Workshop on BSDEs and SPDEs

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Contents

Plenary Talks

All the plenary talks will take place in **Room: Ashworth Labs LT1**

Francois Delarue, *Monday 9.00 – 10.00*

McKean–Vlasov FBSDEs with a common noise

Motivated by the theory of mean field games, I will address the solvability and the smoothness of the flow of fully coupled McKean–Vlasov forward-backward SDEs with a common noise. Part of this presentation is taken from a joint work with Chassagneux and Crisan.

Jianfeng Zhang, *Tuesday 14.00 – 15.00*

Some new types of path dependent PDEs

A path dependent PDE is a PDE where the value (or solution) depends on the path of the state process, and is a convenient tool for studying path dependent (and non-Markovian) problems. In the standard literature, the PPDE is typically due to the (exogenous) path dependence imposed on the coefficients of the problem. In this talk, we present two examples where state dependent problems lead naturally to (new types of) PPDEs. The first example is that the state process is a fractional Brownian motion B^H (or more generally a solution to a Volterra SDE). In this case, the value function of the backward problem will take the form $u(t, (B^H_s)_{0\le s<t} \otimes_t (E_t[B^H_s])_{t\le s\le T})$. In particular, instead of being flat after t as in standard literature, in this case an $\mathcal{F}_t$-measurable path is involved over $[t, T]$. The second example is the stochastic control problem with information delay, namely the control α_t at time t depends on $\mathcal{F}_{t-\delta}$ for some delay parameter δ. This problem leads naturally to a path dependent master equation, and under the optimal control (if it exists), the state process solves some McKean–Vlasov SDE. We remark that in the latter example, the original control problem is in a standard setting without involvement of the laws. We also note that both examples involve some type of time inconsistency issue. The talk is based on two joint works, one with Frederi Viens and the other with Yuri Saporito.

Alison Etheridge, *Thursday 9.00 – 10.00*

Modelling populations under fluctuating selection

It has been recognised for a very long time that natural selection is not necessarily a constant force acting on a population; for example the genetic types favoured in a wet year may be different from those favoured in a dry year. As a result, fluctuating environmental conditions can maintain a balance between the different genotypes over extended periods. If, for example, we suppose that selection is acting on a single gene, then it is straightforward to write down a stochastic (ordinary) differential equation that captures the evolution of the frequencies of the different types of that gene (alleles) in the population. Crucially, such models can also capture genetic drift — the randomness due to reproduction in a finite population. What is much less studied is the evolution of allele frequencies in a population that is spatially structured. Here we discuss one such model, based on the so-called spatial Lambda-Fleming–Viot process, that can capture something of the interplay between fluctuating selection and genetic drift. We shall see that when viewed over sufficiently large spatial and temporal scales, in at least two spatial dimensions, allele frequencies are dominated by the fluctuations due to the environment and can be captured by an SPDE. Ideally, one would be able to capture family structure in the population. As time permits we shall explain some partial results in this direction.

Invited Talks

All the invited talks will take place in **Room: Ashworth Labs LT1**

Thaleia Zariphopoulou, *Monday 13.30 – 14.00*,

Mean field and n-agent games for optimal investment under relative performance criteria

I will discuss the optimal portfolio management of a population of fund managers who trade in a common horizon $[0, T]$, aiming at maximizing their expected utility but are also concerned about their relative performance. I will present the n-agent model as well as the mean field game under both CARA and CRRA risk preferences, construct the equilibria explicitly and provide conditions for their existence and uniqueness. (Joint work with D. Lacker.)

Panagiotis Souganidis, *Monday 14.00 – 14.30*,

Regularizing effects and long time behavior of stochastic viscosity solutions

I will review briefly the notion of stochastic viscosity solution and discuss some regularizing effects as well as results about the long time behavior of the solutions. This is joint work with P. L. Lions.

Anis Matoussi, *Monday 14.30 – 15.00*,

Reflected second order BSDEs under weak regularity conditions

We present well-posed solutions of reflected second order BSDEs in a general filtration and under weak regularity conditions on the terminal value, the generator and the obstacle (lower and upper cases). We introduce also a well-posed Skorokhod-solution to the second order BSDEs.

Shige Peng, *Tuesday 9.00 – 9.30*,

Quantifying nonlinear expected value by BSDE and PDE

Frank Knight challenged the feasibility of using probability to treat uncertainty. The term "Knightian uncertainty" or "ambiguity" is now widely accepted, especially among economists, to refer situations where there is no objective probability, or distribution, available for making decisions. Nonlinear expectation (e.g. g(and G)-expectation) provided a new framework to deal with this type of uncertainties. But a challenging problem is: how can we use real-world data to construct the corresponding nonlinear expectation hidden behind? We have introduced a new algorithm for computing the nonlinear expected value based on a given historical data under a much weaker assumption than the classical i.i.d. one. The method is fundamentally based on our framework of nonlinear expectation theory. It also reveals a deep relation between robust limit theory and statistics and quasilinear and fully nonlinear PDEs.

Juan Li, *Tuesday 9.30 – 10.00*,

Mean-field forward and backward SDEs with jumps. Associated nonlocal quasi-linear integral-PDEs

In this talk we consider a decoupled mean-field backward stochastic differential equation (BSDE) driven by a Brownian motion and an independent Poisson random measure. The existence and the uniqueness of the solution $(Y^{t,x,P_\xi}, Y^{t,\xi})$ of the decoupled equation are proved. We prove that under our assumptions the value function $V(t,x,P_\xi) := Y_t^{t,x,P_\xi}$ is regular, and it is the unique classical solution of the related quasi-linear integral-partial differential equation of mean-field type with the help of a new Itô formula.

Nizar Touzi, *Tuesday 10.00 – 10.30*,

Continuous-time Principal-Agent problem: a Stackelberg stochastic differential game

We provide a systematic method for solving general Principal-Agent problems with possibly infinite horizon. Our main result reduces such Stackelberg stochastic differential games to a standard stochastic control problem, which may be addressed by the standard tools of control theory. Our proofs rely on the backward stochastic differential equations approach to non-Markovian stochastic control, and more specifically, on the recent extensions to the second order case. The infinite horizon setting requires an extension of second order BSDEs to the random horizon setting.

Jin Ma, *Wednesday 9.30 – 10.00*,

Time-Consistent Approaches in Time-inconsistent Problems

Time-inconsistency appears naturally and frequently in economics and finance, and has been noticed for more than half a century. Many attempts have been made in recent years to break through the major barrier in this problem: the failure of the dynamic programming principle, and various methods have been proposed to find optimal strategies for these problems that are time-inconsistent" by nature. In this talk we discuss possible dynamic approaches to study time-inconsistent optimization problems that are time-consistent. Originated on the idea of dynamic programming, these approaches will focus more on the value of the optimization problem rather than on the optimal control, which may not even exit. In particular, we shall present a newly discovered fact that a commonly known time-inconsistent nonlinear expectation under probability distortion can actually be turned into a non-sub-additive, but time-consistent (or filtration-consistent) one, with a careful localization of the distortion impact. More interestingly, we show it has many features that resembles Peng's theory of non-linear expectation, and we shall derive the corresponding PDE and BSDE representing such a nonlinear expectation. We hope that such an observation will lead to some new developments in the study of time-inconsistent problems. This is based on joint works with Chandrasekhar Karnam, Ting-Kam Leonard Wong, and Jianfeng Zhang.

Annie Millet, *Wednesday 10.00 – 10.30*,

On generalized KdV equation subject to some random perturbation

We will describe global well-posedness and orbital stability for the generalized KdV equation in the subcritical case, subject to some additive random perturbation. The critical and super-critical cases will also be discussed. This is joint work with S. Roudenko.

Rainer Buckdahn, *Thursday 13.30 – 14.00*,

Representation formulas for long run averaging stochastic optimal control problems

We study a stochastic control problem with long run averaging cost for both the Cesaro and also the Abel means. The asymptotic behaviour of the associated value functions is a classical and meanwhile well studied problem in ergodic stochastic control. But unlike ergodic control problems we don't make assumptions here which lead to a limit value function which is a constant independent of the initial condition; our limit value functions can depend on the initial condition. While we have shown in previous work under the so-called non expansion condition the existence of a limit value function (together with Dan Goreac and Marc Quincampoix for classical cost functionals, and recently also Juan Li with Hui Min for cost functionals defined through controlled backward SDEs with infinite time horizon), we study now representation formulas for accumulation points of the value functions — in the sense of the uniform convergence — when the time horizon of the Cesaro mean tends to infinity and the discount factor of the Abel mean converges to zero. The representation formulas are given by using occupational measures on the state spaces and invariant measures on the state path spaces, respectively. The independence of the converging subsequence of value functions has as consequence that the limit of the value functions defined through the Cesaro or Abel means exists as soon as these value functions are equicontinuous. This is the case, for instance, under the non expansion condition. The talk is based on joint work with Juan Li (Shandong University, Weihai, P.R.China), Marc Quincampoix (Universite de Bretagne Occidentale, Brest, France) and Jerome Renault (Toulouse School of Economics, Toulouse, France).

Peter Imkeller, *Thursday 14.00 – 14.30*,

On the Hausdorff dimension of a very rough Weierstrass curve whose components are not controlled

We investigate geometric properties of Weierstrass curves with two components, representing series based on trigonometric functions. They are seen to be $\frac{1}{2}$-Hölder continuous, do not possess a Lévy area, and are not (para-)controlled with respect to each other in the sense of the recently established Fourier analytic approach of rough path analysis. Their graph is represented as an attractor of a smooth random dynamical system. Our argument that its graph has Hausdorff dimension 2 is in the spirit of Ledrappier-Young's approach of the Hausdorff dimension of attractors.

Joint work with G. dos Reis (U. of Edinburgh) and O. Pamen (U. Liverpool and AIMS Ghana).

Monique Jeanblanc, *Thursday 14.30 – 15.00*,

BSDE and enlargement of filtration

We present two problems where one considers BSDE in various filtrations. The first one corresponds to an optimisation problem, when the measurability of the strategies varies. The second one is a comparison between two BSDEs written in different filtrations.

Etienne Pardoux, *Friday 11.00 – 11.30*,

Nonlinear filtering with degenerating noise

We consider a new class of filtering problem, where the observation lives in a bounded set, and the observation noise vanishes when the observation visits the boundary of the set where it lives. We show that an adaptation of the classical arguments yields a version of the traditional filtering equations. In the case where the observation is scalar and the observation and signal noises are independent, we obtain a robust version of the Zakai equation, which has a unique measure valued solution, thanks to a duality argument. We extend the uniqueness result via duality between a forward and a backward SPDE, beyond the "robust case".

Saïd Hamadene, *Friday 11.30 – 12.00*,

Existence and uniqueness of viscosity solutions for second order integro-differential equations without monotonicity condition

In this talk, we discuss a new existence and uniqueness result of a continuous viscosity solution for integro-partial differential equation (IPDE in short). The novelty is that we relax the so-called monotonicity assumption on the driver which is classically assumed in the literature of viscosity solution of equation with a nonlocal term. Our method is based on the link of those IPDEs with backward stochastic differential equations (BSDEs in short) with jumps for which we already know that the solution exists and is unique.

Michael Röckner, *Friday 12.00 – 12.30*,

Nonlinear Fokker–Planck equations driven by Gaussian linear multiplicative noise

Existence and uniqueness of a strong solution in $H^{-1}(\mathbb{R}^d)$ is proved for the stochastic nonlinear Fokker–Planck equation

$$dX - div(DX)dt - \triangle\beta(X)dt = X\ dW \text{ in } (0,T)\times\mathbb{R}^d,\ X(0) = x,$$

via a corresponding random PDE. W is a Wiener process in $H^{-1}(\mathbb{R}^d), D \in C^1(\mathbb{R}^d, \mathbb{R}^d)$ and β is a Lipschitz continuous monotonically increasing function. The solution is pathwise Lipschitz continuous with respect to initial data in $H^{-1}(\mathbb{R}^d)$ and preserves positivity. In case of initial conditions bounded below by a constant $\rho > 0$, the Lipschitz condition on β can be relaxed. Stochastic equations with nonlinear drift of the form $dX - div(a(X))dt + \triangle\beta(X)dt = XdW$ are also considered for Lipschitzian functions $a : \mathbb{R} \to \mathbb{R}^d$. The main technique for the proof is a variant of the Crandall–Liggett approach applied to the random PDE, but with time dependent coefficients. This is joint work with Viorel Barbu.

Contributed Talks

Elena Bandini, *Tuesday 17.30 – 18.00*, **Room: JCMB Lecture Theatre A**

Existence and uniqueness for BSDEs driven by a general random measure, possibly non quasi-left-continuous

We study a backward stochastic differential equation on finite time horizon driven by an integer-valued random measure μ on $R_+ \times E$ (where E is a Lusin space) with compensator $\nu(dt, dx) = dA_t \varphi_t(dx)$; the generator of the equation satisfies as usual a uniform Lipschitz condition with respect to Y and Z. In the literature, the existence and uniqueness for BSDEs in this general setting has only been established when A is continuous or deterministic. The general case, i.e. A is a right-continuous nondecreasing predictable process, is addressed here. These results are particularly useful in the study of control problems related to piecewise deterministic Markov processes by means of BSDEs methods.

Fulvia Confortola, *Tuesday 17.30 – 18.00*, **Room: JCMB Lecture Theatre B**

BSDEs and point processes

We formulate and solve a class of BSDEs driven by the compensated random measure associated to a given marked point process on a general state space. We present basic well-posedness results in L^2 and in L^1. We show that in the setting of point processes it is possible to solve the equation recursively, by replacing the BSDE by an ordinary dierential equation in between jumps. Finally we address applications to optimal control of marked point processes, where the solution of the BSDE allows to identify the value function and the optimal control. The talk is based on joint works with Marco Fuhrman and Jean Jacod.

Martin Jönsson, *Tuesday 17.30 – 18.00*, **Room: JCMB Lecture Theatre C**

European option pricing with stochastic volatility models under parameter uncertainty

We consider stochastic volatility models under parameter uncertainty and investigate how model derived prices of European options are affected. We let drift parameters evolve dynamically over time within a specified region and formalise the approach as a control problem where the control acts on the parameters to maximise/minimise the option value. Through a dual representation with backward stochastic differential equations, we obtain explicit equations for Heston's model and investigate numerical solutions thereof. In an empirical study, we apply our results to market data from the S&P 500 index where the parameter uncertainty is inferred from historical index prices.

Eyal Neuman, *Tuesday 17.30 – 18.00*, **Room: JCMB Room 5326**

Uniqueness and blowup properties for SDEs

As the first step for approaching the uniqueness and blowup properties of the solutions of the stochastic wave equations with multiplicative noise, we analyze the conditions for the uniqueness and blowup properties of the solution (X_t, Y_t) of the equations $dX_t = Y_t dt$, $dY_t = |X_t|^\alpha dB_t$, $(X_0, Y_0) = (x_0, y_0)$. In particular, we prove that solutions are nonunique if $0 < \alpha < 1$ and $(x_0, y_0) = (0, 0)$ and unique if $1/2 < \alpha < 1$ and $(x_0, y_0) \neq (0, 0)$. We also show that blowup in finite time holds if $\alpha > 1$ and $(x_0, y_0) \neq (0, 0)$.

