ABSTRACTS FOR ADDITIVE COMBINATORICS 2024

Green (Monday 10am)

A survey of higher-order Fourier analysis

I will give a survey of higher-order Fourier analysis, discussing a little of the history and motivation and some of the key notions. I will briefly survey the state of the art and recent progress, details of which will feature in several other talks. Finally, I will comment on potential goals for the future.

Manners (Monday 11:30am)

Marton's polynomial Freiman–Ruzsa conjecture

Suppose $f : \mathbb{F}_2^n \to \mathbb{F}_2^n$ is a function. If $f(x+y) = f(x) + f(y)$ for all pairs (x, y) then certainly f is linear. Say f is "1% linear" if $f(x+y) = f(x) + f(y)$ for 1% of pairs (x, y). Is such an f close to a true linear function ψ, in the sense that $f(x) = \psi(x)$ for a positive proportion of $x \in \mathbb{F}_2^n$?

Obtaining good bounds in this statement is one of many equivalent forms of Marton's "polynomial Freiman-Ruzsa" conjecture. This family of "inverse" questions plays a central role in additive combinatorics. I will discuss ideas that go into the recent proof of this conjecture (joint work with Timothy Gowers, Ben Green and Terence Tao).

Leng (Monday 2:30pm)

Improved bounds for the Gowers $U^{s+1}[N]$ inverse Theorem, Part I

We discuss recent progress on Szemerédi's theorem, equidistribution of nilsequences, and bounds on the Gowers U^{s+1} inverse theorem. During this talk we discuss the relations between these various ingredients and discuss further applications to results regarding counts for linear equations in the primes. We will then zoom in on a crucial ingredient in these proofs, the equidistribution of nilsequences and discuss the specific case of step 2 nilsquences using bracket polynomials.

Based on work with Ashwin Sah and Mehtaab Sawhney.

Sawhney (Monday 4pm)

Improved bounds for the Gowers $U^{s+1}[N]$ inverse Theorem, Part II

We discuss the proof of the U^{s+1} inverse theorem. For the purposes of the talk, we will restrict attention to the U^4 case and perform the analysis with bracket polynomials. The key theme will be how equidistribution theory is sufficient to provide "linear relations between phases" and the proof of the inverse theorem consists of glueing these linear relations together.

Based on work with James Leng and Ashwin Sah.

Moreira (Tuesday 10am)

Recent developments in Ergodic Ramsey theory

Ever since the ergodic theoretic proof of Szemerédi's theorem appeared, Furstenberg's correspondence principle has seen numerous applications to problems in additive combinatorics and Ramsey theory. Ergodic theoretic methods are particularly well suited for qualitative problems, as they typically yield no interesting bounds/rates, but in this regime, there are several results which were first proved using ergodic theory (some of which have no ergodic-free proofs yet).

I will survey some of the main achievements in the last few years and mention some open problems.

Richter (Tuesday 11:30am)

Uniformity norms and Hindman's conjecture

In this talk we will discuss how configurations of the form $\{x + y, xy\}$ are controlled by the local Host-Kra uniformity norms. This will allow us to derive a density analogue of (a special case of) a theorem of Moreira and resolve a conjecture of Moreira.

Lifshitz (Tuesday 2:30pm)

Product mixing in groups

Let A, B, C be subsets of the special unitary group $\mathrm{SU}(n)$ of Haar measure $\geq e^{-n^{1/3}}$. Then $ABC = \mathrm{SU}(n)$. In fact, the product abc of random elements $a \sim A$, $b \sim B$, $c \sim C$ is equidistributed in $\mathrm{SU}(n)$.

This makes progress on a question that was posed independently by Gowers studying nonabelian variants of questions from additive combinatorics and settles a conjecture of physicists studying quantum communication complexity.

To prove our results we introduce a tool known as 'hypercontractivity' to the study of high rank compact Lie groups. We then show that it synergies with their representation theory to obtain our result. Time permitting, I will also discuss corresponding results for finite simple groups, where the relevant representation theoretic notion of 'tensor rank' was introduced only recently independently by Gurevich-Howe and Guralnick-Larsen-Tiep.

(Based on joint work with Ellis-Kindler-Minzer, Filmus-Kindler-Minzer, Keevash, Evra-Kindler, and Evra-Kindler-Lindzey.)

Tao (Wednesday 10am)

Additive combinatorics and the primes

We survey ways in which additive combinatorics has been used in recent years to attack problems in analytic number theory concerning additive problems involving the primes.

Teräväinen (Wednesday 11:30am)

On quantitative Gowers uniformity and applications

Recent breakthroughs of Leng, Sah and Sawhney established strong bounds for the inverse theorem for the Gowers norms. Leng used these bounds to obtain polylogarithmic bounds for the Green-Tao-Ziegler Gowers uniformity of the primes theorem, improving on an earlier result of Tao and myself. I will describe how to pass from strong bounds for the inverse theorem to strong bounds for the Gowers norms of the primes. I will then discuss some applications of quantitative Gowers norm bounds, in particular to polynomial progressions in the primes and to pointwise convergence of multiple ergodic averages.

