Proportional Representation Beyond Elections

Markus Brill

University of Warwick

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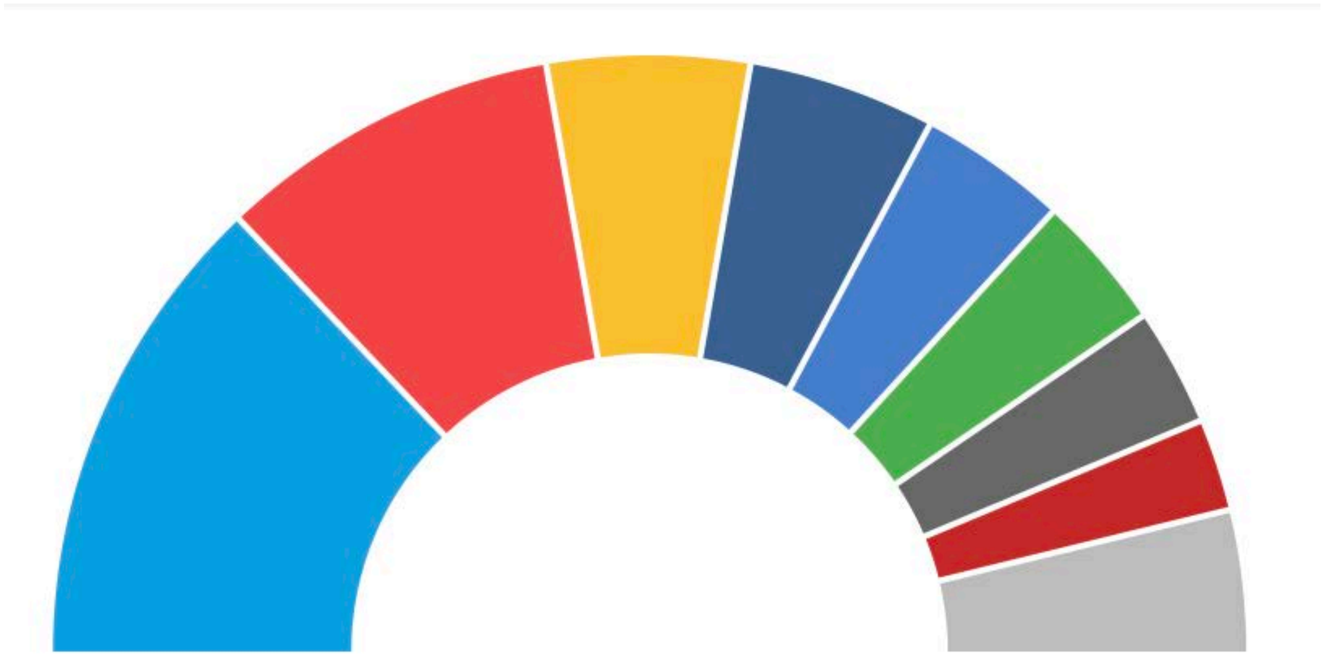
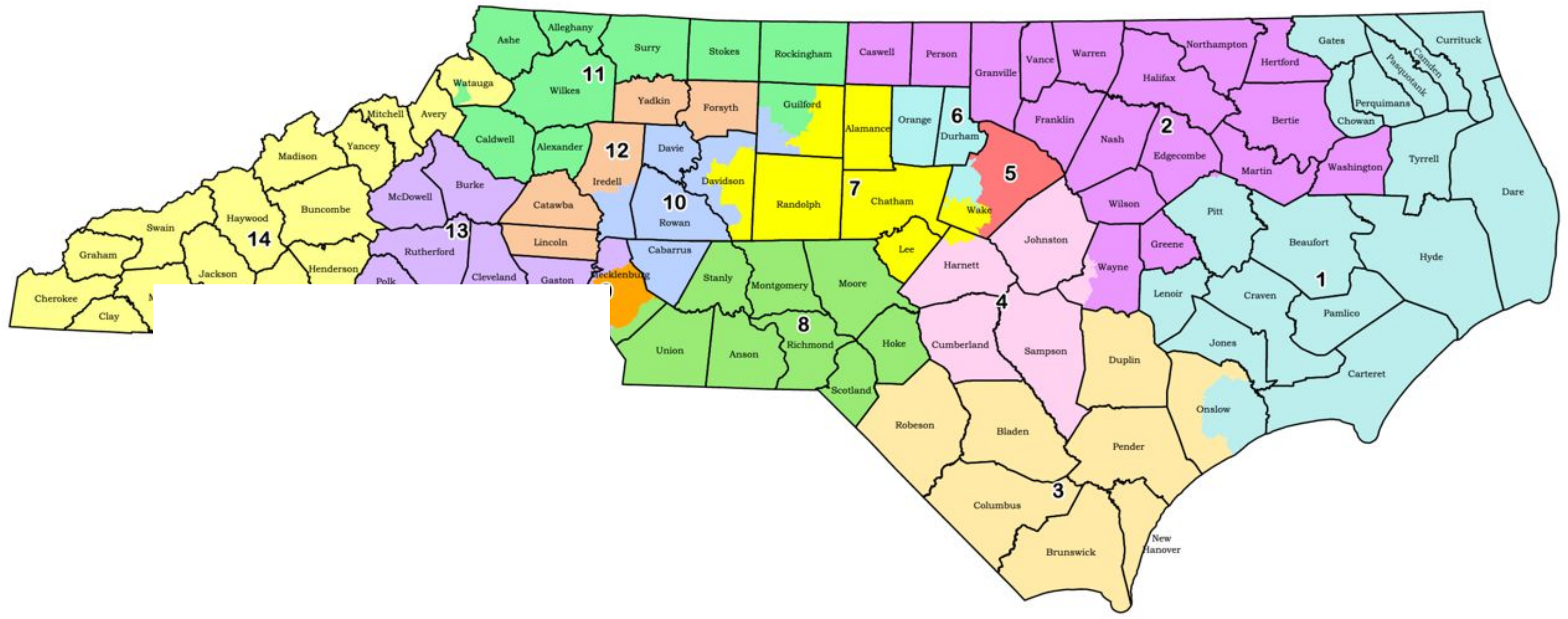
Proportional Representation

- **Goal:** Choose a subset of candidates (“committee”) that is “representative” of the preferences of a set of voters.
- Traditional approaches to achieve PR:
  - partitioning the set of candidates
    ➜ party-list elections
  - partitioning the set of voters
    ➜ district-based elections
  - mix of the above