Giuseppina Guatteri, *Tuesday 17.30 – 18.00*, **Room: JCMB Room 5327**

Optimal control of two scale stochastic systems in infinite dimensions: the BSDE approach

In this paper, joint with Gianmario Tessitore University of Milano Bicocca, we study, by probabilistic techniques, the convergence of the value function for a two-scale, infinite-dimensional, stochastic controlled system as the ratio between the two evolution speeds diverges. The value function is represented as the solution of a *backward stochastic differential equation* (BSDE) that it is shown to converge towards a *reduced* BSDE. The noise is assumed to be additive both in the slow and the fast equations for the state. Some non degeneracy condition on the slow equation are required. The limit BSDE involves the solution of an *ergodic* BSDE and is itself interpreted as the value function of an auxiliary stochastic control problem on a reduced state space.

More precisely we study the convergence of the value function of an optimal control problem for a singularly perturbed infinite dimensional state equation as

$$\begin{cases} dX_t = AX_t^{\varepsilon,\alpha} + b(X_t^{\varepsilon,\alpha}, Q_t^{\varepsilon,\alpha}, \alpha_t)dt + Rd\mathcal{W}_t^1, & X_0^{\varepsilon,\alpha} = x_0, \\ \varepsilon dQ_t^{\varepsilon,\alpha} = (BQ_t^{\varepsilon,\alpha} + F(X_t^{\varepsilon,\alpha}, Q_t^{\varepsilon,\alpha}))dt + G\rho(\alpha_t)dt + \varepsilon^{\frac{1}{2}} G d\mathcal{W}_t^2, & Q_0^{\varepsilon} = q_0, \end{cases}$$

where the state processes X and Q are Hilbert valued, A and B are unbounded linear operators, α represents the control, $(\mathcal{W}_t^1)_{t\geq 0}$, $(\mathcal{W}_t^2)_{t\geq 0}$ are infinite dimensional cylindrical Wiener processes, b, F, are functions satisfying suitable assumptions. We notice that the presence of the constant ε in the second equation corresponds to the fact that Q evolves with a speed which is larger by a factor $1/\varepsilon$ than the speed of evolution of the component X. In other words the above equation is a good model for a so called *two scale system*. The optimal control problem is then completed by a standard cost functional of the form:

$$J^\varepsilon(x_0, q_0, \alpha) := \mathbb{E}\Big(\int_0^1 l(X_t^{\varepsilon,\alpha}, Q_t^{\varepsilon,\alpha}, \alpha_t)dt + h(X_1^{\varepsilon,\alpha})\Big).$$

Several authors have studied the convergence of singular stochastic control problems in finite dimensions, see for instance [1], [2], [13], [14], [15]. In particular [1] has been an inspiration for the present work. In that paper authors represent the value function of a singular stochastic control problem, in finite dimensions, by the solution, in viscosity sense, of an Hamilton Jacobi Bellman equation. Then they show, by PDE methods their convergence towards the solution, again in viscosity sense, of a *reduced* parabolic PDE with smaller state space and a new nonlinearity usually called *effective Hamiltonian*. Such analysis is performed in the case of periodic boundary conditions. Although PDE techniques perfectly fit the finite dimensional case allowing to cover general situations, including state equations with control dependent diffusions that require introduction of fully non-linear HJB equations, they seem not to be adaptable to the infinite dimensional case, and consequently to the case of two scale stochastic control problems for stochastic PDEs. The reason essentially is the difficulty of handling, by analytic tools and viscosity solutions, parabolic equations in infinite variables, see the discussion in the introduction of [11].

The purpose of the present paper is twofold. On one side we wish to show that Backward Stochastic Differential Equations (BSDEs) are, in general, an efficient way to represent the limit of the value functions of two scale systems when the ratio between the two evolutions' speed diverge. On the other hand we wish to show that, in such a way, we can cover the case of infinite dimensional state equations (that is the case of two scale systems described by stochastic PDEs) that, to our best knowledge, was not considered in the existing literature. As a counterpart we notice that we consider state equation in which the control only acts the drift and in which the noise of the slow component is assumed to be non-degenerate. Such restrictions seem not to be intrinsic in the BSDEs approach but allows essential technical simplifications. To be more specific, our main result will be to prove that if

$$v^\varepsilon(x_0, q_0) := \inf_\alpha J^\varepsilon(x_0, q_0, \alpha)$$

then

$$v^\varepsilon(x_0, q_0) \to Y_0$$

where $(X, \bar{Y}, \bar{Z})$ is the unique solution of the following decoupled forward backward system of stochastic differential equations

$$\begin{cases} dX_t = AX_t dt + T d\mathcal{W}_t^1, \\ -d\bar{Y}_t = \lambda(X_t, \bar{Z}_t)dt - \bar{Z} d\mathcal{W}_t^1, \\ X_0 = x_0, \quad \bar{Y}_1 = h(X_1). \end{cases}$$

It is important to notice that the 'reduced nonlinearity' λ is itself a component of the unique solution $(\check{Y}, \check{Z}, \lambda)$ of a parametrized *ergodic* BSDE as they were introduced in [9]. The function λ can itself be interpreted as the optimal cost of an ergodic optimal control problem. Moreover, as it happens in the infinite dimensional case, the space in which the above reduced BSDE lives is a subspace of the original one (corresponding to the slow evolution). As a by-product of our main result, using the Bismut Elworthy formula in [12] we immediately get that the solution of the reduced BSDE, and therefore the limit value function, depends on x_0 in a differentiable way and is linked to the unique *mild* solution of a semilinear parabolic PDE in infinite dimensional spaces:

$$\begin{cases} \frac{\partial v(t,x)}{\partial t} + \frac{1}{2}\mathrm{Tr}[\nabla_x^2 v(t,x)] = \lambda(x, \nabla v(t,x)), \quad t \in [0,1],\ x \in H, \\ v(1,x) = h(x). \end{cases}$$

Finally, exploiting the concavity of λ, we give a representation of $\bar{Y}_t$ as the value function of an auxiliary stochastic control problem on a reduced state space.

References

[1] O. Alvarez and M. Bardi. Viscosity solutions methods for singular perturbations in deterministic and stochastic control. *SIAM J. Control Optim.* 40 No.4 (2001), pp. 11591188

[2] A. Bensoussan. *Perturbation Methods in Optimal Control.* Wyley/Gauthiers-Villars, U.K., 1988.

[3] P. Briand and Y. Hu Stability of BSDEs with random terminal time and homogenization of semilinear elliptic PDEs. *J.Func. Anal.*, 155 (1998), pp. 455-494.

[4] V. I. Bogachev. *Measure Theory.* Springer-Verlag, Berlin, Heidelberg, 2007

[5] G. Da Prato, S. Kwapien and J. Zabczyk. Regularity of solutions of linear stochatsic equations in Hilbert spaces. *Stochastics*, 23, 1-23.

[6] G. Da Prato and J. Zabczyk. *Stochastic Equations in Innite Dimensions.* Cambridge University Press, Cambridge, 1992.

[7] G. Da Prato and J. Zabczyk. *Ergodicity for innite-dimensional systems.* London Mathematical Society Lecture Note Series, 229. Cambridge University Press, Cambridge, 1996.

[8] A. Debussche, Y. Hu and G. Tessitore. Ergodic BSDEs under weak dissipative assumptions. *Stoc. Pro. Appl.* 121 (2011), 407-426.

[9] M. Fuhrman, H. Ying, G. Tessitore, Ergodic BSDES and optimal ergodic control in Banach spaces. *Siam Journal On Control and Optimization*, 48(3) (2009), 1542-1566.

[10] M. Fuhrman, G. Tessitore, Generalized directional gradients, backward stochastic dieren- tial equations and mild solutions of semilinear parabolic equations. *Applied Matematics and Optimization*, 51(3) (2005), 279-332.

[11] M. Fuhrman and G. Tessitore. Nonlinear Kolmogorov equations in innite dimensional spaces: The backward stochastic dierential equations approach and applications to optimal control. *Ann. Probab.* 30 (2002), 1397-1465.

[12] M. Fuhrman and G. Tessitore. The Bismut-Elworthy formula for backward SDE's and applications to nonlinear Kolmogorov equations and control in infinite dimensional spaces. *Stochastics and Stochastic Reports* 74 (2002), 429-464.

[13] Y. Kabanov and S. Pergamenshchikov. *Two-scale Stochastic Systems.* Applications of Mathematics, 49. Springer-Verlag, Berlin, Heidelberg, New York, 2003.

[14] Y. Kabanov, W. Runggaldier, On Control of Two-scale Stochastic Systems with Linear Dynamics in the Fast Variables. *Math. Control Signals Systems* 9 (1996), 107-122.

[15] H. Kushner. *Weak convergence Methods and Singularly Perturbed Stochatsic Control And Filtering Problem.* Birkauser, Boston, 1990.

Dimitra Antonopoulou, *Thursday 16.00 – 16.30*, **Room: JCMB Room 5326**

On stochastic dynamics and sharp interface limit for a class of SPDEs from phase separation problems

A class of stochastic PDEs of Cahn Hilliard and Allen–Cahn type with mass conservative additive noise is considered. These equations involve a small positive parameter which stands as a measure for the width of transition layers that may be generated during the phase separation process. Due to mass conservation, the stochastic solution is approximated to a manifold of solutions. We apply Itô calculus and derive the stochastic dynamics of the layers in a proper coordinate system. Moreover, we present briefly some recent results for the stochastic Cahn–Hilliard equation on the sharp interface limit. (This talk presents some main results on SPDEs derived in collaboration with P. Bates, D. Bloemker and G. Karali.)

Ananta Kumar Majee, *Thursday 16.30 – 17.00*, **Room: JCMB Room 5326**

On Stochastic Optimal Control in Ferromagnetism

In this presentation, we study an optimal control problem for the stochastic Landu-Lipschitz-Gilbert equation on a bounded domain in $\mathbb{R}^d$ ($d = 1, 2, 3$). We establish existence of a relaxed optimal control for relaxed version of the problem. As the control acts in the equation linearly, we then establish existence of an optimal control for the underlying problem. Furthermore, convergence of a structure preserving finite element approximation for $d = 1$ and physically relevant computational studies will be discussed.

Hyungbin Park, *Thursday 17.30 – 18.00*, **Room: JCMB Room 5327**

Sensitivity analysis of long-term cash flows

This talk discusses a sensitivity analysis of long-term cash flows. The price of the cash flow at time zero is given by a pricing operator of a Markov diffusion acting on the cash flow function at the payoff time. We study the extent to which the price of the cash flow is affected by small perturbations of the underlying Markov diffusion. The main tool is Hansen–Scheinkman decomposition, which is a technique that expresses the cash flow in terms of the eigenvalue and eigenfunction of the pricing operator. By combining the results of Fournie et al. (1999), we conclude that the sensitivities of long-term cash flows can be represented via simple expressions in terms of the eigenvalue and the eigenfunction.

Ivan Yaroslavtsev, *Thursday 17.30 – 18.00*, **Room: JCMB Lecture Theatre A**

L^q-valued Burkholder–Rosenthal inequalities and sharp estimates for stochastic integrals

In this talk we present the Burkholder–Rosenthal inequalities for L^q-valued martingales. This type of inequality connects the L^p-norm of a martingale with a norm that depends on the conditional moments of the martingale differences. We use these inequalities to obtain Itô isomorphisms for L^q-valued stochastic integrals with respect to a random measure belonging to a broad class of compensated random measures, that for example contains compensated Poisson random measures.

The Burkholder–Rosenthal inequalities will provide us also with Itô isomorphisms for L^q-valued stochastic integrals with respect to a general (Hilbert space-valued) martingale. This talk is based on joint work with Sjoerd Dirksen (RWTH Aachen University).

Federica Masiero, *Thursday 17.30 – 18.00*, **Room: JCMB Lecture Theatre B**

Well-posedness of semilinear stochastic wave equations with Hölder continuous coefficients

We prove that semilinear stochastic abstract wave equations, including wave and plate equations, are well-posed in the strong sense with an α-Hölder continuous drift coefficient, for some $\alpha < 1$. The uniqueness may fail for the corresponding deterministic PDE and well-posedness is restored by adding an external random forcing of white noise type. This shows a kind of regularization by noise for the semilinear wave equations. To prove the result we introduce an approach based on backward stochastic differential equations (BSDEs) and we prove some new results on BSDEs in infinite dimensions. Joint work with E. Priola, arXiv:1607.00029.

Thomas Önskog, *Thursday 17.30 – 18.00*, **Room: JCMB Lecture Theatre C**

Reflected stochastic differential equations on non-smooth time-dependent domains

We consider the setting of single-valued, smoothly varying directions of reflection and non-smooth time-dependent domains whose boundary is $W^{1,p}$ in time. In this setting, we prove existence and uniqueness of strong solutions to stochastic differential equations with oblique reflection. Our approach is based on the Skorohod problem and two classes of test functions. The results generalize two articles by Dupuis and Ishii to time-dependent domains.

Yuhong Xu, *Thursday 17.30 – 18.00*, **Room: JCMB Room 5326**

Multidimensional g-risk measure with interacting agents

This talk is about multidimensional dynamic risk measures induced by conditional g-expectations. Uniqueness theorem and viability on a rectangle of solutions to multidimensional backward stochastic differential equations, some necessary and sufficient conditions are given for the constancy, monotonicity, positivity and translatability properties of multidimensional conditional g-expectations and multidimensional dynamic risk measures; we prove that a multidimensional dynamic g-risk measure is nonincreasingly convex if and only if the generator g satisfies a quasi-monotone increasingly convex condition. A general dual representation is given for the multidimensional dynamic convex g-risk measure in which the penalty term is expressed more precisely. It is shown that model uncertainty leads to the convexity of risk measures. As to applications, we show how this multidimensional approach can be applied to measure the insolvency risk of a firm with interacted subsidiaries; optimal risk sharing for γ-tolerant g-risk measures and risk contribution for coherent g-risk measures are investigated. Insurance g-risk measure and other ways to induce g-risk measures are also studied.

Xiling Zhang, *Friday 14.00 – 14.30*, **Room: JCMB Lecture Theatre A**

A Multi-Dimensional Central Limit Bound Applied to the Euler Approximation of Lévy-SDEs

Given a sequence of i.i.d. random variables $\{X_i \in \mathbb{R}\}_{i\geq 1}$ with variance σ^2 and $Y_m := m^{-1/2}\sum_{i=1}^m X_i$ for $m \in \mathbb{Z}^+$, Rio showed that the Wasserstein distances $W_p(Y_m; N(0,\sigma^2))$ are bounded by a ratio between some higher moments of X_1 and some powers of σ. Fournier showed that this central limit bound leads to the normal approximation of the small jumps of Levy processes, and one can avoid the possible blow-up of the computational load when simulating Lévy-SDEs by the Euler scheme. However neither result applies to vector-valued X_i's. In this talk I will show that, by considering some polynomial perturbations of the normal distribution, one can generalise Rio's result to the multi-dimensional case, and the number of moments needed for X_1 is independent of the dimension. I will also give two different approaches to generalise Fournier's result: one as a corollary to the central limit bound and the other derived from the Lévy–Khinchine formula.