Klurman (Wednesday 2:30pm)

Partition Regularity of Squares

Is there a partition of the natural numbers into finitely many pieces, none of which contains a Pythagorean triple (i.e. a solution to the equation $x^2 + y^2 = z^2$)? This is one of the simplest questions in arithmetic Ramsey theory, which is still widely open. I will discuss recent progress on a weaker version of this problem, addressing the partition regularity of pairs for the general quadratic equation $ax^2 + by^2 = cz^2$, where x, y belong to the same cell and $z \in \mathbb{N}$. The talk is based on joint works with Frantzikinakis and Moreira.

Milićević (Wednesday 4pm)

Good bounds for sets lacking skew corners

A skew corner is a triple of points in $\mathbb{Z} \times \mathbb{Z}$ of the form $(x, y), (x, y + a)$ and $(x + a, y')$. Pratt posed the following question: for a given n, how large can a set $A \subseteq [n] \times [n]$ be, provided it contains no non-trivial skew corner? In this talk I will discuss a proof of Behrend-type bounds for this problem, which, along with a construction of Beker, essentially resolves Pratt's question. The approach is based on a two-dimensional variant of a method of Kelley and Meka, which they used in their proof of Roth's theorem.

Conlon (Thursday 10am)

Removal lemmas, dense and sparse

In this talk, we survey some of what is and isn't known about removal lemmas and related questions such as the Brown-Erdő-Sós problem. We will discuss both the classical 'dense' variants and more recent work in the sparse setting, as well as how they relate to arithmetic problems.

Wolf (Thursday 11:30am)

The structure of sets of bounded VC_2-dimension

In joint work with Caroline Terry, we showed that subsets of bounded VC_2-dimension in a high-dimensional vector space of a fixed finite field can be approximated by a union of atoms of a high-rank, bounded-complexity quadratic factor. This generalised prior work of Alon-Fox-Zhao, Sisask, and Conant-Pillay-Terry for subsets of bounded VC-dimension, and is analogous to joint work with Terry and qualitative results of Chernikov-Towsner in the setting of hypergraphs. In this talk we give a new perspective on the proof and explore the quantitative aspects of the argument.

Fox (Thursday 2:30pm)

Additive Combinatorics without addition

A fundamental problem in additive combinatorics is to better understand sets with small doubling. To what extent is group structure important in such problems? Through a purely combinatorial approach, we can begin to address this problem. In doing so, we better understand cliques in random Cayley graphs, make progress on Alon's conjecture that every finite group has a Ramsey Cayley graph, and solve a problem of Alon and Orlitsky in information theory. The talk is based on joint work with David Conlon, Huy Tuan Pham, and Liana Yepremyan. The subsequent talk by Huy Tuan Pham will include more advanced applications of this approach.

Pham (Thursday 4pm)

Ruzsa's conjecture from a combinatorial approach

Extending a seminal result of Freiman, Green and Ruzsa proved a structural result characterizing subsets in abelian groups with bounded doubling $K = |A + A|/|A|$. The characterization takes a particularly simple form over abelian groups with bounded exponent, in which case a set A with doubling K must be contained in a subgroup of size not much bigger than A. Quantitative aspects of such structural results are important themes motivating numerous developments in additive combinatorics. Over abelian groups with bounded exponent, Ruzsa conjectured a tight quantitative dependence, which has remained open since his original paper in 1999.

I will discuss a resolution of Ruzsa's conjecture in joint work with Jacob Fox. Surprisingly, the approach is crucially motivated by purely combinatorial graph-theoretic insights which abstract away the group structure. Similar combinatorial ideas are useful for several other problems involving sets with small doubling, including the study of Ramsey properties of random Cayley graphs in general groups, which are discussed extensively in the talk by Jacob Fox. Time permitting, I will discuss further refinement and development in this direction.

Rudnev (Friday 10am)

Some recent progress apropos of the sum-product conjecture

The talk will review the results on the Erdős-Szemerédi sum-product conjecture and then focus on the specific set-up where a sharp answer has recently been proven for the product set versus additive energy variant of the question.

Namely, it was asked by Szemerédi if the state of the art sum-product estimates could be improved for a set of N integers under the constraint that each integer has a small number of prime factors. We prove, if the maximum number of prime factors for each integer is sub-logarithmic in N, the sum-product exponent $5/3 - o(1)$. This is derived from the product set versus additive energy estimate, which is optimal up to the $o(1)$ term.

The estimate is based on a scheme of Burkholder-Gundy-Davis martingale square function inequalities in p-adic scales, followed by an application of a variant of the Schmidt subspace theorem.

Jing (Friday 11:30am)

Measure Doubling for Small Sets in $\mathrm{SO}(3, \mathbb{R})$

Let $\mathrm{SO}(3, \mathbb{R})$ be the 3D-rotation group equipped with the real-manifold topology and the normalized Haar measure mu. Confirming a conjecture by Breuillard and Green, we show that if A is an open subset of $\mathrm{SO}(3, \mathbb{R})$ with sufficiently small measure, then $\mu(A^2)$ is greater than $3.99\mu(A)$. This is joint work with Chieu-Minh Tran (NUS) and Ruixiang Zhang (Berkeley).