Orthogonal rings and discrete integrable systems

Alexander Bobenko

We introduce orthogonal ring patterns consisting of pairs of concentric circles. They generalize orthogonal circle patterns which can be treated as conformal limit. It is shown that orthogonal ring patterns in euclidean and hyperbolic planes and in a sphere are governed by integrable equations (Q4. The variational description is given in terms of elliptic generalizations of the dilogarithm function. It is used to prove existence and uniqueness results, and also to compute ring patterns with classical boundary conditions. We define discrete constant mean curvature (cmc) surfaces in the three-dimensional Euclidean and Lorentz spaces in terms of sphere packings with orthogonally intersecting circles. These discrete cmc surfaces can be constructed from orthogonal ring patterns. The data used for the construction is purely combinatorial - the combinatorics of the curvature line pattern. Virtual and printed models as well as animation movies will be demonstrated.