Lukas Wresch, *Friday 14.00 – 14.30*, **Room: JCMB Lecture Theatre B**

Path-by-Path Uniqueness for SDEs in Hilbert spaces

Consider the following SDE in a separable Hilbert space

$$\mathrm{d}X_t = -AX_t\mathrm{d}t + f(t, X_t)\mathrm{d}t + \mathrm{d}W_t,$$

where A is a positive, linear operator, f is a bounded Borel measurable function and W a cylindrical Wiener process. If the components of f decay to 0 in a faster than exponential way we establish path-by-path uniqueness for mild solutions of this SDE. This extends A. M. Davie's result from $\mathbb{R}^d$ to Hilbert space-valued stochastic differential equations. In this talk we consider the so-called path-by-path approach where the above SDE is considered as a random integral equation with parameter $\omega \in \Omega$. We show that there is a set Ω' of measure 1 such that for every $\omega \in \Omega'$ the corresponding integral equation for this ω has at most one solution. This notion of uniqueness (called path-by-path uniqueness) is much stronger than the usual pathwise uniqueness considered in the theory of SDEs.

James-Michael Leahy, *Friday 14.00 – 14.30*, **Room: JCMB Lecture Theatre C**

Navier–Stokes Equation with Rough Transport Noise

We study a Navier–Stokes equation on the d-dimensional torus with a transport type noise term. The noise is smooth in space but rough in time, so we can use the recently developed theory of "Unbounded Rough Drivers" to study the equation. We prove existence and uniqueness of a solution for the 2-d case and existence of a solution for the 3-d case.

Alessandro Calvia, *Friday 14.00 – 14.30*, **Room: JCMB Room 5326**

Filtering and control of time-homogeneous pure jump Markov processes with noise-free observation

In this talk I am going to address a stochastic filtering and optimal control problem with partial observation, mainly characterized by observations not corrupted by noise. Let X and Y be a given couple of stochastic processes, with values in two Borel spaces I and O respectively. The unobserved (or signal) process X is a time-homogeneous pure jump Markov process, whose rate transition measure is known. The observed process Y is defined as $Y_t = h(X_t)$, for all nonnegative t, where h is a known measurable function mapping the space I onto the space O. The first aim is to provide an explicit SDE for the filtering process. As known this process represents at each time t the conditional distribution of the unobserved process X given the information provided by the observed process Y available up to time t. The problem is tackled with the aid of known results from marked point processes theory and a martingale approach. The filtering process is also characterized as a Piecewise Deterministic Markov Process, in the sense of Davis. The second goal is to solve an infinite-horizon optimal control problem. The aim is to minimize a discounted cost functional by controlling the rate transition measure of the unobserved process via the information provided by the observed process. The problem is reformulated into a discrete-time optimal control problem for the filtering process. In the case of a finite-state controlled Markov chain (i.e. when the space I is of finite cardinality), the value function can be characterized as the unique fixed point of a suitably defined operator. In addition, an HJB equation can be explicitly written and the value function is proved to be its unique constrained viscosity solution, in the sense of Soner. Finally, the existence of an optimal control for the discrete-time optimal control problem is shown. In our setting such an optimal control coincides with a piecewise open-loop control in the sense of Vermes.

Junjian Yang, *Friday 14.00 – 14.30*, **Room: JCMB Room 5327**

On L^1 solutions of BSDEs

We consider the existence and uniqueness of L^1 solutions of BSDEs and reflected BSDEs. It was shown that if the generator f is of sublinear growth with respect to z, there exists a solution for $\xi \in L^1$. Here, we show the existence and uniqueness of solutions of a linearly growing BSDE under an integrability condition on ξ and $f_s(0,0)$ uniformly with respect to a family of probability measures.

Flash Talks

Chuying Huang, *Thursday 15.00 – 15.15*, **Room: JCMB Lecture Theatre A**

Symplectic Runge-Kutta methods for Hamiltonian systems driven by rough paths

We consider Hamiltonian systems driven by multi-dimensional Gaussian process in rough path sense, with either bounded or linear vector fields. We indicate that the phase flow of this rough Hamiltonian system preserves the symplectic structure, and this property could be inherited almost surely by symplectic Runge-Kutta methods. Furthermore, we give the solvability condition as well as the convergence rates for Runge-Kutta methods. Numerical experiments verify our theoritical analysis.

Wing Fung (Alfred) Chong, *Thursday 15.00 – 15.15*, **Room: JCMB Lecture Theatre B**

An ergodic BSDE approach to forward entropic risk measures: representation and large-maturity behavior

Using elements from the theory of ergodic backward stochastic differential equations, we study the behavior of forward entropic risk measures. We provide a general representation result and examine their behavior for risk positions of large maturities. We also compare them with their classical counterparts and derive a parity result. This is a joint work with Ying Hu, Gechun Liang and Thaleia Zariphopoulou.

Florian Lemonnier, *Thursday 15.00 – 15.15*, **Room: JCMB Lecture Theatre C**

Ergodic BSDEs with sublinear diffusion in finite dimension

In this talk, we deal with Ergodic Backward Stochastic Differential Equations for which the underlying SDE has a sublinear diffusion. Hu, Madec and Richou have recently studied those equations, but with a constant diffusion (and in infinite dimension). First, we show existence and uniqueness of the solution under assumptions on the driver similar to their work. Then, we obtain the large time behaviour of viscosity solutions of HJB equations.

Edward Kim, *Thursday 15.00 – 15.15*, **Room: JCMB Room 5326**

VALUATION OF GAME OPTIONS IN NONLINEAR MARKETS

We summarise from a working paper by Kim, Nie and Rutkowski. The goal is to re-examine findings from Dumitrescu, Quenez and Sulem (2017) who studied game options within the nonlinear arbitrage-free pricing approach developed in El Karoui and Quenez (1997). We show that unilateral arbitrage-free prices for the game option can be obtained by solving what we coin optimal replication problems. Fairly general assumptions on the nonlinear wealth dynamics of self-financing portfolios are needed in particular, we need a strict backward monotonicity assumption. In the language of BSDEs, this translates to strict comparison theorems. Interestingly, it turns out that neither strict comparison theorems for (doubly-)reflected BSDEs nor the existence of a value for the nonlinear Dynkin game associated with the game contract are essential.

Xu Wang, *Thursday 15.00 – 15.15*, **Room: JCMB Room 5327**

Numerical Analysis on Ergodic Limit of Approximations for Stochastic NLS Equation via Multi-symplectic Scheme

We consider a finite dimensional approximation of the stochastic nonlinear Schrodinger equation driven by multiplicative noise, which is derived by applying a symplectic method to the original equation in spatial direction. Both the unique ergodicity and the charge conservation law for this finite dimensional approximation are obtained on the unit sphere. To simulate the ergodic limit over long time for the finite dimensional approximation, we discretize it further in temporal direction to obtain a fully discrete scheme, which inherits not only the stochastic multi-symplecticity and charge conservation law of the original equation but also the unique ergodicity of the finite dimensional approximation. The temporal average of the fully discrete numerical solution is proved to converge to the ergodic limit with order one with respect to the time step for a fixed spatial step. Numerical experiments verify our theoretical results on charge conservation, ergodicity and weak convergence.

Luca Scarpa, *Thursday 15.15 – 15.30*, **Room: JCMB Lecture Theatre A**

Well-posedness of semilinear SPDEs with singular drift: a variational approach

The talk deals with the class of semilinear SPDEs of the type $dX_t + AX_t dt + \beta(X_t)dt \ni B(t, X_t)dW_t$ in $(0,T) \times D, X = 0$ in $(0,T) \times \partial D, X(0) = X_0$ in D, where $D \subset R^n$ is a smooth bounded domain, A is a linear coercive maximal monotone operator on $L^2(D)$, W is a cylindrical Wiener process on a separable Hilbert space U and B is a random time-dependent Hilbert-Schmidt operator from U to $L^2(D)$ satisfying classical Lipschitz-continuity conditions in the second variable. Here, β is allowed to be a multivalued maximal monotone graph defined on the whole real line, on which no growth restrictions are required. Existence, uniqueness and continuous dependence on the initial datum are proved for the problem. The proof consists in approximating the equation, finding uniform estimates both pathwise and in expectation on the approximated solutions, and then passing to the limit using compactness and lower semicontinuity results: the study is based on a joint work with Carlo Marinelli (University College London).

Alexandros Saplaouras, *Thursday 15.15 – 15.30*, **Room: JCMB Lecture Theatre B**

BSDEs with jumps are stable

In this work we deal with the stability property of BSDEJ. The aforementioned property, roughly speaking, describes the continuity of the operator that maps the input data to the output data. In conjunction with a general result of existence and uniqueness of solutions of BSDEJ, which covers the stochastically non-continuous case, we can translate the robustness property into numerical schemes for BSDEJ.

Nahuel Tomas Foresta, *Thursday 15.15 – 15.30*, **Room: JCMB Lecture Theatre C**

Reflected BSDE driven by marked point process with applications to optimal stopping and optimal switching

We define a class of reflected backward stochastic differential equation (RBSDE) driven by a marked point process (MPP), where the solution is constrained to stay above a given process. Under usual assumptions we obtain existence and uniqueness in the case where only a marked point process is involved. The equation is used to represent the value function of an optimal stopping problem, and we characterize the optimal strategy. We also define a system of BSDE driven by MPP, interconnected by the obstacle conditions. When the generator has a specific form, this can be used to solve an optimal switching problem for MPP where the law of the process depends on the control strategy. Keywords: reflected backward stochastic differential equations, optimal stopping, optimal switching, marked point process.

Liying Sun, *Thursday 15.15 – 15.30*, **Room: JCMB Room 5326**

High order conformal symplectic and ergodic schemes for stochastic Langevin equation via generating functions

We consider the stochastic Langevin equation with additive noises, which possesses both conformal symplectic geometric structure and ergodicity. We propose a methodology of constructing high weak order conformal symplectic schemes by converting the equation into an equivalent autonomous stochastic Hamiltonian system and modifying the associated generating function. To illustrate this approach, we construct a specific second order numerical scheme, and prove that its symplectic form dissipates exponentially. Moreover, for the linear case, the proposed scheme is also shown to inherit the ergodicity of the original system, and the temporal average of the numerical solution is a proper approximation of the ergodic limit over long time. Numerical experiments are given to verify these theoretical results.

Petr Čoupek, *Thursday 15.15 – 15.30*, **Room: JCMB Room 5327**

Stochastic convolution driven by Volterra processes in L^p spaces

Regularity of mild solutions to linear stochastic evolution equations with values in L^p spaces are studied. We consider equations with additive noise of Volterra type which lives in a finite Wiener chaos. The noise need not be Gaussian, Markov or semimartinagle but it must admit a certain covariance structure instead. The main examples include the fractional Brownian motion of $H > 1/2$ and the Rosenblatt process. Sufficient conditions for the mild solution to be Hölder continuous in a suitable function space are given and this allows to show space-time Hölder continuity of solutions to particular SPDEs. The main tool used is a hypercontractivity result on Banach-space-valued random variables from a finite Wiener chaos. The contribution is based on a joint work with B. Maslowski and M. Ondreját.

Special Sessions

BSDEs, Malliavin Calculus, Analytic Methods, and Applications

Organisers: P. Cheridito and S. Geiss, **Room: JCMB Lecture Theatre B**

Khaled Bahlali, *Monday 10.30 – 11.00*

One dimensional BSDEs: Existence by domination

The domination argument expresses that : if (ξ_1, f_1) and (ξ_2, f_2) are two BSDEs which respectively admit two solutions (Y_1, Z_1) and (Y_2, Z_2) such that $(\xi_1, f_1, Y_1) \leq (\xi_2, f_2, Y_2)$, then any Quadratic BSDE (ξ, f) with continuous generator satisfying $(\xi_1, f_1) \leq (\xi, f) \leq (\xi_2, f_2)$ has at least one solution (Y, Z) such that $Y_1 \leq Y \leq Y_2$. This result is presented in a general setting. The solutions Y_1 and Y_2 are assumed continuous but no integrability condition on none of the terminal data of the three involved BSDEs is needed. We explain how the most previous results in one dimensional BSDEs can be deduced from this result. For instance, one can show that the solvability of quadratic BSDEs, with $|H(t, y, z)| \leq \alpha + \beta|y| + \frac{\gamma}{2}|z|^2$ is reduced to the solvability of a simple BSDE which generator is zero. We then deduce, in this case, the existence of an exponentially integrable solution when the terminal value has some exponential moment. Comparing with the previous works, here the exponential integrability of the terminal value is naturally generated. If time permits, we also explain how the solvability of BSDEs which generator is dominated by $a + b|y| + c|y \ln |y||$ is also reduced to the solvability of a BSDE which generator is zero.

Dirk Becherer, *Monday 11.00 – 11.30*

Good deal hedging and valuation under combined uncertainty about drift and volatility: A 2BSDE approach

We derive robust good-deal hedges and valuations under combined model ambiguity about the drift and volatility of asset prices for generically incomplete markets. Solutions for measurable contingent claims are fully characterized by 2nd-order BSDEs with non-convex drivers, by building on recent progess on wellposedness and non-linear kernels by D.Possamai, X.Tan and C.Zhou. Good-deal valuations are determined such that not just opportunities for arbitrage are excluded but also for overly attractive reward-to-risk ratios, by restricting instantaneous Sharpe ratios in any market extension by derivatives. From a finance point of view, this permits for hedges and valuation bounds than are less extreme than those from the more fundamental approach of almost-sure superhedging and its corresponding no-arbitrage bounds. This shows in particular in examples for incomplete markets. In mathematical terms, it demands however that not just ambiguities about the volatility but also about the drift become relevant. This is joint work with Klebert Kentia.

[1] Preprint on ssrn.com/abstract=2951742, Goethe-Univ.Frankfurt am Main.

Christel Geiss, *Monday 11.30 – 12.00*

Malliavin derivative of Lévy driven BSDEs with bounded solutions

Boundedness, existence and uniqueness of solutions to BSDEs was proved in [1] using Malliavin derivatives and comparison theorems. Here we show that this approach also works for BSDEs with jumps

$$Y_t = \xi + \int_t^T \mathbf{f}(s, Y_s, Z_s, U_s)ds - \int_t^T Z_s dW_s - \int_{(t,T]\times\mathbb{R}} U_s(x)\tilde{N}(ds, dx), \quad t \in [0, T],$$

where W denotes the Brownian motion and $\tilde{N}$ a compensated Poisson random measure. In order to compute Malliavin derivatives we require that the generator has the structure

$$\mathbf{f}(s, Y_s, Z_s, U_s) = f\left(s, Y_s, Z_s, \int_{\mathrm{R}\setminus\{0\}} g(s, U_s(x))\kappa(x)\nu(dx)\right)$$

where ν denotes the Lévy measure. Similar to [1] it turns out that responsible for the boundedness of (Y, Z, U) is the boundedness of the terminal condition and its Malliavin derivative. The requirements on the generator $\mathbf{f}(\omega, s, y, z, u)$ are Lipschitz continuity w.r.t. y and certain boundedness of its Malliavin derivative, while only a local Lipschitz property w.r.t. z and u is needed. As an application, we derive a general existence result for BSDEs. This is joint work with Alexander Steinicke (University of Graz).

[1] P. Cheridito and K. Nam, *BSDEs with terminal conditions that have bounded Malliavin derivative*, Journal of Functional Analysis, 266, 1257-1285, 2014.

Stefan Geiss, *Tuesday 11.00 – 11.30*

Permutation invariant functionals of Lévy processes and BSDEs

We study natural invariance properties of functionals defined on Lévy processes and show that they can be described by a simplified structure of the deterministic chaos kernels in Itô's chaos expansion. These structural properties of the kernels relate intrinsically to a measurability with respect to invariant σ-algebras. The results are applied to BSDEs driven by Lévy processes to deduce structural properties of their solutions. This is joint work with Florian Baumgartner (Innsbruck) [1].

