of the classical Strichartz estimates for the Schrödinger equation, and also with the cone multiplier operator.

Christ, Michael
Maximizers of certain multilinear functionals
An inequality of Brascamp-Lieb-Luttinger states that certain functionals are maximized, over tuples of one-dimensional sets with prescribed measures, by tuples of intervals centered at the origin. We show, under some natural hypotheses and some less natural ones, that up to symmetries of these functionals, there are no other maximizers. This inverse theorem extends results of Burchard, and of the speaker and Flock. Related results will be discussed, and the method of proof will be outlined. This is one facet of a subject to which A. Carbery has made seminal contributions.

Cowling, Michael
The Brascamp-Lieb inequality revisited
We review the Brascamp-Lieb inequality from the point of view of optimisation theory. In particular we show that the constants in the inequality vary in a Holder continuous manner with the data, and consider the polytope of indices for which the inequality holds.

Demeter, Ciprian
Recent developments in decoupling theory
I will review some of the recent developments in the field.

Gressman, Philip
Nonlinear geometric averages of multilinear and intermediate dimensional types
We will discuss a pair of recent results based on extensions of the method of refinements which allow one to prove almost-sharp L^p-improving estimates for nonlinear geometric averages of intermediate dimension and almost-sharp weighted estimates for nonlinear Loomis-Whitney inequalities.

Guth, Larry
Pointwise convergence estimates for the Schrödinger equation
Carleson asked how much regularity one needs for the initial data to guarantee that a solution of the Schrödinger equation converges pointwise almost everywhere to the initial data as t goes to zero. For a long time, it seemed plausible that convegence holds for H^s initial data when $s > 1/4$. Recently, more interesting counterexamples have been found. In 2 space dimensions, Bourgain gave an example showing that pointwise convergence can fail almost everywhere with H^s initial data for any $s > 1/3$. Du, Li, and I were able to show that this sharp up to the endpoint: if $s > 1/3$, then the solution converges to the initial data pointwise almost everywhere. The proof is based on two recent developments in restriction theory: polynomial partitioning and decoupling theory.

Hickman, Jonathan
Sharp bounds for oscillatory integral operators via polynomial partitioning
I will discuss recent joint work with Larry Guth and Marina Iliopoulou concerning sharp bounds for Hormander-type oscillatory integral operators.

Hytönen, Tuomas
The matrix-weighted frontier of sharp norm inequalities
There is by now a fairly complete picture of quantitatively sharp norm inequalities for standard Calderon-Zygmund operators in classical weighted spaces, as well as substantial partial extensions of this theory to rougher kernels. There are also several results on vector-valued matrix-weighted generalisations of the theory, but the sharp matrix-weighted bounds are so far only known in two cases: the maximal operator and the dyadic square function. I will discuss these and related recent results.

Iliopoulou, Marina
Sharp bounds for oscillatory integral operators via polynomial partitioning, part II
This talk will be a second part to Jonathan Hickman's talk, regarding our joint work with Larry Guth on Hormander-type oscillatory integral operators with positive definite phase. The talk will primarily consist of an overview of the proof.

Katz, Nets
Kakeya sets in three dimensions
We prove that the Hausdorff dimension of a Kakeya set in R^3 is at least $5/2 + \varepsilon$ for some universal $\varepsilon > 0$. This is joint work with Josh Zahl and our talks will be a series, both covering this topic.

Lee, Sanghyuk

Bilinear restriction estimates for surfaces with co-dimension bigger than 1

In connection with the restriction problem for hypersurfaces such as the sphere and paraboloid, the bilinear (adjoint) restriction estimates have been extensively studied. However, not much is known about such estimates for surfaces with codimension larger than one. Under certain curvature condition we show sharp bilinear $L^2 \times L^2 \to L^q$ restriction estimates for some class of surfaces with higher codimension. We also discuss associated restriction problems. This talk is based on a joint work with Jong-Guk Bak and Jungjin Lee.

Lie, Victor

The pointwise convergence of the Fourier series near L^1

In our talk we will discuss the old and celebrated question regarding the pointwise behavior of Fourier Series near L^1. This presentation will include

- the resolution of Konyagin's conjecture (ICM, Madrid 2006) on the pointwise convergence of Fourier Series along lacunary subsequences;
- the L^1-strong convergence of Fourier Series along lacunary sub- sequences;
- recent progress on the L^1-strong convergence of (the full) Fourier Series.

We end with several considerations on the relevance/impact of the above items on the subject of the pointwise convergence of Fourier series.

Mirek, Mariusz

Dimensional free estimates for maximal functions: discrete and continuous perspective

We prove the dimension-free bounds on $\ell^p(\mathbb{Z}^d)$ with $p > 3/2$ for the discrete maximal function associated with cubes in $\mathbb{Z}^d$. Using similar methods we also give a new simplified proof for the dimension-free bounds on $L^p(\mathbb{R}^d)$ with $p > 3/2$ for maximal functions corresponding to symmetric convex bodies in $\mathbb{R}^d$.

