Positivity in algebraic and complex geometry

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Abstracts

Bertram, Aaron
Finite quot schemes
When the expected dimension of a Grothendieck scheme of quotients is finite, it has an expected length. This length, surprisingly, computes the dimension of the space of sections of a (positive) determinant line bundle on a moduli space of complexes. On a smooth curve, the length is thus given by the Verlinde formula. On surfaces, it seems to also be given by a Verlinde-type formula that has been investigated by Lothar Goettsche and coauthors. This is joint work with Thomas Goller and Drew Johnson.

Cascini, Paolo
Relative semi-ampleness in positive characteristic
We show that given an invertible sheaf on a fibre space between projective varieties defined over a field of positive characteristic, fibre wise semi-ampleness coincides with relative semi-ampleness. Joint work with H. Tanaka.

Castravet, Ana-Maria
The derived category of moduli spaces of stable rational pointed curves
I will report on joint work with Jenia Tevelev on Orlov's question on exceptional collections on moduli spaces of pointed stable rational curves.

Collins, Tristan
The deformed Hermitian-Yang-Mills equation
Mirror symmetry predicts that the moduli space of complex structures/ special Lagrangians on one Calabi-Yau is dual to the moduli space of complexified forms/ stable bundles on the mirror Calabi-Yau. However, the precise definition of a complexified Kähler form/stable bundle has remained mysterious. I will discuss these notions in the setting of Strominger-Yau-Zaslow mirror symmetry, the connection to fully nonlinear PDEs and algebro-geometric stability. This is joint work with A. Jacob and S.-T. Yau.

Dervan, Ruadhaí
A Harder-Narasimhan filtration for unstable Fano varieties
Analogously to the Hitchin-Kobayashi correspondence, it is now known that a Fano manifold admits a Kähler-Einstein metric if and only if it is "stable". We consider Fano manifolds that do not admit a Kähler-Einstein metric. Motivated by the existence of the Harder-Narasihman filtration on an unstable vector bundle, we prove the existence of a "most destabilising" test configuration, with respect to a new stability notion. Our proof is analytic, rather than algebraic. This is joint work with Gábor Székelyhidi.

Ein, Lawrence
Positivity of the conormal sheaves and singularities
We'll describe several examples how to use positivity of the conormal sheaves of the fibers of a resolution to understand the singularities a space.

Fulger, Mihai
Positive cones of numerical cycle classes
I will present progress on problems concerning the structure of the pseudoeffective cone of cycle classes of projective varieties.

Höring, Andreas
The fundamental group of compact Kähler threefolds
In this talk I will report on some recent progress on the bimeromorphic Kodaira problem for compact Kähler threefolds, i.e. the question whether every compact Kähler threefold has a bimeromorphic model that is deformation equivalent to a projective variety. In particular I will explain that the answer is positive on the level of fundamental groups. This is joint work with Benoit Claudon.

Kebekus, Stefan
Varieties with vanishing first Chern class
A famous "decomposition theorem" asserts that any projective manifold with vanishing first Chern class admits an étale cover that decomposes as a product of an Abelian variety, simply-connected Calabi-Yau manifolds, and irreducible holomorphic-symplectic (IHS) manifolds (also called hyperkähler manifold). The three classes can be distinguished by looking at their algebra of holomorphic differential forms. In the Minimal Model Program, we are forced to work with singular varieties. I survey recent progress towards a decomposition in the singular case, and argue that an analogue of the dichotomy Calabi-Yau/IH exists in this more general setting. This is work in progress with Henri Guenancia and Stefan Kebekus.

Lehmann, Brian
Geometric Manin's Conjecture and rational curves
Manin's Conjecture predicts that the number of rational curves on a projective variety is controlled by the curvature of the canonical bundle. I will discuss partial progress toward formulating and proving a version of this conjecture over C. This is joint work with Sho Tanimoto.

Lozovanu, Victor
Syzygies on algebraic surfaces
The aim of the talk is to explain how, by exploring the local positivity at a point, one can obtain strong syzygetic properties in terms of geometry for algebraic surfaces with trivial canonical class.

Maclean, Catriona
Fujita theorems and approximable algebras
Huayi Chen introduced the notion of approximable algebras in order to prove Fujita-type theorems in the arithmetic context, and asked the question if such algebras are always sub algebras of section rings. We give a counter example showing this is not the case.

Macrì, Emanuele
Derived categories of cubic fourfolds and non-commutative K3 surfaces
The derived category of coherent sheaves on a cubic fourfold has a subcategory which can be thought as the derived category of a non-commutative K3 surface. This subcategory was studied recently in the work of Kuznetsov and Addington-Thomas, among others. In this talk, I will present joint work in progress with Bayer, Lahoz, and Stellari on how to construct Bridgeland stability conditions on this subcategory. This proves a conjecture by Huybrechts, and it allows to start developing the moduli theory of semistable objects in these categories, in an analogue way as for the classical Mukai theory for (commutative) K3 surfaces. I will also discuss a few applications of this result.