[1] F. Baumgartner and S. Geiss: Permutation Invariant Functionals of Lévy Processes. Transactions AMS 2017.

Alexander Steinicke, *Tuesday 11.30 – 12.00*

Approximation of Levy driven BSDEs by BSDEs with underlying finite activity Levy processes and application to comparison theorems.

We consider backward stochastic differential equations in the Levy setting with linear growth generators which exceed the Lipschitz condition. In the talk, we discuss how solutions to such BSDEs can be approximated by solutions to BSDEs driven by a finite activity Levy process. All results are shown for generator functions that are subject to an extended monotonicity condition. Together with theorems that grant existence and uniqueness, we apply the approximation procedure mentioned above to prove a comparison theorem for such BSDEs.

Kihun Nam, *Tuesday 12.00 – 12.30*

BSEs, BSDEs and fixed point problems

We introduce a class of backward stochastic equations (BSEs) that extend classical BSDEs and include many interesting examples of generalized BSDEs as well as semimartingale backward equations. We show that a BSE can be translated into a fixed point problem in a space of random vectors. This makes it possible to employ general fixed point arguments to establish the existence of a solution. For instance, Banach's contraction mapping theorem can be used to derive general existence and uniqueness results for equations with Lipschitz coefficients, whereas Schauder type fixed point arguments can be applied to non-Lipschitz equations. The approach works equally well for multidimensional as for one-dimensional equations and leads to results in several interesting cases such as equations with path-dependent coefficients, anticipating equations, McKean-Vlasov type equations and equations with coefficients of superlinear growth. This is a joint work with Kihun Nam.

BSDEs, Reflected BSDEs and the General Theory of Processes

Organiser: Miryana Grigorova, **Room: JCMB Lecture Theatre C**

Miryana Grigorova, *Monday 10:30 – 11:00*

Optimal stopping with non-linear f-expectations and Reflected BSDEs: the completely irregular case

We consider the optimal stopping problem with non-linear f-expectation (induced by a BSDE) without making any regularity assumptions on the payoff process ξ. We show that the value family can be aggregated by an optional process Y . We characterize the process Y as the $\mathcal{E}^f$-Snell envelope of ξ. We also establish an infinitesimal characterization of the value process Y in terms of a Reflected BSDE with ξ as the obstacle. To do this, we first establish some useful properties of Reflected BSDEs with completely irregular obstacles, in particular an existence and uniqueness result and a comparison theorem.

Based on joint works with P. Imkeller, E. Offen, Y. Ouknine and M.-C. Quenez.

Marie-Claire Quenez, *Monday 11.00 – 11.30*

Doubly Reflected BSDEs and $\mathcal{E}^f$-Dynkin games: beyond the right-continuous case

We formulate a notion of doubly reflected BSDE in the case where the barriers ξ and ζ do not satisfy any regularity assumption. Under a technical assumption (a Mokobodzki-type condition), we show existence and uniqueness of the solution. In the case where ξ is right upper-semicontinuous and ζ is right lower-semicontinuous, the solution is characterized in terms of the value of a corresponding $\mathcal{E}^f$-Dynkin game, i.e. a game problem over stopping times with (non-linear) f-expectation, where f is the driver of the doubly reflected BSDE. In the general case where the barriers do not satisfy any regularity assumptions, the solution of the doubly reflected BSDE is related to the value of "an extension" of the previous non-linear game problem over a larger set of "stopping strategies" than the set of stopping times. This characterization is then used to establish a comparison result and *a priori* estimates with universal constants.

Based on a joint work with M. Grigorova, P. Imkeller, and Y. Ouknine.

Nicole El Karoui, *Monday 11.30 – 12.00*

Recovery of forward utility from given monotonic characteristics/Extremal processes

Based on a joint work with Mohamed M'rad.

Martingale Representation, BSDEs and Enlargement of Filtrations

Organisers: P. Di Tella, M. Jeanblanc and H.-J. Engelbert, **Room: JCMB Room 5327**

Barbara Torti, *Monday 10.30 – 11.00*

Martingale representations in progressive enlargement by a general semimartingale: possible applications

Let $\mathbb{F}$ and $\mathbb{H}$ be two filtrations on the same probability space $(\Omega, \mathcal{F}, P)$. Assume that $\mathbf{X}$ and $\mathbf{Y}$ are multidimensional special square integrable semimartingales such that $\mathbf{X}$ enjoys the $\mathbb{F}$-predictable representation property (p.r.p.) and $\mathbf{Y}$ enjoys the $\mathbb{H}$-p.r.p. Denote by $\mathbf{M}$ and $\mathbf{N}$ the martingale parts of $\mathbf{X}$ and $\mathbf{Y}$. If there exists a decoupling measure for $\mathbb{F}$ and $\mathbb{H}$, then, according to different choices of probability on $(\Omega, \mathcal{F})$, two different martingale representations for the enlarged filtration $\mathbb{F} \vee \mathbb{H}$ hold. The first representation is expressed in terms of $\mathbf{M}$, $\mathbf{N}$ and the covariation process $[\mathbf{M}, \mathbf{N}]^V$, which is the vector process defined as any fixed sort order of the family $([M^i, N^j],\ i = 1, \ldots, m, j = 1, \ldots, n)$. The second martingale representation involves $\mathbf{X}$, $\mathbf{Y}$ and $[\mathbf{X}, \mathbf{Y}]^V$.

In this talk we discuss possible applications of the previous results: First of all, in the case of one dimensional semimartingales X and Y, we compute the multiplicity in the sense of Davis and Varaiya of $\mathbb{F} \vee \mathbb{H}$.

Then we propose an extension of the classical Kusuoka's theorem. We assume $\mathbb{H}$ to be the completed natural filtration of the default process associate to a general random time τ and we derive a $(P, \mathbb{F} \vee \mathbb{H})$-basis of martingales. Finally we deal with a case when no decoupling measure for $\mathbb{F}$ and $\mathbb{H}$ may exist. This is when $\mathbb{F}$ is the natural filtration of a Poisson process $\mathcal{N}$ and $\mathbb{H}$ is the completed natural filtration of the default process associated to a random time τ, which has positive probability to be equal to some of the jumps of $\mathcal{N}$.

Victor Nzengang Feunou, *Monday 11.00 – 11.30*

Reflected BSDEs in a progressively enlarged filtration

We study RBSDEs in the setup where the underlying filtration $\mathbb{G} = (\mathcal{G}_t)_{t \geq 0}$ is given by the progressive enlargement of a reference filtration $\mathbb{F} = (\mathcal{F}_t)_{t \geq 0}$ by a default time τ, i.e.

$$\mathcal{G}_t = \cap_{s>t} \left(\mathcal{F}_s \vee \sigma(\tau \wedge s) \right), t \geq 0.$$

Assuming a density hypothesis on the regular conditional distribution of τ given $\mathbb{F}$, we establish a two step algorithm to solve RBSDEs in the filtration $\mathbb{G}$ which consists in solving two alternative weakly coupled systems of RBSDEs: first a RBSDE in the filtration $\mathbb{G}^\tau = (\mathcal{F}_t \vee \sigma(\tau))_{t \geq 0}$ whose solution leads to the value after τ of the solution to our original RBSDE, and then a RBSDE in the filtration $\mathbb{F}$ whose solution is the solution to our original RBSDE before τ. Based on our algorithm, we show for $p > 1$ and for a general filtration F, the existence of L^p-solutions for Lipschitz drivers. We also consider drivers with quadratic growth in the control variable for which we prove existence of a solution in the setup where $\mathbb{F}$ is a Brownian filtration. This talk is based on a joint work with Peter Imkeller.

Alexandre Popier, *Monday 11.30 – 12.00*

BSDEs with jumps in general filtration (and optimal targeting problem)

In this talk we present some results obtained with T. Kruse (Duisburg-Essen, Germany) about BSDEs in a general filtration. Our initial aim was to extend the work of S. Ankirchner, M. Jeanblanc and T. Kruse (2014) on liquidation problem. We wanted to remove the Brownian setting since the representation property was not used. And to enlarge the result to the Knightian uncertainty setting using second order BSDEs, the condition on the filtration has to be the weakest as possible (work in progress with C. Zhou (NUS)). Hence we deal with a BSDE with a generator monotone w.r.t. y, *without any additional condition on the underlying filtration* (only right-continuity and completeness). The driver depends also on the integrands of the stochastic integrals (w.r.t. the Brownian motion and to the Poisson measure).

In this talk we present in details the notion of L^p-solution for $1 < p < 2$. We emphasize that the proof has to be modified in order to control the part of the solution coming from the predictable representation w.r.t. the Poisson measure. The procedure is quite different from the case $p \geq 2$. For the singular solution (and the optimal closure problem), we will discuss the sufficient *left-continuity condition* imposed on the filtration at the terminal time and why we should avoid the thin time case.

Shiqi Song, *Tuesday 11.00 –11.30*

Condition (A) and some consequences: martingale representation property, Markov property and BSDEs

The enlargement of filtration theory is used in modeling the situation of market participants who trade the same securities with different information sources. The most noticeable aspect of such a situation is the potential profits that a market participant may hope to make with the extra information he possesses, with respect to the others. It is precisely the issue of the insider trading problem. However, there exists another aspect of the theory, as important as the first one, that is the invariance principle. Actually, few can claim at his disposal of the whole market information. However, the market composed of various participants remains generally stable and free of (some type of) arbitrage. The mathematical explanation of this situation is the invariance principle which consists to say that, till some degree, the differential in information will not create any arbitrage opportunity. The mathematical work behind the invariance principle is to determine the exact degree under which the invariance principle is valid and to provide the corresponding trading procedure in the reduced information flow. In this talk, we present a particular case of the invariance principle, that we call the condition (A). We speak about notably the martingale representation property, the Markov property and the resolution of BSDEs under the condition (A).

Claudio Fontana, *Tuesday 11.30 – 12.00*

Martingale representation results in initially enlarged filtrations

We study the strong predictable representation property in filtrations initially enlarged with a random variable L. We prove that the strong predictable representation property can always be transferred to the enlarged filtration as long as the classical density hypothesis of Jacod (1985) holds. This generalizes the existing martingale representation results and does not rely on the equivalence between the conditional and the unconditional laws of L. The results are illustrated in the context of hedging contingent claims under insider information.

Hans-Jürgen Engelbert, *Tuesday 12.00 – 12.30*

BSDEs and log-utility maximization in progressively enlarged Lévy filtrations

In this talk we consider an investor who maximizes his preferences in a financial market up to an exogenous event, whose realization time τ cannot be inferred from the available information in the market (e.g., death of the investor or default of the market). To solve this problem, we follow a dynamical approach based on an optimal martingale principle and on BSDEs. The information of the market is modeled by the natural filtration of a general Lévy process. This filtration is progressively enlarged by a random time τ satisfying the density hypothesis. As a first step we show a predictable representation property in the enlarged filtration. Then we prove existence and uniqueness of the solution of BSDEs in the enlarged filtration. Finally we apply these results to the utility maximization problem under the hypothesis that the utility function is logarithmic. We solve the problem in both cases for a continuous price process and a price process with jumps.This talk is based on a joint work with Paolo Di Tella.

Pathwise stochastic calculus

Organisers: G. dos Reis and N. Perkowski, **Room: JCMB Lecture Theatre A**

Vladimir Vovk, *Monday 10.30 – 11.00*

Continuous martingales without probability

In this talk I will define classes of supermartingales, martingales, and semimartingales in idealized financial markets with continuous price paths. After stating probability-free versions of a number of standard results of stochastic calculus in this setting, I will apply them to the equity premium and CAPM. Based on joint work with Glenn Shafer.

Joscha Diehl, *Monday 11.00 – 11.30*

Feyman-Kac formulae for rough PDEs

Using a stochastic representation via (B)SDEs with an additional rough drift, we deduce well-posedness results for a class of (semi)-linear rough PDEs.

Danyu Yang, *Monday 11.30 – 12.00*

Integration of rough paths

The theory of rough path develops a mathematical tool to model the evolution of controlled systems driven by highly oscillating signals, and the continuity theorem explains the convergence of controlled systems. There have been rapid developments in the field. This talk will focus on path integration. We build a connection between rough path theory and a noncommutative combinatorial Hopf algebra, and reduce the integration of rough path to a non-abelian version of the classical Young integration.

Rama Cont, *Thursday 16.00 – 16.30*

Functional calculus and pathwise integration for paths of finite quadratic variation

We present recent results on pathwise integration and functional calculus for functionals of paths with finite quadratic variation, using the approach initiated by H. Foellmer (1981) and extended by Cont & Fournie (2010) to the functional setting. We show that the Foellmer integral, defined as a limit of left Riemann sums, satisfies an isometry property which allows it to be extended as a continuous functional over an appropriately defined space of integrands. This leads to a unique pathwise ('Doob-Meyer') decomposition of functionals of irregular path into a sum of a rough component and a component with zero quadratic variation. We show that these results provide a natural calculus for functionals of controlled rough paths. [1] V Bally, L Caramellino, R Cont (2016) Stochastic integration by parts and Functional Ito calculus (Lectures Notes of the Barcelona Summer School on Stochastic Analysis, Centro de Recerca de Matematica, July 2012), https://link.springer.com/book/10.1007%2F978-3-319-27128-6 [2] Anna Ananova, Rama Cont (2017) Pathwise integration with respect to paths of finite quadratic variation, Journal de Mathematiques Pures et Appliquees, http://doi.org/10.1016/j.matpur.2016.10.004 [3] Anna Ananova, Rama Cont (2017) Functional calculus for controlled rough paths, Working Paper.

Mauro Rosestolato, *Thursday 16.30 – 17.00*

Representation of path-dependent PDEs in Hilbert spaces: relationship between two notions of viscosity solution

Path-dependent PDEs associated with stochastic models with delay can be addressed through viscosity solutions with two main approaches. The first one consists in introducing a notion of solution where tests are path-dependent and the local maximality/minimality condition at the point to test holds in a probabilistic sense. The second one consists in rephrasing the original problem in a function space: in this case, the PDE is no more path-dependent but its domain becomes infinite dimensional, hence viscosity solution theory for Hilbert or, more generally, Banach spaces is used. In this joint work with Z. Ren (Paris Dauphine) and N. Touzi (Ecole Polytechnique), we explore the connection between the two settings and study to which extent they are equivalent.

Pietro Siorpaes, *Thursday 17.00 – 17.30*

Pathwise BDG inequality and applications

The celebrated Burkholder-Davis-Gundy (BDG) inequalities state that the running maximum of a martingale and the square root of its quadratic variation have comparable L^p norms. Here we discuss pathwise versions of the BDG inequalities, which automatically imply the standard ones simply by taking expectations, and have several other consequences; for example, we immediately recover the BDG inequalities for Bessel processes of dimension bigger than 1.

Paul Gassiat, *Friday 9.00 – 9.30*

Existence of densities for the 3D stochastic quantization equation

Many nonlinear stochastic PDEs arising in statistical mechanics are ill-posed in the sense that one cannot give a canonical meaning to the nonlinearity. Nevertheless, Martin Hairer's theory of regularity structures provides us with a good notion of solution for a large class of such equations. An important example is given by the 3D stochastic quantization equation $(\partial_t - \Delta)u = -u^3 + Cu + \xi$, where ξ is space-time white noise. In this talk, we show that finite-dimensional (space-time) projections of the solution u have laws which are absolutely continuous with respect to Lebesgue measure. Our proof is based on combining Malliavin calculus and tools from regularity structures theory. Interestingly, the result still holds if the noise is degenerate in some directions, as long as it is sufficiently rough on small scales. Based on a joint work with Cyril Labbé (Paris-Dauphine).

