DIRECTIONAL SQUARE FUNCTIONS

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Abstract. A well known result of Rubio de Francia states that the square function of frequency projections onto disjoint intervals of the real line is bounded on $L^p(\mathbb{R})$ for $2 < p < \infty$. Higher dimensional versions of this result have different possible realizations, depending on the choice of frequency projections. If the projection sets have a directional element then these operators will have Kakeya counterexamples, and estimates are only possible in the critical range. This is the case for frequency projections onto cones or rectangles in $\mathbb{R}^2$ pointing at a finite set of directions. We prove directional Rubio de Francia estimates in the critical range of L^p-spaces with sharp dependence on the number of directions. The proof uses a directional Carleson embedding theorem coupled with time-frequency analysis techniques. This talk will be partly expository but also referring to joint work with N. Accomazzo, P. Hagelstein, F. Di Plinio and L. Roncal.

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Date: October 17, 2024.