Müller, Detlef

Maximal functions associated to 2*-hypersurfaces with height below* 2

In this talk, I shall report on joint work with S. Buschenhenke, S. Dendrinos and I. Ikromov in which we continue the study of the problem of Lp-boundedness of the maximal operator M associated to averages along isotropic dilates of a given, smooth hypersurface S of finite type in 3-dimensional Euclidean space. An essentially complete answer to this problem (in terms of associated Newton diagrams) had been given about seven years ago in joint work of mine with I. Ikromov and M. Kempe for the case where the height h of the given surface in the sense of Varchenko is at least two. Here I shall discuss recent progress on the case where $h > 2$, which turned out to create a number of substantial new challenges.

Orponen, Tuomas

On the packing dimension of planar Furstenberg sets

Given $0 \leq s \leq 1$, a planar set K is called an s-Furstenberg set, if for every unit vector e, K contains an s-dimensional subset of some line parallel to e. How small can an s-Furstenberg set be? A theorem of Wolff states that a lower bound for the Hausdorff dimension is $2s$, but this is only known to be sharp for $s = 1$. For packing dimension, a small improvement of the form $(2s + \varepsilon)$ can be obtained for $s < 1$, and this is the main topic of the talk.

Ricci, Fulvio

Smoothness of multipliers versus smoothness of convolution kernels in the functional calculus of differential operators on Lie groups

For a homogeneous sub-Laplacian L on a nilpotent Lie group and a bounded multiplier m on the half-line, it is known that the convolution kernel K_m of the operator $m(L)$ is a Schwartz function if and only if m itself is the restriction of a Schwartz function. For multivariate functional calculus on a commuting family of operators which includes a sub-Laplacian, the appropriate reformulation of the "if" implication still holds. The "only if" implication does not hold in general, but is known to hold for rather general classes of operators exhibiting special invariance properties. The problem of a correspondence between Schwartz multipliers and Schwartz kernels can be posed more generally on groups of polynomial volume growth. We present some positive examples on motion groups in low dimensions.

Rogers, Keith

Necessary conditions for pointwise convergence of the Schrödinger equation

We will consider solutions to the Schrödinger equation on R^{n+1} with initial data in Sobolev spaces with s derivatives in L^p. In one dimension, Carleson proved that the solution converges almost everywhere to its data, as time goes to zero, as long as $s \geq 1/4$. Dahlberg and Kenig then showed that this is not the case for certain data with less regularity. It was thought that $s \geq 1/4$ might also be the correct threshold in higher dimensions, however this was disproved in a recent flurry of progress. After presenting the Dahlberg-Kenig example and another due to Lucà and myself (based on an example of Barceló, Bennett, Carbery, Ruiz and Vilela), I will show two ways to combine the examples to prove that $s \geq n/2n + 2$ is in fact necessary. This was first proved by Bourgain. Du, Guth and Li have since shown that $s > n/2n + 2$ is sufficient when $n = 2$, and so this could be correct threshold in all dimensions.

Seeger, Andreas

Almost everywhere convergence results for radial multipliers

In their 1988 paper Carbery, Rubio de Francia and Vega established optimal almost everywhere convergence results for Bochner-Riesz means on $L^p(R^d), p > 2d/(d-1)$, by proving almost sharp bounds on weighted L^2spaces, with power weights. We shall discuss various related endpoint results, including a simple characterization of boundedness of radial convolution operators on $L^2(|x|^{-a}dx)$, for 1.

Stein, Elias

Dimension-free estimates in harmonic analysis

I will present a brief survey of dimension-free estimates, and then describe some recent results of Bourgain, Mirek, Wrobel, and myself in this area. These arise when we consider the r-variation version of maximal function estimates of averages over convex sets.

Stovall, Betsy

Endpoint bounds for weighted averages on polynomial-like curves

We will discuss joint work with Spyridon Dendrinos and Brian Street, in which we establish optimal Lebesgue space bounds for weighted averages on polynomial-like curves.

Tao, Terence

The entropy decrement method

We describe a technique to decouple random variables that we call the "entropy decrement method", which I was able to use to attack the Erdos discrepancy problem and study the statistics of multiplicative functions. Hopefully it will be applicable to other problems as well.

Thiele, Christoph

On the foundations of analysis

Honouring Tony Carbery's efforts to turn difficult matters into simple ones by throwing fresh ideas at them, we revisit the old subject of foundations of analysis and mathematics. We discuss a rather minimalistic model, which nicely illustrates a number of mathematical principles.

Wang, Hong

Bilinear decoupling in R^3

We prove a bilinear decoupling theorem in R^3. The main ingredient of the proof is orthogonality and induction on scales.

Zahl, Joshua

An improved bound on the Hausdorff dimension of Besicovitch sets in R^3

I will discuss a new bound on the Hausdorff dimension of Besicovitch sets in R^3: every Besicovitch set in R^3 must have dimension at least $5/2 + \varepsilon$, where $\varepsilon > 0$ is a small absolute constant. This is joint work with Nets Katz.

Zhang, Ruixiang

Endpoint perturbed Brascamp-Lieb inequalities

We will discuss some ideas behind our proof of endpoint perturbed Brascamp-Lieb inequalities using polynomial partitioning.