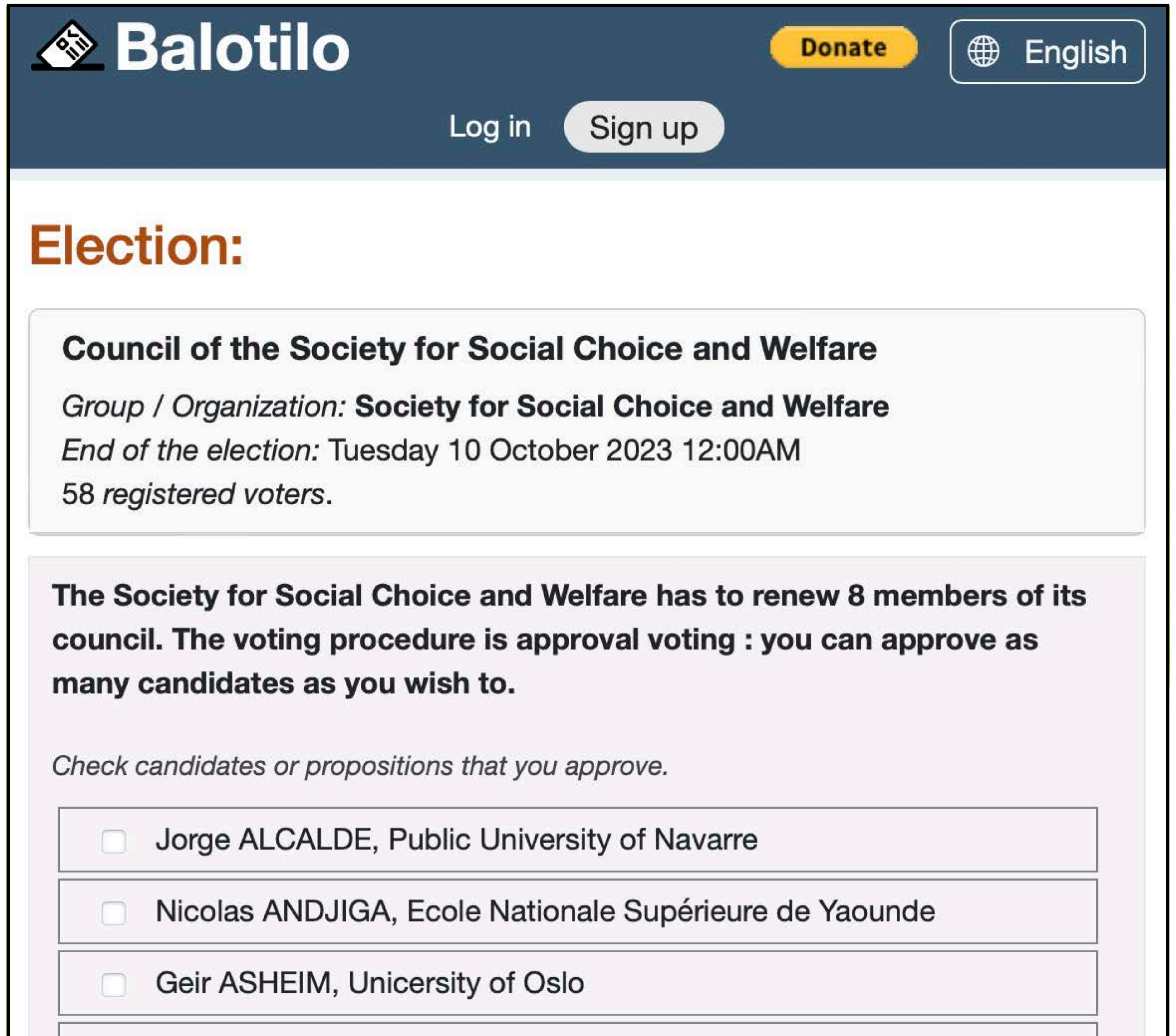




Proportional Representation

- **Goal:** Choose a "representative" subset of candidates without relying on partitioning voters or candidates.

- Applications:
  - Voting for individuals, not parties
  - Elections without parties and districts
  - Validator selection in a blockchain
  - Participatory Budgeting
  - Ranking proposals on online platforms
  - …



Outline

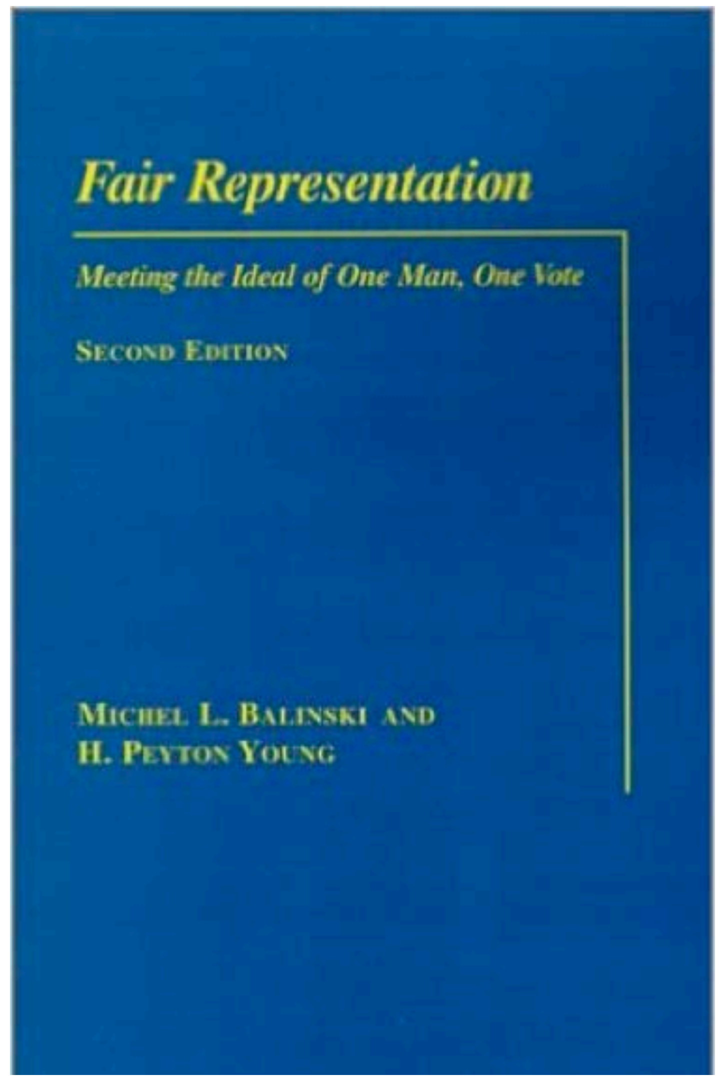
- Apportionment
- Proportionality Axioms
  - ordinal setting: *proportionality for solid coalitions*
  - approval setting: *justified representation*
- Proportional Rules
  - *Thiele's rule* and *Phragmén's rule*
- Applications
  - Blockchain; Digital Democracy; Participatory Budgeting