Benjamin Gess, *Friday 9.30 – 10.00*

Well-posedness by noise for scalar conservation laws

In certain cases of (linear) partial differential equations random perturbations have been observed to cause regularizing effects, in some cases even producing the uniqueness of solutions. In view of the long-standing open problems of uniqueness of solutions for certain PDE arising in fluid dynamics such results are of particular interest. In this talk we will extend some known results concerning the well-posedness by noise for linear transport equations to the nonlinear case.

David Prömel, *Friday 10.00 – 10.30*

Rough path integrals in the spirit of Föllmer integration

Föllmer's pathwise Ito formula and its associated pathwise integration provides a powerful tool in financial mathematics. We justify the application of controlled rough path integrals in finance by showing that it is the limit of non-anticipating classical Riemann sums and not only a limit of so-called compensated Riemann sums. In particular, this demonstrates that rough path integration can be seen as a natural extension of Föllmer's integration. As an application to portfolio theory, we present a model-free version of Cover's "universal" portfolio which is in the long run as good as the best retrospectively chosen portfolio. The talk is based on joint works with Christa Cuchiero and Nicolas Perkowski.

Multidimensional quadratic BSDEs and their applications

Organisers: Michael Kupper, Peng Luo and Hao Xing, **Room: JCMB Room 5326**

Gordan Žitković , *Monday 15.30 – 16.00*

Equilibria in incomplete continuous-time financial markets and systems of BSDEs

The problem of existence of equilibrium prices in incomplete continuous-time financial markets has proved to be one of the most stubborn open problems in financial economics and mathematical finance. While the complete case was settled 30 years ago in the work of Ioannis Karatzas and others, very little is known about what happens when no completeness assumptions are made.

A new approach to this problem - and its positive resolution - in the special case when all agents have exponential utility functions will be presented. It is based on systems of quadratic backward stochastic differential equations (BSDEs) and provides a general existence result for a class of such equations under structural conditions. It is interesting that very similar conditions appear in completely different contexts - e.g, when one tries to construct martingales on Riemannian manifolds or find Nash points of non-zero-sum stochastic games. Joint work with Hao Xing.

Dewen Xiong , *Monday 16.00 – 16.30*

The time-inconsistent α-robust open-loop equilibrium strategy

We consider the log-utility optimization under E^{α}, a nonlinear expectation which is not time-consistent and convex or concave for $\alpha \in (0,1)$. We study the Nash equilibrium open-loop strategy introduced by Hu Jin and Zhou(2012). By introduce a fully coupled quadratic BSDEs system, we characterize the equilibrium strategy and show such BSDEs system has a unique solution as in Xing and Žitković(2017).

Ludovic Tangpi , *Monday 16.30 – 17.00*

Multidimensional Markov FBSDEs with superquadratic growth

We give local and global existence and uniqueness results for systems of coupled FBSDEs in the multidimensional setting and with genera- tors allowed to grow arbitrarily fast in the control variable. Our results are based on Malliavin calculus arguments and pasting techniques.

Kihun Nam , *Tuesday 11.00 – 11.30*

Path-differentiability of multidimensional quadratic BSDE driven by a continuous martingale

We study existence, uniqueness, and path-differentiability of solution for backward stochastic differential equation (BSDE) driven by a continuous martingale M. The driver and the terminal condition depends on the path of M. The path-derivative is defined as a directional derivative with respect to the path-perturbation of M in a similar way to the vertical functional derivative introduced by Dupire (2009), and Cont and Fournie (2013). We first prove the existence, uniqueness, and path-differentiability of solution (Y,Z) in the case where the driver is Lipschitz. After proving that Z is a path-derivative of Y, we extend the results to locally Lipschitz driver. When the BSDE is one-dimensional, we could show the existence and uniqueness of solution. On the contrary, when the BSDE is multidimensional, we show existence and uniqueness only when the quadratic variation of M is small enough: otherwise, we provide a counterexample that has blowing-up solution.

Alexander Fromm , *Tuesday 11.30 – 12.00*

Optimal position targeting via decoupling fields

We consider a variant of the basic problem of the calculus of variations, where the La- grangian is convex and subject to randomness adapted to a Brownian filtration. We solve the problem by reducing it, via a limiting argument, to an unconstrained control problem that consists in finding an absolutely continuous process minimizing the expected sum of the Lagrangian and the deviation of the terminal state from a given target position. Using the Pontryagin maximum principle we characterize a solution of the unconstrained control problem in terms of a fully coupled forward-backward stochastic differential equa- tion (FBSDE). We use the method of decoupling fields for proving that the FBSDE has a unique solution.

Peng Luo , *Tuesday 12.00 – 12.30*

Multidimensional BSDEs with triangularly quadratic generators

Motivated by the recent works of Hu and Tang (2016) and Xing and Žitković (2016), we consider a type of multidimensional BSDEs with triangularly quadratic generators. We obtain the existence and uniqueness of solutions. Some applications in stochastic differential games are given.

McKean-Vlasov SDEs: Control, Regularity and Numerics

Organisers: J.–F. Chassagneux and L. Szpruch, **Room: JCMB Lecture Theatre A**

Andrea Cosso, *Monday 15.30 – 16.00*

Randomization method in stochastic optimal control

The talk is about a recently introduced methodology in stochastic optimal control theory, known as randomization method, firstly developed for classical Markovian control problem in the paper: I. Kharroubi and H. Pham "Feynman-Kac representation for Hamilton-Jacobi-Bellman IPDE", Ann. Probab., 2015. The randomization method consists, in a first step, in replacing the control by an exogenous process independent of the driving noise and in formulating an auxiliary (?randomized?) control problem where optimization is performed over changes of equivalent probability measures affecting the characteristics of the exogenous process. We will discuss the main features of this approach, showing that the randomization method allows for greater generality beyond the Markovian case. In particular, we may consider stochastic control problems with path-dependence in the coefficients (with respect to both state and control), without requiring any non-degeneracy condition on the controlled equation. The talk is based on joint works with E. Bandini, M. Fuhrman, H. Pham.

Etienne Tanré, *Monday 16.00 – 16.30*

Mean-field limit of a network of neurons with correlated synaptic weights: a Large Deviation Principle

Consider a network of neurons in interaction. The neurons are characterized by their membrane potential, say V_t^i for neuron i, solutions of a system of Stochastic Differential Equations. We assume that the membrane potential of each neuron is governed by a Brownian noise, independent of the other neurons. The dynamic of V^i contains an interaction of the form $\sum_j J^{j,i} f(V_t^j)$ where the "synaptic weights $J^{j,i}$ are correlated random variables.?We study the asymptotic behavior when the number of neurons tends to infinity and obtain a Large Deviation Principle. This is a common work with O. Faugeras (Inria) and J. MacLaurin (Univ. of Sydney).

Dan Crisan, *Monday 16.30 – 17.00*

Cubature on Wiener Space for McKean-Vlasov SDEs with Smooth Scalar Interaction

I will present two cubature on Wiener space algorithms for the numerical solution of McKean-Vlasov SDEs with smooth scalar interaction. The first method has be introduced recently by Raynal and Trillos under a uniformly elliptic assumption. For this method we extend the analysis to a uniform strong Hrmander assumption. The second method is new and uses Lagrange polynomial interpolation. This is joint work with Eamon McMurray (Imperial College London).

Mireille Bossy, *Tuesday 11.00 – 11.30*

On two examples of McKean Nonlinear SDEs with some local interaction in the diffusion part

Motivated by models and methodologies in physics and finance, we consider two situations of McKean SDEs that explore existence results when the diffusion coefficient is composed with an irregular McKean Kernel. The first case is a McKean conditional diffusion equation, the second one is a moderated local McKean diffusion with weakened hypotheses on the moderation function. In the two cases, the analysis of the Fokker Planck PDE takes part to the construction of a solution to the SDE. This is a joint work with J.-F Jabir, University of Valpareiso.

Julien Reygner , *Tuesday 11.30 – 12.00*

Mean-field Atlas models

Atlas models are a class of equity market models in which the dynamics of the price of an asset only depends on its rank within the portfolio. When this dynamics exhibits a mean-field scaling, propagation of chaos techniques arising from kinetic theory lead to a functional nonlinear description of the evolution of the market. In particular, one can obtain a fluid limit for the distribution of the capital, which reproduces some features of actual data, such as a Pareto law for capital concentration.

Benjamin Jourdain, *Tuesday 12.00 – 12.30*

Existence to a calibrated regime-switching local volatility model

By Gyongy's theorem, a local and stochastic volatility (LSV) model is calibrated to the market prices of all European call options with positive maturities and strikes if its local volatility function is equal to the ratio of the Dupire local volatility function over the root conditional mean square of the stochastic volatility factor given the spot value. This leads to a SDE nonlinear in the sense of McKean. Particle methods based on a kernel approximation of the conditional expectation, as presented by Guyon and Henry-Labordère (2011), provide an efficient calibration procedure. But so far, no global existence result is available for the limiting SDE. With Alexandre Zhou, we obtain existence in the special case of the LSV model called regime switching local volatility, where the stochastic volatility factor is a jump process taking finitely many values and with jump intensities depending on the spot level.

Mykhaylo Shkolnikov, *Wednesday 11.00 – 11.30*

A predictor of systemic risk and particle systems interacting through hitting times

We propose an interacting particle system model for the mutual exposures of banks. In the model banks may default, possibly triggering cascades of defaults of other banks. When the aggregate losses from default cascades become on the order of the whole banking system, we speak of a systemic event. The main results of the paper show that, when the number of banks is large, the relative asset value profile of banks that are close to failure can be used as a predictor of systemic events.

Sean Ledger, *Wednesday 11.30 – 12.00*

Some McKean–Vlasov problems on the half-line

We present two models of interacting continuous-time particle systems on the half-line and study their large population limits. One is motivated from mathematical finance and the other by mathematical neuroscience.

In the first, we consider a population of diffusions interacting through a correlation that is a function of the proportion of particles that have hit an absorbing threshold. It is shown that the system converges to the unique solution of a non-linear heat equation with random transport and discontinuous coefficients. A useful tool here is the Skorokhod M1 topology on the space of cadlag processes taking values in the tempered distributions.

For the second example, we consider a contagious system where each particle receives a kick towards the absorbing boundary whenever another particle reaches that boundary. The large population limit gives rise to the corresponding PDE and M–V problems. It is shown that solutions must blow-up in finite time if this positive feedback effect is made too strong. From a practical perspective this is desirable, so we consider how to adjust the notion of a solution to accommodate this effect.

Huyên Pham, *Wednesday 12.00 – 12.30*

Stochastic control under partial observation

We study and revisit the optimal control problem of partially observed stochastic systems. By using a control randomization method, we provide a backward stochastic differential equation (BSDE) representation for the value function in a general framework including path-dependence in the coefficients (both on the state and control) and without any non degeneracy condition on the diffusion coefficient. In the standard Markovian case, this BSDE representation has important implications: it allows us to obtain a corresponding randomized dynamic programming principle (DPP) for the value function, which is obtained from a flow property of an associated filter process. This DPP is the key step towards our main result: a characterization of the value function of the partial observation control problem as the unique viscosity solution to the corresponding dynamic programming Hamilton-Jacobi-Bellman (HJB) equation. The latter is formulated as a new, fully non linear partial differential equation on the Wasserstein space of probability measures, and is derived by means of the recent notion of differentiability with respect to probability measures introduced by P.L. Lions in his lectures on mean-field games at the Collge de France. An important feature of our approach is that it does not require any condition to guarantee existence of a density for the filter process solution to the controlled Zakai equation, as usually done for the separated problem. We give an explicit solution to our HJB equation in the case of a partially observed non Gaussian linear quadratic model. Finally, if time permitting, we discuss the issue of numerical treatment of the proposed randomized BSDE for solving partial observation control problem.

Based on joint works with E. Bandini (Luiss University), A. Cosso (Politecnico Milano), and M. Fuhrman (Universita di Milano).

BSDEs and SDEs with mean reflexion and particles systems

Organisers: Ph. Briand and A. Guillin, **Room: JCMB Lecture Theatre B**

Philippe Briand, *Monday 15.30 – 16.00*

BSDEs with mean reflexion

This talk is based on a joint paper with R. Elie and Y. Hu concerning reflected backward stochastic differential equations in the case where the constraint involves the law of the solution to the equation rather than its paths. A typical example of such reflexion consists in solving a BSDE with the constraint that the expected value of a function of the solution remains nonnegative at each time. I will explain how to obtain an existence and uniqueness result for such equations and I will give some basic properties of the solutions. Finally, I will give some recent developments of this theory.

Paul-Éric Chaudru de Raynal, *Monday 16.00 – 16.30*

Mean Reflected SDE

In this talk, I will present a class of Stochastic Differential Equation with Mean Reflection. The main particularity of these equations comes from the non-linearity (in a McKean-Vlasov sense) of the reflection: the constraint acts on the law of the solution itself. Typical examples of applications come from Risk Measure Theory. These equations have been introduced, in their backward form, in a work of Briand, Elie and Hu. Here we will present assumptions that guarantee existence and uniqueness of a solution as well as the existence of a density with respect to the Lebesgue measure for the constraint process. Then, we will show that these equations can be obtain as a limit of particles system interacting in a mean field way via their reflection. This, in particular, will allow us to present a numerical algorithm for their approximation. This is a joint work with P. Briand, A. Guillin and C. Labart.

Hélène Hibon, *Monday 16.30 – 17.00*

Mean Reflected BSDEs - Chaos propagation

In this talk, we deal with Reflected Backward Stochastic Differential Equations for which the constraint is not on the paths of the solution but on its law. With the help of the Snell envelope and all its characterizations, we extend the recent work of Briand, Chaudru de Raynal, Guillin and Labart on the chaos propagation for mean reflected SDEs to the backward framework. We develop two methods - according to whether the constraint is affine or not - to define the path reflected particle system for Lipschitz generators. We study then its convergence towards the square integrable deterministic flat solution to the mean reflected BSDE (MRBSDE).

Games and BSDEs

Organiser: S. Hamadene, **Room: JCMB Lecture Theatre C**

Samuel Cohen, *Monday 15.30 – 16.00*

Fun, Games and Graphs with EBSDEs

When studying ergodic games, or ergodic control systems with jumps, one often struggles with the fact that the comparison theorem does not hold without extra conditions. In this talk, we will look at some settings in which these problems arise, and some methods to get around them.

Boualem Djehiche, *Monday 16.00 – 16.30*

ZERO-SUM GAMES FOR MARKOV CHAINS OF MEAN-FIELD TYPE

We establish existence of a saddle-point for the zero-sum games associated with payoff functionals of mean-field type, under a dynamics driven by a class of Markov chains of mean-field type.

Zhiyong YU, *Monday 16.30 – 17.00*

Saddle Point of Zero-Sum Linear-Quadratic Stochastic Differential Game

In this talk, we consider a two-person zero-sum linear-quadratic stochastic differential game problem. From a new viewpoint, we construct a saddle point for the game in feedback control-strategy pair form based on the solution of a Riccati equation. A key part of our analysis involves proving the global solvability of this Riccati equation, which is interesting in its own right. Moreover, we demonstrate an indefinite phenomenon arising from the linear-quadratic game.