Martinez-Garcia, Jesus
Positivity of the twisted CM line bundle for log pairs
We introduce a dynamic twisted CM line bundle for pairs depending on one parameter, generalizing Paul-Tian CM line bundle for varieties. We use this dynamic twisted CM line bundle to show that K-stable polarized log pairs are GIT stable. We apply our setting to study moduli of log pairs formed by a hypersurface and a hyperplane section and discuss the positivity of our line bundle. We will illustrate the setting by discussing the case of log pairs formed by a cubic surfaces and anti-canonical linear section. This is joint work with Patricio Gallardo and Cristiano Spotti.

Naumann, Philipp
Curvature of higher direct image sheaves
The differential geometric properties of the classical Hodge bundles were first studied by Griffiths in the context of the period map and variation of Hodge structures. This can be used to show the hyperbolicity of the moduli space of polarized Calabi-Yau manifolds. In the talk we consider generalized Hodge bundles which are twisted by a relative ample line bundle. An intrinsic curvature formula can be given. This generalizes a result of Berndtsson on the Nakano positivity of the direct image of the ample twisted relative canonical bundle of a fibration as well as the curvature formula for higher direct images of Schumacher in the canonically polarized case.

Ottem, John
Positivity of the diagonal
A natural approach to classifying varieties X is via the positivity properties of the diagonal in the self-product X x X. For instance, if the tangent bundle of X is nef, then the diagonal is nef (as a cycle). We analyze when the diagonal is big, movable or nef, with special emphasis on the case of surfaces. We also give several criteria for establishing rigidity of the diagonal. This is joint work with Brian Lehmann.

Patakfalvi, Zsolt
On projectivity of the moduli space of stable surfaces in characteristic p > 5
Stable varieties are higher dimensional generalizations of stable curves. Their moduli space contains an open locus parametrizing varieties of general type up to birational equivalence, just as the space of stable curves contains the space of smooth curves in dimension one. Furthermore, also similarly to the one dimensional picture, it provides a compactification of the above locus, which is known in characteristic zero but it is only conjectural in positive characteristic in dimension at least two. I will present the statement that every proper subspace of the moduli space of stable surfaces in characteristic greater than 5 is projective. This eventually boils down to proving a positivity result on Hodge type bundles.

Popa, Mihnea
Positivity for direct images of pluricanonical bundles
I will present recent results on direct images on pluricanonical bundles obtained jointly, in various combinations, with Hacon, Lombardi and Schnell. I will explain a number of applications to complex and birational geometry, with a heavy emphasis on morphisms to abelian varieties.

Toma, Matei
Moduli spaces of semistable sheaves with respect to irrational polarizations
Wall crossing phenomena for Gieseker-Maruyama stability over higher dimensional projective manifolds may give rise to purely irrational walls inside the ample cone. In recent work together with Daniel Greb and Julius Ross we show in the threefold case that projective moduli spaces of semistable sheaves with respect to irrational polarizations exist. The method uses GIT to first construct a moduli space of representations of suitable quivers. In this talk we present an alternative GIT-free construction method of such moduli spaces over projective manifolds which is supposed to generalize to Kähler manifolds as well. The method uses local moduli and stack theoretical results recently obtained by Alper, Hall and Rhyd. This is part of an ongoing research project together with Daniel Greb and Peter Heinzner.

Witt Nyström, David
Okounkov bodies and the Kähler geometry of projective manifolds
A typical polarized manifold will not admit any torus symmetries. Nevertheless, in this talk I will explain how using Okounkov bodies any polarized manifold can (in a precise sense) be approximated by torus-invariant domains in Euclidean space. In particular the domains can be chosen to be ellipsoids.

Xu,Chenyang
Stability on valuations of a singularity
In higher dimensional geometry, it has been known that from many perspectives a log terminal singularity is a local analogue of Fano varieties. Many statements of Fano varieties have a counterpart for log terminal singularities. One central topic on the geometry of a Fano variety is its stability which for instance reflects whether the Fano variety carries a canonical metric. In this talk, we will discuss a recent joint work with Chi Li (some part still in progress) in which we want to establish a local stability theory of a fixed log terminal singularity. Inspired by the study from differential geometry, (e.g. tangent cone, Sasakian-Einstein metric), for any log terminal singularity, we investigate the valuation which has the minimal normalized volume. Our goal is to prove various properties of this valuation which enable us to canonically degenerate the singularity to a T-singularity (i.e., with a torus action) with 'stability'.