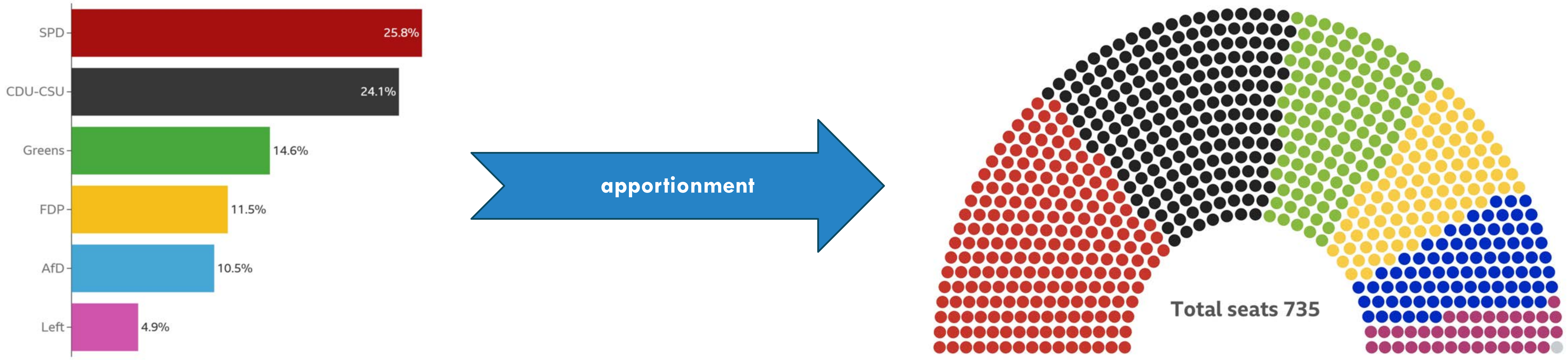
Apportionment

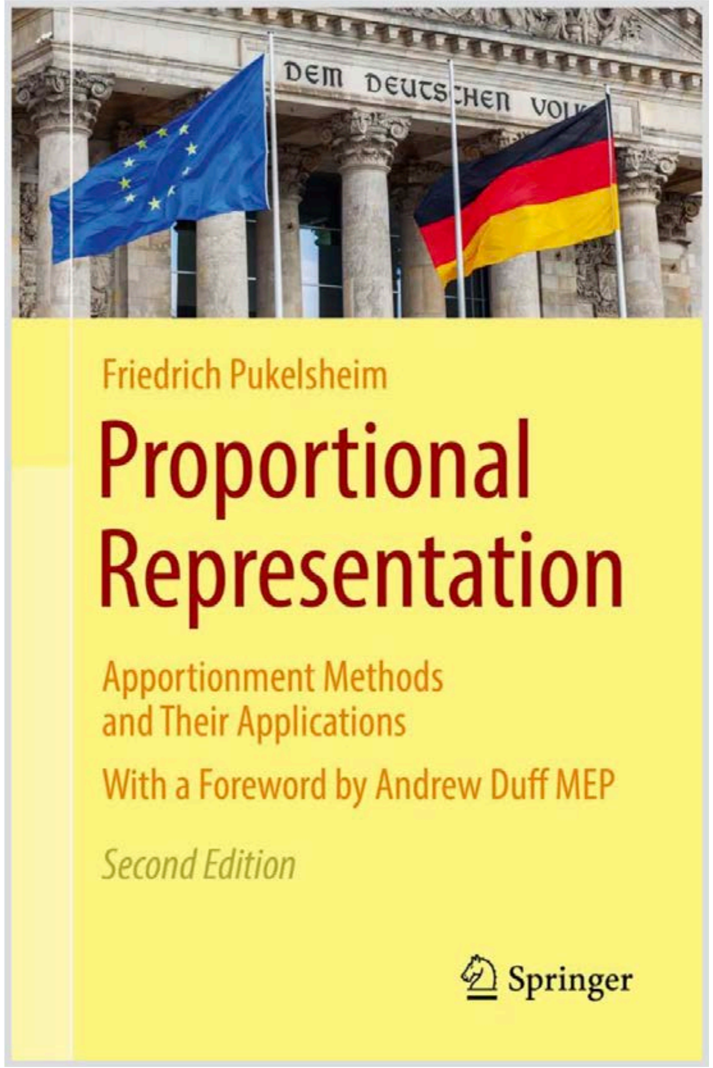
Declaration of Independence, by John Trumbull

Apportionment as Baseline

- The *apportionment problem* is well-studied
- Prominent apportionment methods:
  Jefferson/D'Hondt, Webster/Sainte-Laguë, Hamilton, …
- We consider more general settings that have apportionment as a special case







Axioms

Setting 1: Ordinal Preferences

- Finite set C of candidates
- Finite set N of voters; $|N| = n$
- Each voter $i \in N$ submits a rank-order over (a subset of) C
- **Task:** Select committee $W \subseteq C$ of size $|W| = k$.
- **Intuition:** each committee seat corresponds to n/k voters

Proportionality for Solid Coalitions (PSC)

Definition: A subset $N' \subseteq N$ of voters forms a **solid coalition** over a set $C' \subseteq C$ of candidates if $C' \succ_i C \setminus C'$ for all $i \in N'$.

Sir Michael Dummett
(1925—2011)

Example: $n = 6$ voters

1: $a > b > c > d > e$
2: $b > c > a > d > e$
3: $c > a > b > d > e$
4: $d > e > c > a > b$
5: $d > e > c > b > a$
6: $e > d > c > b > a$

Proportionality for Solid Coalitions (PSC)

Definition: A committee W satisfies **PSC** if for any solid coalition N' over C' with $|N'| \geq \ell \frac{n}{k}$ it holds that $|C' \cap W| \geq \min\left(|C'|, \ell\right)$.