Continuous time contract theory and BSDEs

Organisers: H. Xing, **Room: JCMB Room 5327**

Dylan Possamaï, *Tuesday 15.30 – 16.00*

Volatility demand management for electricity: a moral hazard approach

In this work, we propose a model of electricity demand management through a principal-agent problem, allowing to obtain almost explicit optimal compensations for the consumer. We then illustrate our findings through several numerical experiments, putting the emphasis on the practical implementation of the contracts. This is a joint work with René Aïd and Nizar Touzi.

Yiqing Lin, *Tuesday 16.00 – 16.30*

Second-order BSDEs on random horizon with the application in Principle-Agent problem

In this talk, we shall introduce our recent progress on the second-order BSDEs on random horizon. In particular, we first generalize results of (reflected) BSDEs on random horizon. Then we study a control problem on the solutions of BSDEs and prove the existence and uniqueness of the corresponding second-order BSDEs. We afterwards apply this result to solve a Principle-Agent problem with moral hazard when the principle can decide the retirement of the agent. Joint work with Zhenjie Ren, Nizar Touzi and Junjian Yang.

Hao Xing, *Tuesday 16.30 – 17.00*

Optimal contracting with unobservable managerial hedging

We develop a continuous-time model where a risk-neutral principal contracts with a CARA agent to initiate a project. The agent can increase the drift of the project's output by exerting costly hidden effort. In addition, the agent can trade the market portfolio and a risk-free bond in an unobservable private account (the managerial hedging behaviour). We formulate the problem as a regular-singular control problem and solve principal's optimal contracting problem. We show that unobservable managerial hedging under absolute performance evaluation is costly for incentive provision in that the principal's value generally decreases with the easiness of managerial hedging. Replacing an absolute performance evaluation contract by a relative one can improve both efficiency and value. Joint work with Yu Huang and Nengjiu Ju.

SPDEs and PDEs on singular spaces

Organisers: M. Hinz, **Room: JCMB Lecture Theatre C**

Ben Hambly, *Tuesday 15.30 – 16.00*

Some linear SPDEs on p.c.f. fractals

We discuss the stochastic heat and wave equations on p.c.f. self-similar sets, a class of finitely ramified fractals. We will focus on the case where the noise is additive and consider the question of the existence and regularity of random field solutions. By using suitable Kolmogorov continuity theorems we establish the existence and Hölder continuity properties of such solutions to these SPDEs.

Xuan Liu, *Tuesday 16.00 – 16.30*

Semi-linear parabolic PDEs on the Sierpinski gasket

In this talk, we consider a type of non-linear parabolic PDEs on the Sierpinski gasket and related BSDEs driven by Brownian motion on the gasket. In contrast to PDEs on regular spaces such as smooth manifolds, non-linear PDEs on Sierpinski gasket involve mutually singular measures due to the fractal nature of the Sierpinski gasket. Singularity between measures immediately brings difficulties in obtaining energy estimates for solutions of the PDEs. We will give two approaches, one probabilistic and the other analytic, to address this difficulty. The first approach uses techniques of BSDEs. We first establish existence and uniqueness of solutions of the associated BSDEs and then translate this result to PDEs by showing a representation formula for solutions of parabolic PDEs by solutions of the associated BSDEs. The second approach relies on a new type of Sobolev inequalities involving singular measures on the Sierpinski gasket. Using these Sobolev inequalities, we derive energy estimates for solutions of non-linear parabolic PDEs on the gasket and show the existence and uniqueness of solutions. Further results on such Sobolev inequalities will also be presented in this talk.

Elena Issoglio, *Tuesday 16.30 – 17.00*

Non-linear parabolic PDEs on metric measure spaces

In this talk I will investigate a class of non-linear parabolic PDEs living on a metric measure space and perturbed with a non differentiable space-time noise. I will recall results on existence and uniqueness of a mild solution and then discuss the regularity of the solution in space and time. The main tools used to give a meaning to the perturbed term in the PDE include fractional integrals and derivatives, and smoothing properties of semigroups. We also use pointwise-type products defined by means of pairing in suitable generalised Bessel potential spaces.

The PDE lives on a sigma-finite metric measure space: this setting includes classical linear spaces but also various classes of fractals. Moreover the (deterministic) perturbation can be seen as a realization of a suitable stochastic field, hence this theory can be applied path by path to a class of stochastic PDEs. Examples of metric measure spaces (e.g. fractals) and stochastic noises (e.g fractional Brownian fields) will be discussed.

This is a joint work with Martina Zähle.

Mohammud Foondun, *Wednesday 11.00 – 11.30*

Comparison principles for a class of stochastic heat equations driven by coloured noise

In this talk, we will look at a class of stochastic heat equations driven by coloured noise. We will present a few comparison results and explore the consequences of these results.

Joe Chen, *Wednesday 11.30 – 12.00*

Local ergodicity in the exclusion process on an infinite weighted graph

In deriving the hydrodynamic limit of a microscopic fluid model, one often needs to identify a local ergodic (or coarse graining) argument. On Euclidean lattices such argument is available by virtue of the underlying spatial symmetry. Without such symmetry it is not clear how to implement the coarse graining.

The goal of my talk is to describe a coarse graining argument for the (boundary-driven) symmetric exclusion process on a general weighted graph, by exploiting the resistance structure of the graph. In particular, this argument works on any strongly recurrent graph, in the sense of Barlow, Delmotte, and Telcs, which includes the Sierpinski gasket (SG) and other fractals. A key ingredient of the proof is the moving particle lemma which I proved recently using the octopus inequality of Caputo, Liggett, and Richthammer.

As a consequence of this argument, in joint works with Michael Hinz and Alexander Teplyaev, we are able to prove the hydrodynamic limit of the (weakly a)symmetric exclusion process on SG "heads-on."

Alexander Teplyaev, *Wednesday 12.00 – 12.30*

Integral estimates and Girsanov densities for singular perturbations of Ornstein-Uhlenbeck processes

The talk will consist of two parts: the nonlinear heat equation arising in the study of particle systems on fractals, and an infinite-dimensional Ornstein-Uhlenbeck operator perturbed by a singular (non-linear and non-smooth) drift. This is a preliminary report on joint works with J.P. Chen and M. Hinz, and with M. Gordina and M. Röckner.

Numerical approximations of high-dimensional BSDEs and PDEs

Organisers: M. Hutzenthaler and A. Jentzen, **Room: JCMB Lecture Theatre B**

Christa Cuchiero, *Tuesday 15.30 – 16.00*

Non-linear differential equations and affine processes

Affine processes have been used extensively to model financial phenomena since they are very tractable from an analytic point of view, in the sense that their marginal distributions can be obtained by solving a non-linear differential equation. It is well known by works of Dynkin-McKean-LeJan-Sznitman that one can turn this around and represent solutions of non-linear (P)DEs by certain affine processes. Recent advances in Mathematical Finance in this direction have been contributed by Henry-Labordere, Tan and Touzi. We contribute some general theory and some new aspects to this field, in particular by providing stochastic representations to large classes of non-linear ODEs. The talk is based on joint work with Josef Teichmann.

Thomas Kruse, *Tuesday 16.00 – 16.30*

Exponential time integrators for strong approximations of nonlinear SPDEs

In this talk we present a family of new approximation methods for high-dimensional PDEs and BSDEs. A key idea of our methods is to combine multilevel approximations with Picard fixed-point approximations. Thereby we obtain a class of multilevel Picard approximations. Our error analysis proves that for semi-linear heat equations, the computational complexity of one of the proposed methods is bounded by $O(d\,\varepsilon^{-(4+\delta)})$ for any $\delta > 0$, where d is the dimensionality of the problem and $\varepsilon \in (0, \infty)$ is the prescribed accuracy. We illustrate the efficiency of one of the proposed approximation methods by means of numerical simulations presenting approximation accuracy against runtime for several nonlinear PDEs from physics (such as the Allen-Cahn equation) and financial engineering (such as derivative pricing incorporating default risks) in the case of $d = 100$ space dimensions.

The talk is based on joint work with W. E, M. Hutzenthaler, and A. Jentzen.

Tan Xiaolu, *Tuesday 16.30 – 17.00*

Solving semilinear parabolic PDEs by branching diffusion processes.

We provide a probabilistic representation for a class of semilinear parabolic PDEs, using branching diffusion processes. We then discuss how to use this representation to obtain a numerical algorithm to solve these PDEs.

Mathematical Finance

Organisers: JM. Shkolnikov, L.Szpruch and T. Zariphopoulou, **Room: JCMB Lecture Theatre A (Tuesday), JCMB Lecture Theatre C (Thursday)**

Martin Schweizer , *Tuesday 15.30 – 16.00*

Arbitrage theory revisited

We look at the classical topic of absence of arbitrage and reconsider its formulation. The existing theory has some shortcomings, and we aim to address these by a new look at basic concepts and definitions. This is based on (ongoing) work (in progress) with Dániel Bálint.

Sigrid Källblad, *Tuesday 16.00 – 16.30*

A dynamic programming principle for distribution-constrained optimal stopping

We consider an optimal stopping problem where a constraint is placed on the distribution of the stopping time. Reformulating the problem in terms of so-called measure-valued martingales allows us to transform the marginal constraint into an initial condition and view the problem as a stochastic control problem; we establish the corresponding dynamic programming principle.

Beatrice Acciaio, *Tuesday 16.30 – 17.00*

McKean-Vlasov control problems and non-anticipative optimal transport

In my talk I will present a new approach to McKean-Vlasov control problems based on a mass transport perspective. In particular, I will provide a characterization of weak solutions to McKean-Vlasov control problems via dynamic optimal transport of non-anticipative nature. The talk is based on ongoing projects with J. Backhoff, R. Carmona, and P. Wang.

Tomoyuki Ichiba, *Thursday 16.00 – 16.30*

Backward stochastic differential equations in a financial network model

In this talk we shall consider a backward stochastic differential equation for a node coupled with neighboring nodes in a large financial network. We start with a finite system of linear stochastic equations for the nodes in the network to discuss existence and uniqueness of the system, and study its limit, as we let the number of nodes go to infinity. Then we shall consider the backward equation with Lipschitz coefficients in the limiting system and examine its properties.

Sergey Nadtochiy , *Thursday 16.30 – 17.00*

Control-stopping games for market microstructure

In this talk, I present a framework for modeling market microstructure based on continuous time control-stopping games. In this framework, the shape and dynamics of a Limit Order Book (LOB) arise as the outcome of an equilibrium between multiple agents. The agents make dynamic decisions on whether to provide or to consume the liquidity (i.e. submit limit or market orders, respectively), based on their beliefs about the size and direction of the next trade. On the mathematical side, we formulate the problem as a control-stopping game with a continuum of players. One of the main steps in constructing an equilibrium naturally leads to a system of Reflected Backward Stochastic Differential Equations (RBSDEs) with Oblique Reflection (i.e. in which the reflecting barriers depend on the solution). The latter problem has a very natural economic interpretation but does not fall within any known classes of equations, as it does not possess the important monotonicity properties of the classical RBSDEs with Oblique Reflection. I will discuss the mathematical challenges associated with such systems and will prove the existence of their solutions in several cases. The theory will be accompanied with applications and numerical examples.

Gechun Liang, *Thursday 17.00 – 17.30*

Exponential utility maximization with unbounded payoffs and its application to indifference valuation

We solve an exponential utility maximization problem with unbounded random endowments under portfolio constraints by using the elements from the theory of quadratic backward stochastic differential equations with unbounded terminal data, thus we generalize the previous result of Hu et al. (2005) [Ann. Appl. Probab., 15, 1691–1712] from bounded to unbounded case. We then apply our results to study utility indifference valuation of financial derivatives with unbounded payoffs, obtaining a new convex dual representation of the prices and their asymptotics for the risk aversion parameter. This is a joint work with Ying Hu and Shanjian Tang.

Recent advances on Optimal switching problems (via BSDE methods)

Organisers: Marie-Amélie Morlais, **Room: JCMB Room 5326**

Randall Martyr, *Wednesday 11.00 – 11.30*

A brief survey on results for optimal switching problems with signed switching costs.

The traditional optimal switching (or alternating control) problem features a decision maker (controller) that controls a dynamical system over time by sequentially choosing an operating mode (or regime) from a finite set of such modes. The controller has an objective function that measures the performance of an alternating control, which includes a profit function and a cost for each time the operating mode is switched. The controller's objective is to maximise this objective function over all feasible alternating controls. Optimal switching problems often arise in the analysis of flexible industrial projects (Real Options Analysis), with examples related to investment in electricity generation and valuation of energy storage. It is typical to assume that the switching costs are not beneficial to the controller and, given the nature of the objective function, are therefore taken to be positive. However, negative switching costs are important in models where the controller can partially recover its investment, or receive a subsidy/grant for investing in a new technology such as renewable (green) energy production. Unfortunately, in continuous-time models the allowance for negative switching costs may result in unbounded performance of an alternating control, which makes this case generally difficult to deal with theoretically. This talk presents an overview of the theoretical results on optimal switching problems with negative switching costs in continuous time and on a finite time horizon. Various probabilistic solution methods will be presented, including systems of coupled Snell envelopes, multidimensional Backward SDEs with oblique reflection, and constrained Backward SDEs with jumps. There will be a short discussion on the author's own recent contribution, highlighting the particular assumptions made and the manner of proof.

Marie-Amélie Morlais, *Wednesday 11.30 – 12.00*

Max-min and min-max systems of variational inequalities: connection with multidimensional BSDEs with bilateral obstacles and switching game

This talk is based on two joint works (2014 and 2017) with Pr S. Hamadne (LMM Le Mans) , Pr B. Djehiche (KTH Stockholm Sweden) and X. Zhao (former pHD student of S. Hamadne) . It is focused on the connection between systems of interconnected PDEs, related multidimensional BSDEs and the general switching game. We shall explain the main steps allowing to construct solutions to both the min-max and max-min systems of variational inequalities. Due to the non linearities of those systems and the presence of interconnected obstacles, those solutions are irregular and it requires the use of viscosity theory. In a second step (result of the second paper) the connection with a multidimensional BSDE with bilateral obstacles is established (existence and uniqueness holds provided we are in a Markovian framework of randomness. In a last part of the talk and as an application, we shall provide the relationship with the switching game (the existence of the value and of an optimal mixed strategy is established under quite restrictive conditions) and some of the open questions remaining unsolved.

Adrien Richou, *Wednesday 12.00 – 12.30*

Some existence and uniqueness results for obliquely reflected BSDEs

In this talk, we present some recent results on obliquely reflected BSDEs. In particular we are able to deal with assumptions on the generator weaker than in currently known results. An existence and uniqueness result is obtained in a bounded framework (i.e. the terminal condition and the random part of the generator are bounded) while a general existence result is obtained in the Markovian framework. Some examples of applications to switching problems are given. This is a joint work with Jean-Franois Chassagneux (University of Paris 7).

BSDE techniques for XVA calculations

Organisers: S. Crépey and S. Sturm, **Room: JCMB Room 5327**

Chao Zhou, *Wednesday 11.00 – 11.30*

The Sustainable BlackScholes Equations

In incomplete markets, a basic BlackScholes perspective has to be complemented by the valuation of market imperfections. In this paper we consider the sustainable BlackScholes equations that arise for a portfolio of options if one adds to their trade additive BlackScholes price, on top of a nonlinear funding cost, the cost of remunerating at a hurdle rate the residual risk left by imperfect hedging. We assess the impact of model uncertainty in this setup. This is a joint work with Yannick Armenti and Stéphane Crépey.

