The Disc-structure space

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Classically, one studies the (enriched) category of manifolds and their diffeomorphisms by first comparing them to a simplified category of block-manifolds and then comparing the latter to the category of Poincaré complexes. The information lost in both of these passages is encoded in certain structure spaces that are approximated in terms of K- and L-theory by pseudo-isotopy and surgery theory.

More recent developments related to factorisation homology and manifold calculus suggest a different approach, namely to compare the category of manifolds to a variant of the derived category of modules over the little d-discs operad. Again, this amounts to studying certain structure spaces that encode the difference: the Disc-structure spaces.

In this talk, I will explain the above and describe aspects of joint work with A. Kupers in which we show that, in most cases, these Disc-structure spaces are nontrivial infinite loop spaces that depend only little on the underlying manifolds. This resolves questions and conjectures of Ayala–Francis–Tanaka, Dwyer–Hess, and Weiss, but also suggests many new ones of which I will discuss some during the talk.