Example: $n = 6$ voters, $k = 4$

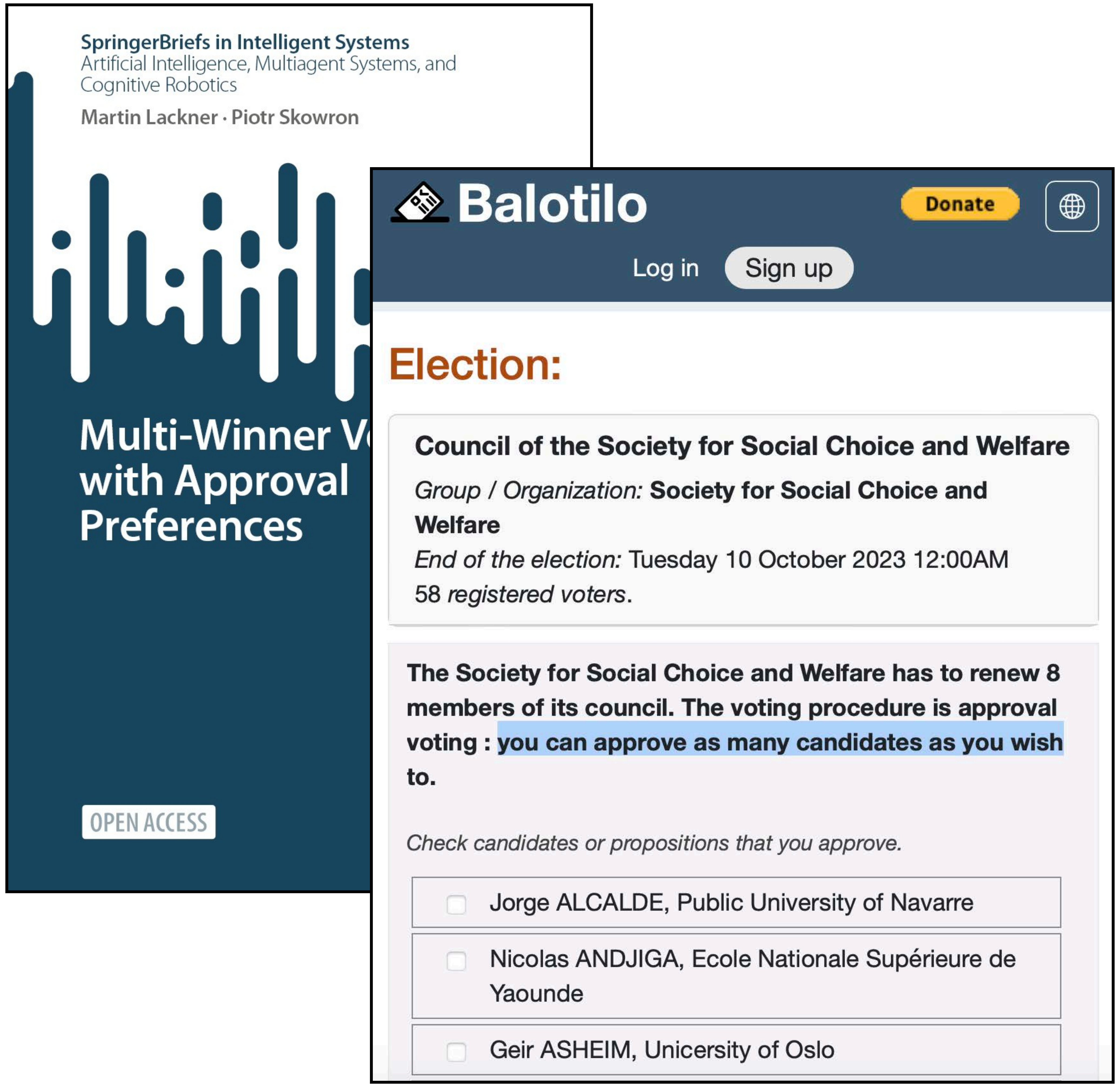
1: $a > b > c > d > e$
2: $b > c > a > d > e$
3: $c > a > b > d > e$
4: $d > e > c > a > b$
5: $d > e > c > b > a$
6: $e > d > c > b > a$

$\ell = 2$
at least 2 of a, b, c need to be selected

$\ell = 2$
both d and e need to be selected

Setting 2: Approval Preferences

- Finite set C of candidates
- Finite set N of voters
- Each voter $i \in N$ has an approval set $A_i \subseteq C$
- **Task:** Select committee $W \subseteq C$ of size $|W| = k$.



Why not simply count approvals?

Multiwinner Approval Voting:

Select the k candidates with highest approval scores.

Example: $n = 9$, $k = 3$

$5 \times \{ a, b, c \}$

$4 \times \{ r, s, t \}$

$3 \times \{ x, y, z \}$

Election:

Council of the Society for Social Choice and Welfare

Group / Organization: **Society for Social Choice and Welfare**

End of the election: Tuesday 10 October 2023 12:00AM

58 *registered voters*.

The Society for Social Choice and Welfare has to renew 8 members of its council. The voting procedure is approval voting : you can approve as many candidates as you wish to.

Check candidates or propositions that you approve.

- [ ] Jorge ALCALDE, Public University of Navarre
- [ ] Nicolas ANDJIGA, Ecole Nationale Supérieure de Yaounde
- [ ] Geir ASHEIM, Unicersity of Oslo
- [ ] Dorothea BAUMEISTER, Heinrich Heine Universität Dusseldorf
- [ ] Antoinette BAUJARD, Univesité de Saint Etienne

Cohesive Groups

Definition: A group $N' \subseteq N$ is **ℓ-cohesive** if $|N'| \geq \ell \frac{n}{k}$ and $|\bigcap_{i \in N'} A_i| \geq \ell$.

Example: $n = 6$ voters, $k = 4$

1: { a, b, c }
2: { b, c, d } → 2-cohesive group
3: { b, c, e }
4: { x, y, z }
5: { x, y, z } → 2-cohesive group
6: { x, y, z }

What do Cohesive Groups Deserve?

Definition: A group $N' \subseteq N$ is **ℓ-cohesive** if $|N'| \geq \ell \frac{n}{k}$ and $|\bigcap_{i \in N'} A_i| \geq \ell$.

First attempt: For each ℓ-cohesive group $N' \subseteq N$, the committee needs to contain at least ℓ candidates from $\cap_{i \in N'} A_i$, i.e., $|W \cap \bigcap_{i \in N'} A_i| \geq \ell$.

This is *too demanding*:

Example: $n = 4$, $k = 2$

1: $\{ a \}$

2: $\{ a, b \}$

3: $\{ b, c \}$

4: $\{ c \}$

Justified Representation Axioms

Definition: A committee $W \subseteq C$ satisfies

- *Proportional Justified Representation* (**PJR**) if, for each ℓ-cohesive group N', we have $|\bigcup_{i \in N'} A_i \cap W| \geq \ell$.
- *Extended Justified Representation* (**EJR**) if, for each ℓ-cohesive group N', there is a voter $i \in N'$ with $|A_i \cap W| \geq \ell$.

EJR implies **PJR**.

Example: $k = 4$

1: { a, b, c }
2: { b, c, d }
3: { b, c, e }
4: { x, y, z }
5: { x, y, z }
6: { x, y, z }

