On geometric properties of conformally invariant curves

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How to construct a canonical random conformally invariant path in two dimensions? Motivated by Loewner's classical theory of dynamics of slit domains, Schramm introduced random Loewner evolutions to model canonical random curves via evolutions of conformal maps. While the initial usage of such Schramm-Loewner evolutions (SLEs) was to describe critical interfaces in statistical physics models and their relation to conformal field theory (CFT), SLE type curves quickly turned out to be ubiquitous in various problems in probability theory and mathematical physics, and to have intricate connections to complex geometry and beyond.

This talk highlights the geometric aspects of SLE type curves. As examples, we shall mention versions of the Loewner energy (the anticipated action functional of these canonical curve models, or more rigorously, the rate function in large deviations principles for the random curves), the classification of real rational functions with prescribed critical points
(a special case of the Shapiro-Shapiro conjecture in real enumerative geometry, first proven by Eremenko and Gabrielov), and the emergence of the Virasoro algebra (the symmetry algebra of CFTs) from complex deformations of boundary components of bordered Riemann surfaces.

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