An “Almost Schiffer” Problem on Neumann Eigenfunctions of Planar Domains

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Schiffer’s conjecture in spectral geometry asserts that the only planar domains with a Neumann eigenfunction that is constant on the boundary are disks. If the hypothesis is relaxed so that the Neumann eigenfunction is only assumed to be locally constant on the boundary, we show that there are nontrivial doubly connected domains with this property, in spite of the fact that that the relaxed problem shares many rigidity properties with Schiffer. Furthermore, we shall see that this has a nice application to stationary solutions of the incompressible Euler equations.