SPECTRAL THEORY: WHAT'S IT ALL ABOUT AND WHAT IS ITS USE

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Spectral theory deals with the study of natural vibrations of physical/mechanical systems. These occur when a system is set in motion with an initial input and allowed to vibrate freely, without further input from an external force.

The simplest example of natural vibrations is the pendulum. The pendulum has one degree of freedom and one natural frequency which can be easily computed.

A more complicated example is a string. Think of a guitar string. A string is a system with an infinite number of degrees of freedom and it has an infinite number of natural frequencies. The natural frequencies of a string can also be computed explicitly and the set of all these natural frequencies is called the *spectrum*. The lowest natural frequency of a string is called the *fundamental frequency*, whereas higher natural frequencies are called *overtones*.

Spectral theory becomes really interesting when one starts looking at more complicated physical/mechanical systems such as acoustic resonators, electromagnetic resonators, fuselages of airplanes, hulls of ships, blades of turbines etc. Even the study of atoms reduces to spectral theory: the behaviour of an electron in an atom is, effectively, a natural vibration described by the Schrödinger equation, or, more precisely, by the Dirac equation.

I will give an overview of spectral theory, charting its development from the non-rigorous works of physicists to modern rigorous mathematical results.

I will also illustrate industrial applications of spectral theory based on my previous work for the Soviet navy as well as a Soviet company designing infrared guidance systems for anti-aircraft missiles.