Maxim Bichuch, *Wednesday 11.30 – 12.00*

Robust XVA

Since the financial crisis of 2008, practitioners in derivative pricing recognized that linear pricing rules are no longer realistic. They started to add pricing adjustments to the clean price, stemming among others from counterparty credit risk, differential interest rates and capital requirements. As summary description for all those adjustment the term XVA has been established. In this talk we want to address the question how robust the calculations of XVA are, given the uncertainty of parameter estimation. Specifically, we will consider the example of hedging an exposure of Credit Default Swaps (CDS) with a counterparty prone to default, but where the exact default intensity is unknown. We provide price bounds on the XVA using the bounds on the unknown parameter, as well as bounds on the associated hedging strategies. Moreover, we show that these bounds are tight. This is joint work with Agostino Capponi and Stephan Sturm.

Wissal Sabbagh, *Wednesday 12.00 – 12.30*

When Capital is a Funding Source: The XVA Anticipated BSDEs

Economic capital (EC) can be used as a funding source by banks, at a risk-free cost instead of the additional risky spread of the bank in the case of unsecured borrowing. This results in a very significant reduction of funding costs and an FVA (funding valuation adjustment) that would ignore it would be grossly overestimated. Mathematically the intertwining of EC and FVA leads to an anticipated BSDE (ABSDE) for the FVA, where the driver entails a conditional risk measure of the one-year-ahead increment of the martingale part of the solution. Accounting further for the KVA (capital valuation adjustment) component of economic capital, with the ensuing feedback condition that EC must be greater than KVA, yields a system of ABSDEs for the FVA and the KVA processes considered simultaneously.

We show that the ensuing (FVA, KVA) ABSDE system is well-posed and we establish the convergence of a Picard approximation scheme. This is first done for a bank without debt. In the realistic case of a defaultable bank, the resulting ABSDEs, which are stopped before the default of the bank, are solved by projection on a reference filtration, assuming the default of the bank modeled by an invariance time.

Junbeom Lee, *Thursday 10.30 – 11.00*

Recovering Linear Equations of XVA in Bilateral Contracts

We investigate conditions to represent derivative price under XVA explicitly. As long as we consider different borrowing/lending rates, XVA problem becomes a non-linear equation and this makes finding explicit solution of XVA difficult. It is shown that the associated valuation problem is actually linear under some proper conditions so that we can have the same complexity in pricing as classical pricing theory. Moreover, the conditions mentioned above is mild in the sense that it can be obtained by choosing adequate covenants between the investor and counterparty.

Stéphane Crépey, *Thursday 11.00 – 11.30*

Invariance Times and BSDEs Stopped Before a Random Time

An invariance time is a stopping time such that local martingales with respect to some reduced filtration and changed probability measure, once stopped before that time, are local martingales with respect to the original model filtration and probability measure. Supposing an invariance time satisfying few additional practical conditions, we establish conditional expectation, martingale and semimartingale characteristics transfer formulas between the original and changed stochastic basis. These results are applied to the study of nonstandard BSDEs stopped before their terminal time, which appear in the context of counterparty risk.

Ankush Agarwal, *Thursday 11.30 – 12.00*

Rare event simulation related to financial risks: efficient estimation and sensitivity analysis

In this work, we develop the reversible shaking transformation methods on path space to estimate the rare event statistics arising in different financial risk settings which are embedded within a unified framework of isonormal Gaussian process. Namely, we combine splitting methods with both Interacting Particle System (IPS) technique and ergodic transformations using Parallel-One-Path (POP) estimators. We also propose an adaptive version for the POP method and prove its convergence. We demonstrate the application of our methods in various examples which cover usual semi-martingale stochastic models (not necessarily Markovian) driven by Brownian motion and, also, models driven by fractional Brownian motion (non semi-martingale) to address various financial risks. Interestingly, owing to the Gaussian process framework, our methods are also able to efficiently handle the important problem of sensitivities of rare event statistics with respect to the model parameters. This is a joint work with Stefano De Marco, Emmanuel Gobet and Gang Liu.

Stephan Sturm, *Friday 14.30 – 15.00*

Indifference Pricing of XVA in Stochastic Volatility Models

We consider the pricing of XVA in markets with stochastic volatility. Assuming that the hedger's risk preferences are given by a convex dynamic risk measure, we study the hedger's indifference price for entering the underlying contract with value adjustments due to credit risk, collateralization and differential rates.

Marek Rutkowski, *Friday 15.00 – 15.30*

American Options in Nonlinear Markets

The goal is to re-examine and extend results from the recent work by Dumitrescu et al. (2015) who generalised the nonlinear arbitrage-free pricing approach developed in El Karoui and Quenez (1997). We provide a detailed study of pricing, hedging and exercising problems for the two counterparties in a contract of American style options. To this end, we use the equivalence between a solution to the nonlinear optimal stopping problem and a solution to the reflected BSDE. A solution to the superhedging problem for the issuer and the superhedging/exercising problem for the holder of an American option are solved using the BSDE approach. We address various questions regarding the values of unilateral prices, exercise and cancellation policies, and breakeven times for particular classes of contracts.

Advanced numerical methods for non-linear equations

Organisers: E. Gobet and M. Mrad, **Room: JCMB Lecture Theatre B (Wednesday), JCMB Lecture Theatre C (Friday)**

Jacques Printems, *Wednesday 11.00–11.30*
Weak approximation of SPDEs

TBA

Romain Poncet, *Wednesday 11.30–12.00*
Metastable vortex dynmamics in rotating Bose-Einstein condensates

We will present in this talk a stochastic model to study the dynamics of Bose-Einstein condensates at positive temperature. It constitutes a generalization of the Gross-Pitaevskii equation that aims to take into account some spontaneous and incoherent processes. This model takes the form of a nonlinear SPDE that can actually be be seen as an ordinary SDE in high dimension. More precisely, it takes the form of a nonreversible overdamped Langevin equation that contains several time scales. The fastest corresponds to the purely Hamiltonian dynamics modeled by the Gross-Pitaevskii equation at zero temperature. Metastable dynamics may appear over longer time scales. This is the case when the energy of the system has several distinct local minima, which can be the case when the condensate rotates. In this case, and in the small temperature limit, the system stays for a long time close to a local minimum before changing of basin of attraction for another local minimum. In our case, these local minima are characterized by the number of vortices (which are singularities of the phase of the wave function) inside the condensate. In order to study this complex dynamics we will present a suitable numerical scheme that will be used with the Adaptive Multilevel Splitting (AMS) algorithm to numerically study its metastable behavior.

José Germán López Salas, *Wednesday 12.00 –12.30*
Parallel Stratified Regression Monte-Carlo Scheme For BSDEs

In this work we design a novel algorithm based on Least-Squares Monte Carlo (LSMC) in order to approximate the (Y, Z) components of the solution to the decoupled forward-backward stochastic differential equation (BSDE)

$$\begin{aligned} Y_t &= g(X_T) + \int_t^T f(s, X_s, Y_s, Z_s)ds - \int_t^T Z_s dW_s, \\ X_t &= x + \int_0^t b(s, X_s)ds + \int_0^t \sigma(s, X_s)dW_s, \end{aligned}$$

where W is a $q \geq 1$ dimensional Brownian motion. The algorithm will also approximate the solution to the related semilinear, parabolic partial differential equation.

In recent times, there has been an increasing interest to have algorithms which work efficiently when the dimension d of the space occupied by the process X is large. This interest has been principally driven by the mathematical finance community, where nonlinear valuation rules are becoming increasingly important. Currently available algorithms [1, 2, 3, 4] rarely handle the case of dimension greater than 8.

The main constraint is not only due to the computational time, but mainly due to memory consumption requirements by the algorithms.

The purpose of this work is to drastically rework the algorithm of [4] to first minimize the exposure to the memory due to the storage of simulations. This will allow computation in larger dimension d. Secondly, in this way the algorithm can be implemented in parallel on GPU processors which enables us to obtain substantial speedups compared to CPU implementations. We present several numerical examples in order to illustrate the performance of the scheme. For further details, see [5]. This is joint work with E. Gobet, P. Turkedjiev, C. Vázquez

References

[1] P. Briand and C. Labart, Simulation of BSDEs by Wiener Chaos Expansion, *Annals of Applied Probability*, **24**, pp. 1129–1171, 2014.

[2] E. Gobet and C. Labart, Solving BSDE with adaptive control variate, *SIAM Numerical Analysis*, **48** pp. 257–277, 2010.

[3] E. Gobet and P. Turkedjiev, Approximation of BSDEs using Malliavin weights and least-squares regression, *Bernoulli*, **22** pp. 530-562, 2016.

[4] E. Gobet and P. Turkedjiev, Linear regression MDP scheme for discrete backward stochastic differential equations under general conditions, *Mathematics of Computation*, **85** pp. 1359-1391, 2016.

[5] E. Gobet, J. G. López-Salas, P. Turkedjiev and C. Vázquez, Stratified regression Monte-Carlo scheme for semilinear PDEs and BSDEs with large scale parallelization on GPUs, *SIAM Journal of Scientific Computing*, **38(6)** pp. C652–C677, 2016.

Charles-Edouard Bréhier, *Friday 9.00–9.30*

Kolmogorov equations and weak order analysis for SPDEs with multiplicative noise

We consider the analysis of weak rates of convergence of numerical approximation for parabolic, semilinear, SPDEs, of the following form

$$dX_t = AX_t dt + G(X_t)dt + \sigma(X_t)dW_t,$$

where W is a cylindrical Wiener process on $H = L^2(0,1)$, A is a linear operator, typically the Laplace operator with Dirichlet boundary conditions. The abstract framework above includes stochastic reaction-diffusion and Burgers-type equations. Consider $(X_n^{\delta t})_{n=0,1,\ldots}$ the approximation obtained using the linear-implicit Euler scheme, with time-step size δt. We are interested in the rate of convergence of the weak error

$$\mathbb{E}[\varphi(X_T)] - \mathbb{E}[\varphi(X_N^{\delta t})]$$

when $\delta t \to 0$, for a fixed $T = N\delta t > 0$, where φ is a sufficiently regular real-valued function. We generalize the approach based on the decomposition of the weak error using the solution $u(t,x) = \mathbb{E}_x[\varphi(X_t)]$ of the associated Kolmogorov equation

$$\frac{\partial u}{\partial t} = \langle Ax + G(x), Du(t,x)\rangle + \frac{1}{2}\mathrm{Tr}\big(\sigma(x)\sigma(x)^{\star}D^2u(t,x)\big).$$

The difficulty when σ is not constant is to justify the well-posedness of all the terms in the right-hand side of the equation above, and to prove appropriate regularity bounds on Du and D^2u to get the same rate of convergence as in the additive noise case (σ constant).

I will first explain how to obtain the required regularity results for Du and D^2u. Our approach is original: it uses a new family of two-sided stochastic integrals.Then these regularity results are applied to prove that the weak error convergence converges with rate 1/2.This is joint work with A. Debussche (ENS Rennes).

Mohamed Mrad, *Friday 9.30 – 10.00*

Convergence rate of strong approximations of compound random maps, application to SPDEs

We derive sufficient conditions to tightly estimate the strong convergence rate of composition of random maps (possible dependent). This question of convergence naturally appears in numerical schemes: simulation of a process at a random time, composition of stochastic flows (characteristics) to solve SPDEs. Deriving convergence rates is important for assessing the efficiency of a numerical scheme but it is also fundamental to perform the recent unbiased simulation scheme of Glynn-Rhee. Here we mainly focus on the application to composition of stochastic flows and SPDEs, some examples from the classical PDE framework and the characteristics method are also given .

BSDEs in Game and Control theory

Organisers: R. Buckdahn and J. Li, **Room: JCMB Lecture Theatre C**

Huaizhong Zhao, *Thursday 10.30 – 11.00*

Ergodicity on Sublinear Expectation Spaces

In this paper, we first develop an ergodic theory of an expectation-preserving map on a sublinear expectation space. Ergodicity is defined as any invariant set either has 0 capacity itself or its complement has 0 capacity. We prove, under a general sublinear expectation space setting, the equivalent relation between ergodicity and the corresponding transformation operator having simple eigenvalue 1, and also with Birkhoff type strong law of large numbers if the sublinear expectation is strongly regular. We also study the ergodicity of invariant sublinear expectation of sublinear Markovian semigroup. We prove that its ergodicity is equivalent to the generator of the Markovian semigroup having eigenvalue 0 and the eigenvalue is simple in the space of continuous functions. As an example we show that G-Brownian motion on the unit circle has an invariant expectation and is ergodic. Moreover, it is also proved in this case that the invariant expectation is strongly regular and the canonical stationary process has no mean-uncertainty under the invariant expectation.Mon

It is based on a joint work with Chunrong Feng (Loughborough University).

Shuai Jing, *Thursday 11.00 – 11.30*

Mean-field SDE driven by a fractional Brownian motion and related stochastic control problem

We study a class of mean-field stochastic differential equations driven by a fractional Brownian motion with Hurst parameter $H \in (1/2, 1)$ and a related stochastic control problem. We derive a Pontryagin type maximum principle and the associated adjoint mean-field backward stochastic differential equation driven by a classical Brownian motion, and we prove that under certain assumptions, which generalise the classical ones, the necessary condition for the optimality of an admissible control is also sufficient.

Wenqiang Li, *Thursday 11.30 – 12.00*

Nash equilibrium payoffs for nonzero-sum stochastic differential games without Isaacs condition

We mainly investigate the existence of the Nash equilibrium payoffs for nonzero-sum stochastic differential games without assuming Isaacs condition in this paper. Along the partition π of the time interval $[0, T]$, we choose a suitable random non-anticipative strategy with delay to study our nonzero-sum stochastic differential game. We prove for the corresponding both zero-sum stochastic differential games without Isaacs condition the existence of the value functions. With the help of these value functions we give the characterization of the Nash equilibrium payoffs. This characterization allows to prove the existence of Nash equilibrium payoffs.

My talk is based on a joint work with Juan Li (SDU, Weihai).

SPDEs for limit order book models

Organisers: U. Horst and D. Kreher, **Room: JCMB Lecture Theatre A**

Xuefeng Gao, *Thursday 10.30 – 11.00*

Hydrodynamic limit of order-book dynamics

In this paper, we establish a fluid limit for a two-sided Markov order book model. The main result states that in a certain asymptotic regime, a pair of measure-valued processes representing the sell-side shape and buy-side shape of an order book converges to a pair of deterministic measure-valued processes in a certain sense. We also test the fluid approximation on data. The empirical results suggest that the approximation is reasonably good for liquidly traded stocks in certain time periods.

Marvin Müller, *Thursday 11.00 – 11.30*

A tractable class of macroscopic order book models

Motivated by observations in high frequency markets we introduce a model for the order book density based on parabolic stochastic partial differential equations. While the celebrated free boundary model for price formation of Lasry-Lions (2007) was starting point for a wide range of price-time continuous models for limit order books, tractability is one of the main issues when working with infinite dimensional systems. We study a class of tractable models which can be applied to market data. Following empirical observations by Cont, Kukanov and Stoikov (2014), the so called order flow imbalance induces naturally a model for short term price dynamics which becomes explicit in the particular framework.

Wei Xu, *Thursday 11.30 – 12.00*

Limit Order Books Driven by Infinite-dimensional Hawkes Processes

In this talk, infinite-dimensional Hawkes processes will be introduced firstly. Then we introduce a new kind of continuous-time order book models driven by the infinite-dimensional Hawkes processes, which is motivated by the non-negligible cross-dependencies (self and mutual excitation) among different types of orders. Finally, we show that their rescaled limits can be described by a coupled SDE-ODEs system. Adjoint work with Ulrich Horst.