{ d, e, x, y } satisfies PJR, but not EJR

H. Aziz, M. Brill, V. Conitzer, E. Elkind, R. Freeman, and T. Walsh. Justified representation in approval-based committee voting. SCW 2016
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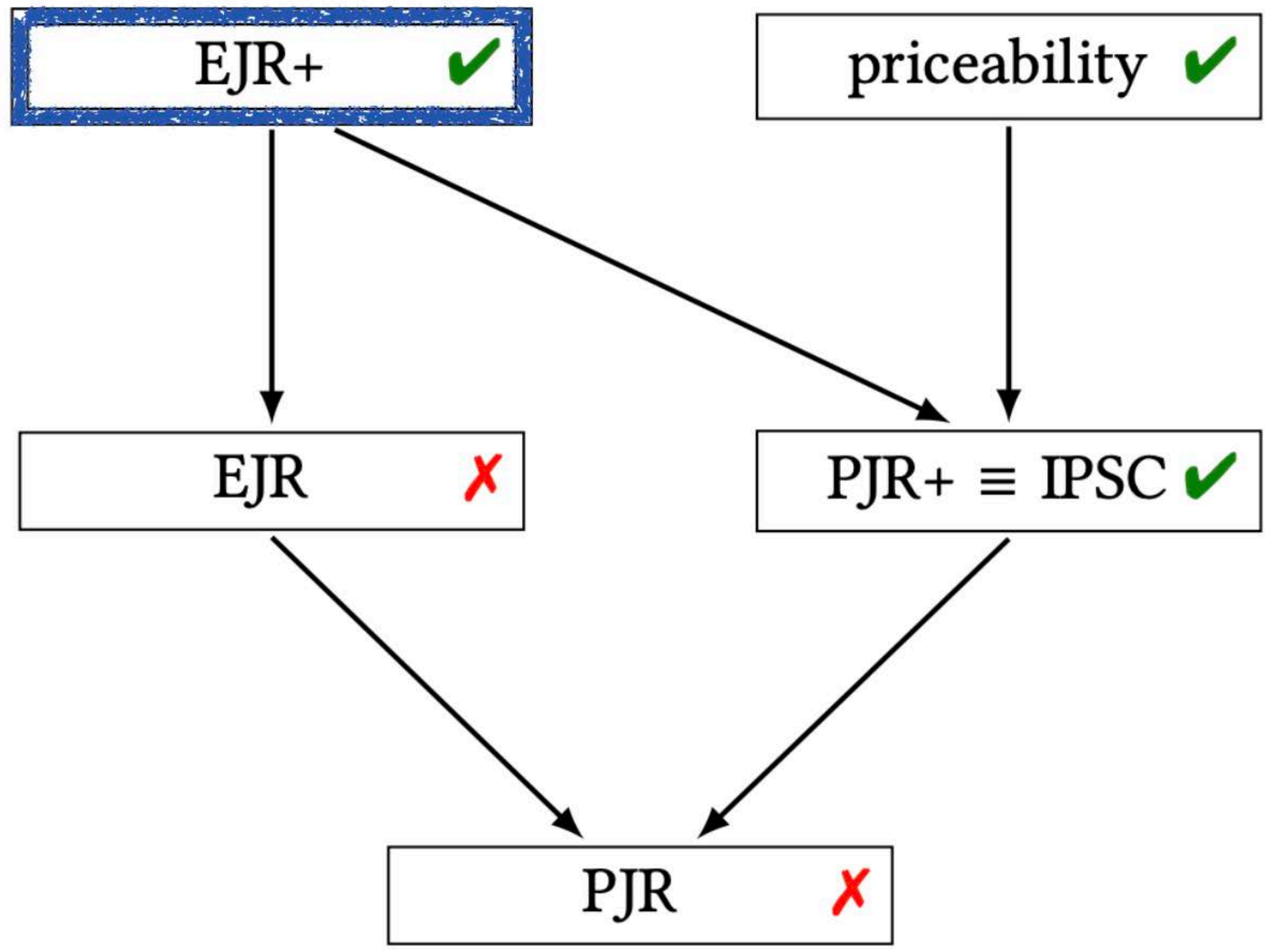
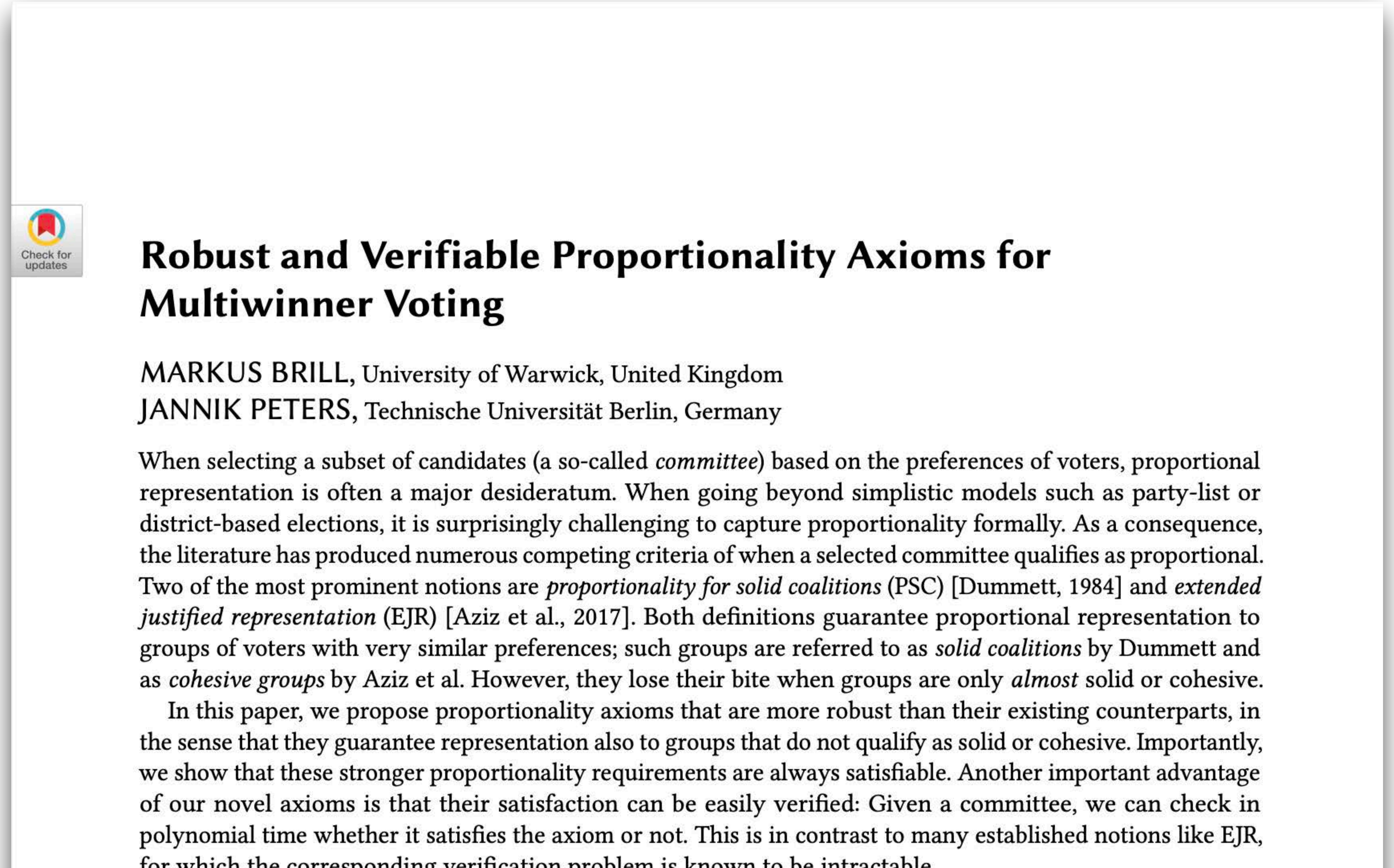
Improved Versions of PJR and EJR

In order to address issues with **PJR** and **EJR**, we propose stronger axioms with computational and other advantages: **PJR+** and **EJR+**.

