

A FINITENESS CONJECTURE FOR ABELIAN VARIETIES OVER NUMBER FIELDS (JOINT WORK WITH CHRISTOPHER RASMUSSEN)

AKIO TAMAGAWA

The study of the pro- l outer Galois representation associated to the arithmetic fundamental group of the projective line minus three points over the rationals, initiated by Y. Ihara in mid-1980s, has been playing an important role in arithmetic geometry. (For example, M. Kim's striking alternative proof of Siegel's theorem in mid-2000s, which is the monumental first step of Kim-Chabauty theory, is based on this representation.) In Ihara's work in 1980s, he posed a question on the description of the kernel of this representation, now often referred to as Ihara's question. A certain approach to Ihara's question using abelian varieties was initiated in C. Rasmussen's thesis work under his advisor Kim in the first half of 2000s. Related to this approach, Rasmussen and I jointly reached a certain finiteness conjecture for abelian varieties over number fields in the second half of 2000s, now often referred to as the Rasmussen-Tamagawa conjecture. In this talk, I will overview various aspects of this conjecture, especially including backgrounds, definitions, statements, results so far (mainly including those of Rasmussen and myself), and so on.