Marie-Therese Wolfram, *Friday 14.30 – 15.00*

Parabolic free boundary price formation models under market size fluctuations

In this talk we propose an extension of the Lasry-Lions price formation model which includes fluctuations of the numbers of buyers and vendors. We analyze the model in the case of deterministic and stochastic market size fluctuations and present results on the long time asymptotic behavior and numerical evidence and conjectures on periodic, almost periodic and stochastic fluctuations. The numerical simulations extend the theoretical statements and give further insights into price formation dynamics.

Dörte Kreher, *Friday 15.00 – 15.30*

A functional central limit theorem for a state-dependent limit order book model

In this talk we study a second order approximation of a microscopic state-dependent limit order book model, when the impact of an individual order as well as the tick size goes to zero and the rate of order arrivals tends to infinity. While in a first order approximation the order book dynamics can be described by a system of deterministic non-linear differential equations, its fluctuations around this first order approximation converge, when properly scaled, to a one-dimensional stochastic differential equation describing the price fluctuations coupled with a measure-valued stochastic differential equation for the volume fluctuations. While the drift of this measure-valued stochastic differential equation depends linearly on the fluctuations of the price process and the fluctuations of some state variable, its volatility coefficient is a function of the first order approximation only. The talk is based on joint work with U. Horst.

Thorsten Rheinländer, *Friday 15.30 – 16.00*

On the stochastic heat equation with multiplicative noise

We provide a solution to the one-dimensional heat equation with multiplicative noise in terms of a local time functional, involving last exit times. This allows us to calculate the moment generating function of the solution. The results have been achieved via economic considerations on the order volume distribution of the limit order book. As an application, we discuss various types of trading times.

Numerical approximations and regularity of SPDEs

Organisers: M. Hutzenthaler and A. Jentzen, **Room JCMB Lecture Theatre B**

Erika Hausenblas, *Thursday 10.30 – 11.00*

The Stochastic Gray Scott system

Reaction and diffusion of chemical species can produce a variety of patterns, reminiscent of those often seen in nature. The Gray Scott system is a coupled equation of reaction diffusion type, modelling these kind of patterns. Depending on the parameter, stripes, waves, cloud streets, or sand ripples may appear.

These systems are the macroscopic model of microscopic dynamics. Here,in the derivation of the equation the random fluctuation of the molecules are neglected. Adding a stochastic noise, the inherit randomness of the microscopic behaviour is modelled. In particular, we add a time homogenous spatial Gaussian random field with given spectral measure. In the talk we first analyse the regularity of the solution of the stochastic Gray Scott system. Then, we will speak about its numerical approximation by a splitting method. This is a joint work with Thalhammer and Randrianasolo.

Mihály Kovács, *Thursday 11.00 – 11.30*

Existence, uniqueness and regularity for a class of semilinear stochastic Volterra equations with multiplicative noise

We consider a class of semilinear Volterra type stochastic evolution equation driven by multiplicative Gaussian noise. The memory kernel, not necessarily analytic, is such that the deterministic linear equation exhibits a parabolic character. Under appropriate Lipschitz-type and linear growth assumptions on the nonlinear terms we show that the unique mild solution is mean-p Hölder continuous with values in an appropriate Sobolev space depending on the kernel and the data. In particular, we obtain pathwise space-time (Sobolev-Hölder) regularity of the solution together with a maximal type bound on the spatial Sobolev norm. As one of the main technical tools we establish a smoothing property of the derivative of the deterministic evolution operator family. This is a joint work with B. Baeumer (Otago) and M. Geissert (Darmstadt).

Felix Lindner, *Thursday 11.30 – 12.00*

Poisson Malliavin calculus in Hilbert space and applications to weak approximation of SPDEs with Lvy noise

In this talk I present a simple approach to Malliavin calculus for Poisson random measures in a Hilbert space setting. It is solely based on elementary principles from the theory of point processes and basic moment estimates. The theory is used to analyze the weak order of convergence of space-time approximations for a class of linear SPDEs with alpha-stable noise and a class of semi-linear SPDEs with square-integrable Lvy noise. The talk is based on joint work with Adam Andersson (Gothenburg).

Gilles Vilmart, *Thursday 16.00 – 16.30*

High-order integrators for sampling the invariant distribution of stochastic ordinary and partial differential equations.

We highlight the role that some geometric integration tools (backward error analysis, processing) that were originally introduced in the deterministic setting, play in the design of high-order integrators to sample the invariant distribution of ergodic systems of stochastic ODEs and PDEs. In particular, we introduce a modification with negligible overhead of the linearized implicit Euler-Maruyama method for the accurate sampling of the invariant measure of a class of semilinear parabolic stochastic PDEs driven by an additive space-time noise.

This talk is based on joint works with Assyr Abdulle (EPF Lausanne), Charkes-Edouard-Brhier (Univ. Lyon), Kostas Zygalakis (Univ. Edinburgh).

Sara Mazzonetto , *Thursday 16.30 – 17.00*

Existence, uniqueness, and approximation (for stochastic Burgers equation).

In this talk we present a recently introduced explicit full-discrete numerical approximation scheme for some stochastic partial differential equations with additive white noise and non-globally monotone nonlinearities. (see Hutzenthaler, Jentzen, and Salimova (2016).) The scheme allows a proof of existence and uniqueness of the mild solution as well as strong convergence of the numerical approximation for example in the case of stochastic Burgers equations. The talk is based on a joint work with Arnulf Jentzen and Diyora Salimova.

Christoph Reisinger, *Thursday 17.00-17.30*

Analysis of Multi-Index Monte Carlo Estimators for a Zakai SPDE

In this talk, we propose a space-time Multi-Index Monte Carlo (MIMC) estimator for a one-dimensional parabolic stochastic partial differential equation (SPDE) of Zakai type. We compare the complexity with the Multilevel Monte Carlo (MLMC) method of Giles and Reisinger (2012), and find, by means of Fourier analysis, that the MIMC method: (i) has suboptimal complexity of $O(\epsilon^2|\log\epsilon|^3)$ for a root mean square error (RMSE) ϵ if the same spatial discretisation as in the MLMC method is used; (ii) has a better complexity of $O(\epsilon^2|\log\epsilon|)$ if a carefully adapted discretisation is used; (iii) has to be adapted for non-smooth functionals. Numerical tests confirm these findings empirically. Joint work with Zhenru Wang.

Sonja Cox, *Friday 9.00 – 9:30*

Weak approximations for semi-linear SPDEs

In numerical analysis for stochastic differential equations, a general rule of thumb is that the optimal weak convergence rate of a numerical scheme is twice as large as the optimal strong convergence rate. However, for SPDEs the optimal weak convergence rate is difficult to establish theoretically. Recently, progress was been made by Jentzen, Kurniawan and Welti for semi-linear SPDEs using the so-called mild Itô formula. We consider this approach for wave equations, and discuss the necessity of extending the theory to the Banach space setting in order to obtain optimal rates when the non-linear terms are given by Nemytskii operators.

Joint work with Arnulf Jentzen, Ryan Kurniawan, and Timo Welti

Petru A. Cioica-Licht, *Friday 09.30 – 10.00*

How does a corner singularity influence the regularity of the stochastic heat equation?

Although there exists an almost fully-fledged regularity theory for (semi-)linear second order stochastic partial differential equations (SPDEs, for short) on smooth domains, very little is known about the regularity of these equations on domains with non-smooth boundaries that have singularities, such as corners or edges.

In this talk we present an approach for analysing the behaviour of the solution to SPDEs in the direct vicinity of corner singularities. We consider the stochastic heat equation with additive noise and zero Dirichlet boundary condition on a planar angular domain. In order to capture the singular behaviour of the solution and its derivatives at the vertex, we use weighted Sobolev spaces, where the weights are appropriate powers of the distance to the vertex. Our approach relies on suitable Green function estimates, which we use to prove a fundamental weighted L_p-estimate for the stochastic convolution associated to the corresponding equation.

This is joint work with Kyeong-Hun Kim (Korea University, Korea), Kijung Lee (Ajou University, Korea), and Felix Lindner(TU Kaiserslautern, Germany).

Xiaojie Wang, *Friday 10.00 – 10.30*

Exponential time integrators for strong approximations of nonlinear SPDEs

The talk is on a kind of exponential Euler-type time-stepping scheme for parabolic stochastic partial differential equation with additive space-time white noise and polynomial nonlinearity. The space discretization is done by a spectral Galerkin method. Strong convergence rates of order $1/2 - \epsilon$ both in space and in time is obtained for the proposed scheme.

Philipp Harms, *Friday 14.30 – 15.00*

Weak rates for noise discretizations of SPDEs

In many important SPDEs the linear part of the drift has no regularizing effect. For example this is the case for the wave equation, HJM equation, Schrödinger equation, and Korteweg–de Vries equation. We prove essentially sharp weak convergence rates for the noise discretization of semilinear SPDEs of this type.

David Cohen, *Friday 15.00 – 15.30*

Exponential integrators for nonlinear Schrdinger equations with white noise dispersion

This presentation deals with the numerical integration in time of the nonlinear Schrdinger equation with power law nonlinearity and random dispersion. We introduce a new explicit exponential integrator for this purpose that integrates the noisy part of the equation exactly. We prove that this scheme is of mean-square order 1 and we draw consequences of this fact. We compare our exponential integrator with several other numerical methods from the literature. We finally propose a second exponential integrator, which is implicit and symmetric and, in contrast to the first one, preserves the L2-norm of the solution.

The presentation is based on a joint work with Guillaume Dujardin (Inria Lille Nord-Europe)

Gabriel Lord, *Friday 15:30 – 16.00*

Adaptive Time Stepping for SPDEs

We introduce adaptive time stepping techniques to control growth in the numerical solution of SPDEs. This can be thought of as an alternative to proving moment bounds for the numerical method and to using a fixed step taming method. Ideas and the convergence result will be illustrated with some numerical experiments that also show that the adaptivity leads to more accurate solutions. If time permits we will introduce a new exponential based method for time stepping for SPDEs with multiplicative noise which have an improved rate of convergence in specific circumstances.

Joint work with Conall Kelly (UWI) and Utku Erdogan (Usak).

Path-dependent PDEs, non-Markovian stochastic control, and applications

Organisers: D. Possamaï and X. Tan, **Room: JCMB Room 5327 (Thursday), JCMB Lecture Theatre C (Friday)**

Thibaut Mastrolia, *Thursday 16.00 – 16.30*

Moral hazard with mean field type interactions

In this talk, we investigate a moral hazard problem in finite time with lump-sum and continuous payments, involving infinitely many Agents with mean field type interactions, hired by one Principal. By reinterpreting the mean-field game faced by each Agent in terms of a mean field forward backward stochastic differential equation (FBSDE for short), we are able to rewrite the Principal's problem as a control problem of McKean-Vlasov SDEs. We solve completely and explicitly the problem in special cases, going beyond the usual linear-quadratic framework.

Christian Keller, *Thursday 16.30 – 17.00*

Path-dependent Hamilton-Jacobi equations in infinite dimensions

We propose notions of minimax and viscosity solutions for a class of fully nonlinear path-dependent PDEs with nonlinear, monotone, and coercive operators on Hilbert space. Our main result is well-posedness (existence, uniqueness, and stability) for minimax solutions. A particular novelty of our approach is a suitable combination of minimax and viscosity solution techniques. Thereby, we establish a comparison principle for path-dependent PDEs under conditions that are weaker even in the finite-dimensional case. In contrast to most of the related works on PDEs in infinite dimensions, perturbed optimization is entirely avoided. The path-dependent setting itself enables us to circumvent the lack of compactness in infinite-dimensional Hilbert spaces. As an application, our theory makes it possible to employ the dynamic-programming approach to study optimal control problems for a fairly general class of (delay) evolution equations in the variational framework. Furthermore, differential games associated to such evolution equations can be investigated following the Krasovskii-Subbotin approach similarly as in finite dimensions. Joint work with Erhan Bayraktar

Zhenjie Ren, *Thursday 17.00 – 17.30*

Viscosity solution to path-dependant PDE

In this talk, we review the recent development of the theory of the viscosity solutions to the path-dependant PDEs. We will go through the definition and the main properties of such solutions, e.g. the comparison result, the stability etc., and give a simple application of the theory.

Julien Claisse, *Friday 14.30 – 15.00*

Optimal control of branching diffusion processes

In this work, we formulate and analyze a stochastic control problem on branching diffusion processes where both the movement and the reproduction of the particles depend on the control. More precisely, we consider the problem of minimizing the expected value of the product of individual costs penalizing the final position of each particle. In this setting, we show that the value function is the unique viscosity solution of a nonlinear parabolic PDE, that is, the Hamilton-Jacobi-Bellman equation corresponding to the problem. A crucial step toward this result is to derive a weak form of the branching property which allows to reduce the state space from an infinite dimensional space of measures to a finite dimensional one.

Numerical methods for BSDEs

Organiser: Arnaud Lionnet, **Room: JCMB Room 5327**

Arnaud Lionnet, *Friday 9.00 – 9.30*

Numerical approximation of BSDEs with polynomial growth drivers

I discuss the numerical approximation of BSDEs with non-Lipschiz drivers having polynomial growth in the primary variable.

Unlike in the Lipschitz case, the Euler-Bouchard-Touzi-Zhang (EBTZ) schemes do not necessarily converge. We show that the implicit scheme converges, as do the theta-schemes for theta greater or equal to 1/2, where theta is the degree of implicitness of the scheme. However, the explicit scheme can explode.

But explicit schemes have the general advantage that they are computationally less expensive than an implicit scheme. We therefore seek modified explicit schemes so as to recover convergence. We present such modifications, based on "taming" the driver, the terminal condition, or the whole dynamics of the scheme. Interestingly, they allow to also obtain some numerical stability properties.

Céline Labart, *Friday 9.30 – 10.00*

Approximation of BSDEs using a random walk

For the FBSDE

$$X_t = x + \int_0^t b(r, X_r)dr + \int_0^t \sigma(r, X_r)dB_r$$

$$Y_t = g(X_T) + \int_t^T f(s, X_s, Y_s, Z_s)ds - \int_t^T Z_s dB_s, \quad 0 \le t \le T$$

Briand, Delyon and Memin have shown in [1] a Donsker-type theorem: If one approximates the Brownian motion B by a random walk B^n, the according solutions (X^n, Y^n, Z^n) converge weakly to (X, Y, Z).
We investigate under which conditions (Y_t^n, Z_t^n) converges to (Y_t, Z_t) in L_2 and compute the rate of convergence in dependence of the Hölder continuity of the terminal condition function g.
This is joint work with Christel Geiss and Antti Luoto (University of Jyväskylä, Finland).

[1] P. Briand, B. Delyon, J. Memin, *Donsker-Type theorem for BSDEs.* Electron. Comm. Probab. 6, 1 – 14 (2001).

Jean-Franois Chassagneux, *Friday 10.00 – 10.30*

Cubature method to solve BSDEs: error expansion and complexity control

In this work, we prove error expansions for the approximation of BSDEs when using the cubature method. To profit fully from these expansions, e.g. to design high order approximation methods, we need however to control the complexity growth of the cubature method. In our work, this is achieved by using a sparse grid representation. We present several numerical results that confirm the efficiency of our new method. This is a joint work with C. Garcia Trillos (UCL)