APPROVAL PREFERENCES



M. Brill and J. Peters. Robust and verifiable proportionality axioms for multiwinner voting. ACM-EC 2023.



Robust and Verifiable Proportionality Axioms for Multiwinner Voting

MARKUS BRILL, University of Warwick, United Kingdom
JANNIK PETERS, Technische Universität Berlin, Germany

When selecting a subset of candidates (a so-called *committee*) based on the preferences of voters, proportional representation is often a major desideratum. When going beyond simplistic models such as party-list or district-based elections, it is surprisingly challenging to capture proportionality formally. As a consequence, the literature has produced numerous competing criteria of when a selected committee qualifies as proportional. Two of the most prominent notions are *proportionality for solid coalitions* (PSC) [Dummett, 1984] and *extended justified representation* (EJR) [Aziz et al., 2017]. Both definitions guarantee proportional representation to groups of voters with very similar preferences; such groups are referred to as *solid coalitions* by Dummett and as *cohesive groups* by Aziz et al. However, they lose their bite when groups are only *almost* solid or cohesive.

In this paper, we propose proportionality axioms that are more robust than their existing counterparts, in the sense that they guarantee representation also to groups that do not qualify as solid or cohesive. Importantly, we show that these stronger proportionality requirements are always satisfiable. Another important advantage of our novel axioms is that their satisfaction can be easily verified: Given a committee, we can check in polynomial time whether it satisfies the axiom or not. This is in contrast to many established notions like EJR, for which the corresponding verification problem is known to be intractable.

From Approval Axioms to Ordinal Axioms

Example: $n = 4, k = 2$

1: $a \succ b \succ c \succ d \succ e \succ f$

2: $e \succ b \succ c \succ d \succ f \succ a$

3: $d \succ c \succ b \succ f \succ a \succ e$

4: $f \succ c \succ b \succ d \succ a \succ e$

Idea:

- Convert ordinal instance into $m = |C|$ approval instances

From Approval Axioms to Ordinal Axioms

Example: $n = 4, k = 2$

1: $a > b > c > d > e > f$

2: $e > b > c > d > f > a$

3: $d > c > b > f > a > e$

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From Approval Axioms to Ordinal Axioms

Example: $n = 4, k = 2$

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4: $f > c > b > d > a > e$

Idea:

- Convert ordinal instance into $m = |C|$ approval instances

From Approval Axioms to Ordinal Axioms

Example: $n = 4, k = 2$

1:	a	$\succ b$	$\succ c$	$\succ d$	$\succ e$	$\succ f$
2:	e	$\succ b$	$\succ c$	$\succ d$	$\succ f$	$\succ a$
3:	d	$\succ c$	$\succ b$	$\succ f$	$\succ a$	$\succ e$
4:	f	$\succ c$	$\succ b$	$\succ d$	$\succ a$	$\succ e$

Idea:

- Convert ordinal instance into $m = |C|$ approval instances
- Require approval-based proportionality axiom to hold for all those instances

Ranked Versions of EJR+ and PJR+

- **Definition:** Let **A** be an "approval axiom." A committee W satisfies **rank-A** if, for any $r \in \{1,2,\ldots,m\}$, W satisfies **A** in the in approval instance in which every voter approves their top r candidates.

- Using this approach, we define a strengthening of PSC that is always satisfiable but violated by STV.

M. Brill and J. Peters. Robust and verifiable proportionality axioms for multiwinner voting. ACM-EC 2023.

Proportional Rules

Swedish Electoral Reform Commission, by Anton Blomberg (CC BY 3.0 via Commons)

Thiele and Phragmén

- Thiele's rule (aka *Proportional Approval Voting*):
  - choose committee maximising $\sum_{i \in N} score(i, W)$, where

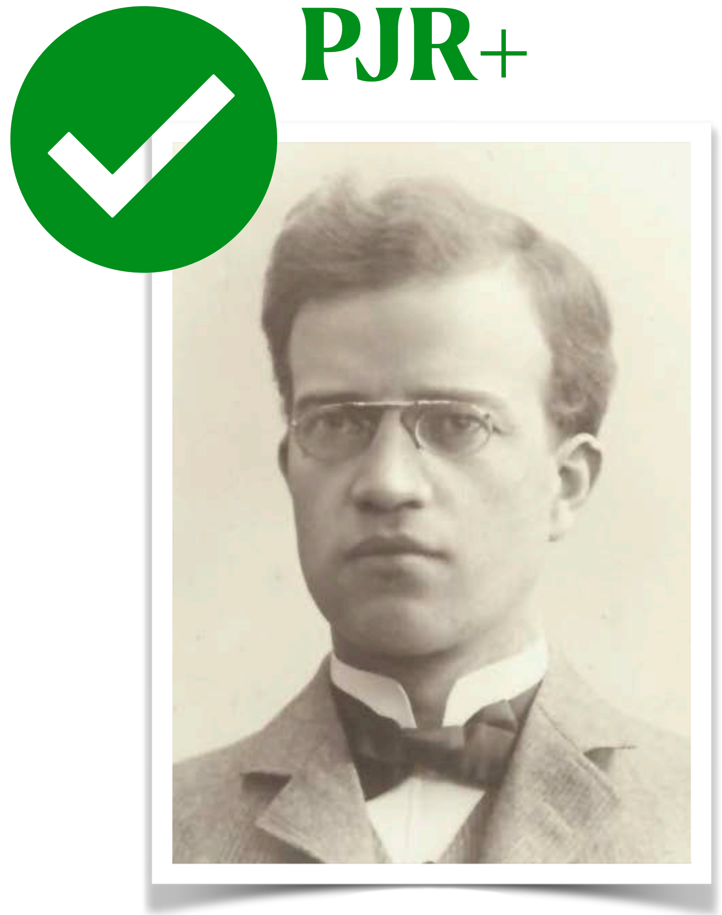
$$score(i, W) = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{|W \cap A_i|}$$

- Phragmén's rule:
  - choose candidates sequentially and let voters "pay" for the selection of approved candidates
  - voters start without any money and earn money over time
  - as soon as approvers jointly own $1, they can buy their candidate

EJR+ & PJR+

Thorvald N. Thiele
(1838–1910)

PJR+



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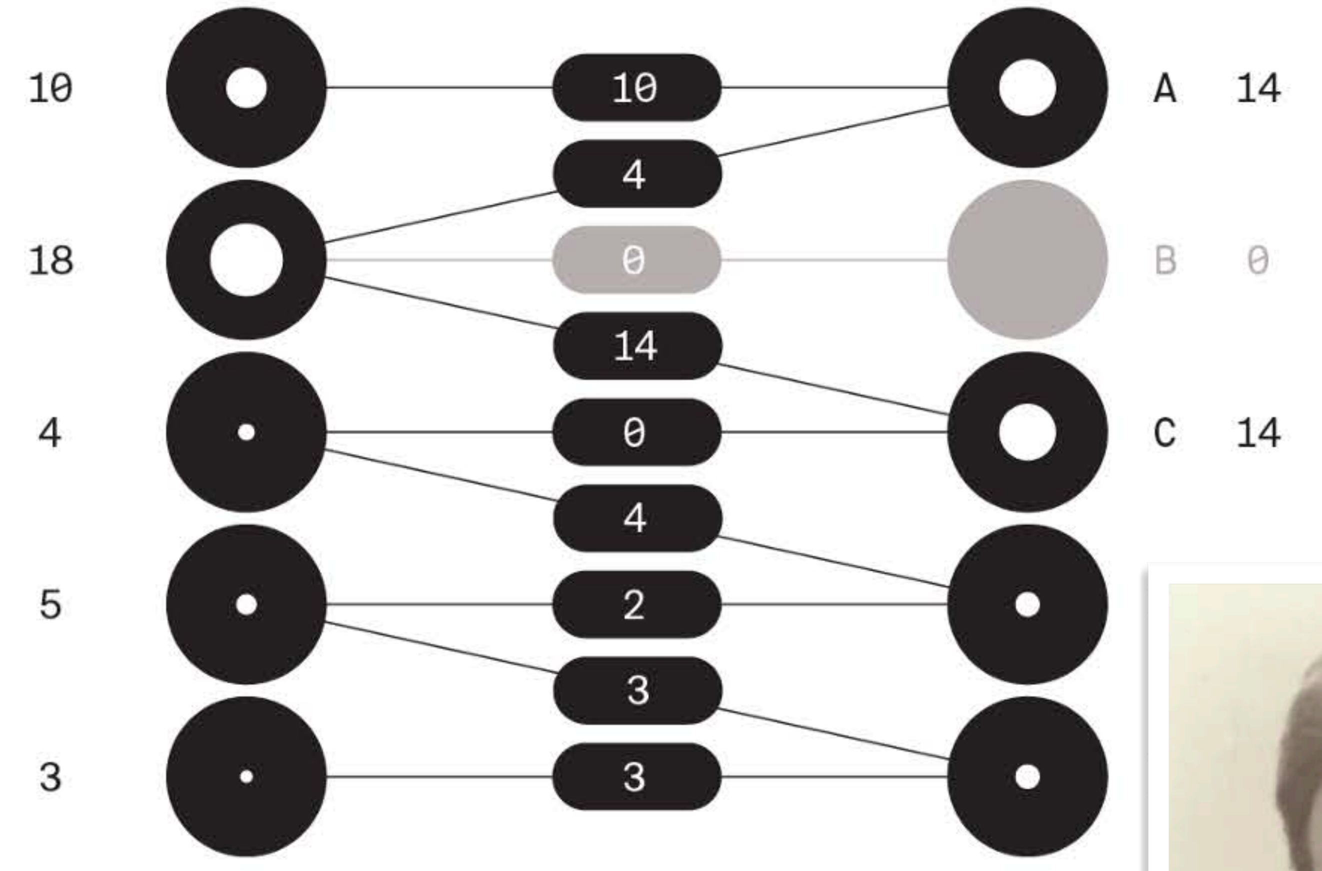
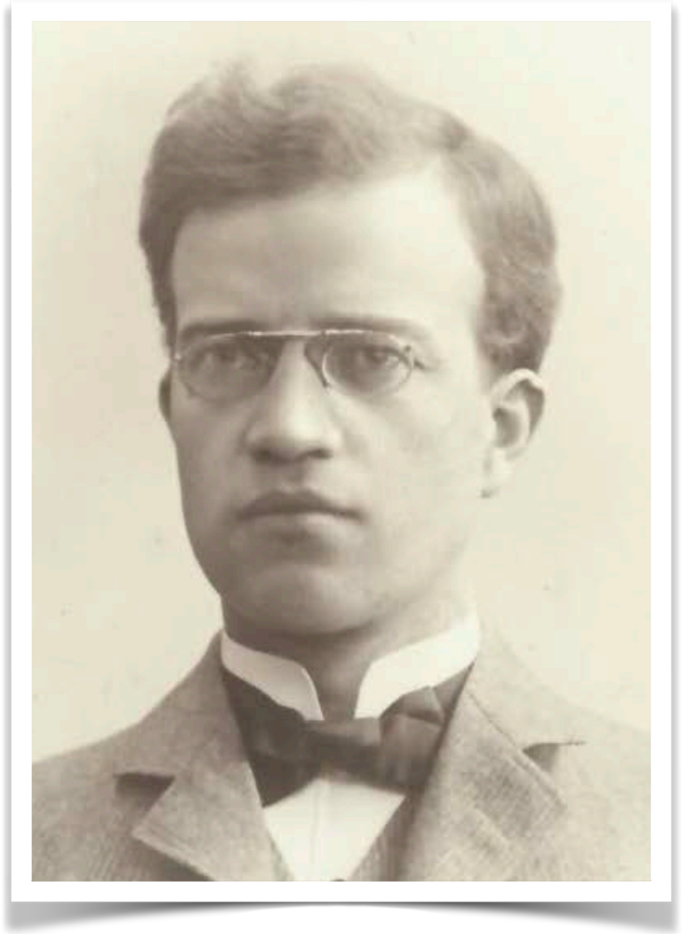
Applications

Polkadot

- *Nominated proof-of-stake* (NPoS): blockchain community elects committee of validators to participate in consensus protocol
- This is a committee election with approval preferences!
  - ~1,000 candidates and >10,000 voters
  - runs every 24 hours



- Polkadot uses *Phragmén's rule*, which satisfies PJR+ and also limits the *over*representation of voters



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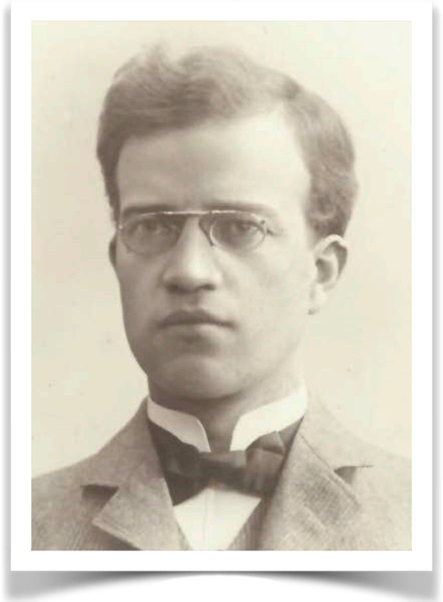
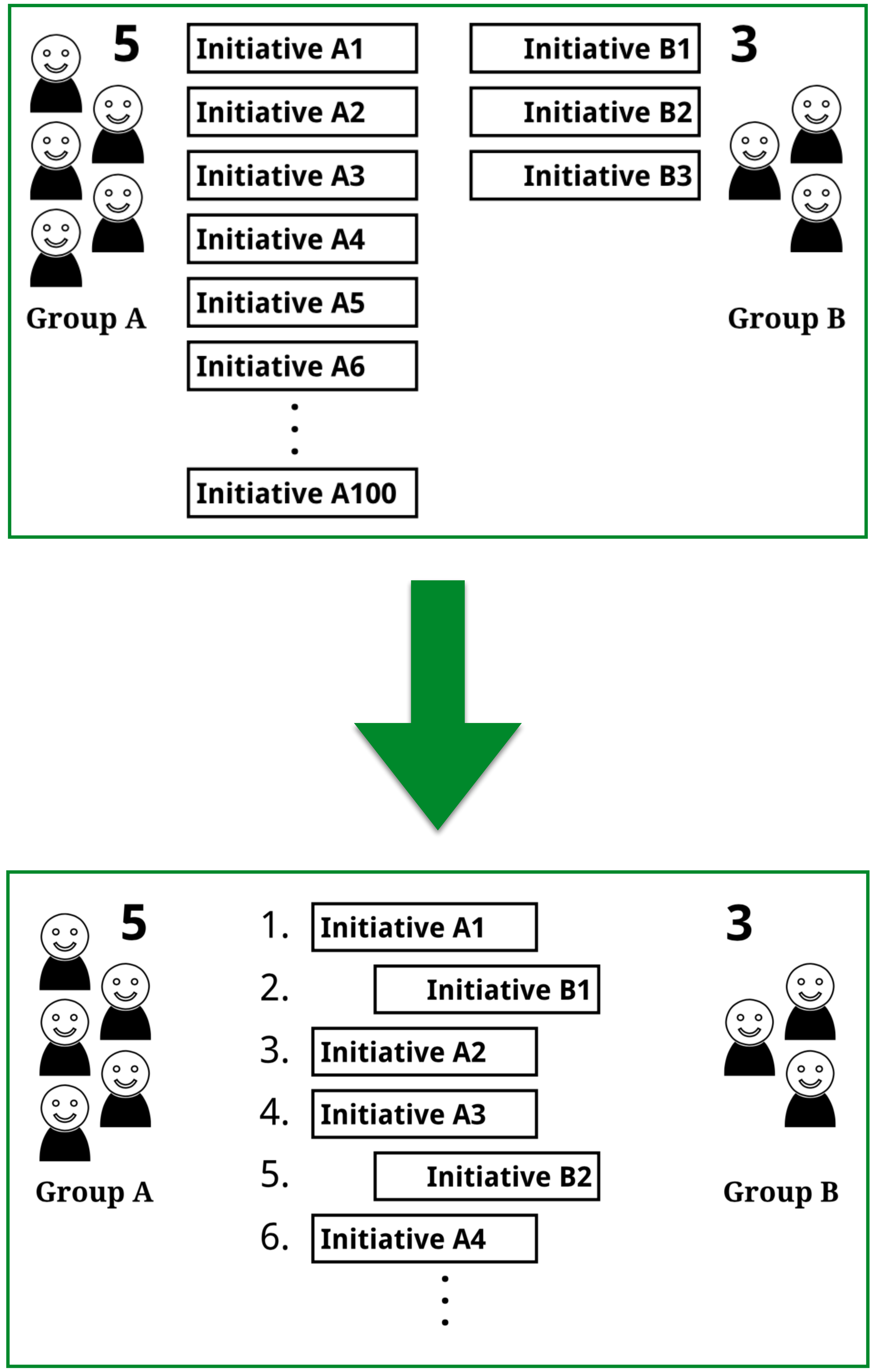
Approval-based committee voting in practice: A case study of (over-)representation in the Polkadot blockchain. AAAI 2024.

LiquidFeedback

- *LiquidFeedback* is a digital democracy platform for proposal development and decision making
- Users can add proposals and approve ("like") proposals of others
- Proportional rankings represent preferences better than simply ranking by approval count
  - *Phragmén's rule* produces rankings s.t. every prefix satisfies PJR+





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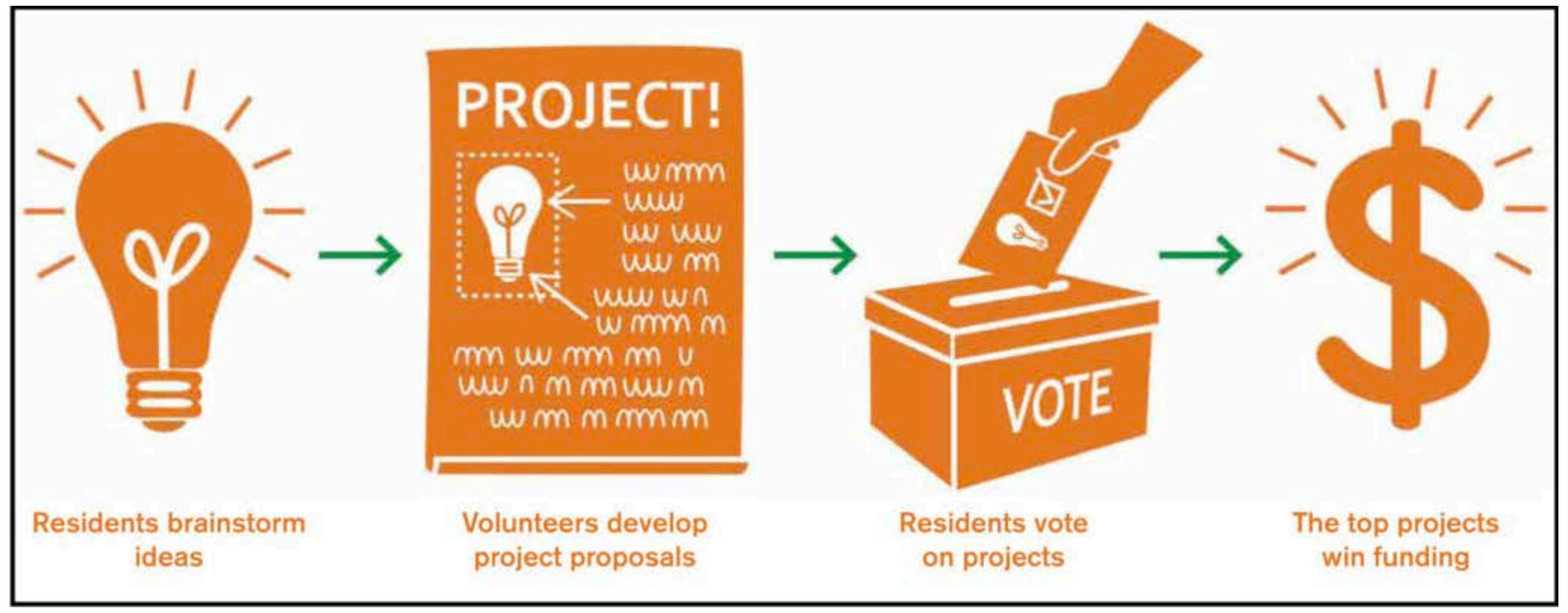
Participatory Budgeting

- Democratic innovation that lets residents of a city decide on how local budget is spent
- *Method of Equal Shares* satisfies appropriate generalization of EJR+
  - used in practice since 2023



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Conclusion

- Proportional representation can be defined and achieved in general settings and has many applications beyond elections.
- Directions for future work
  - Is the core of an approval-based committee election always non-empty? Is there a ranking rule that satisfies EJR(+)?
  - Overrepresentation and generalisations of Sainte-Laguë?
  - Are these proportionality axioms too permissive?