Thresholds of Spectral Computations: Exploring Provability and Phase Transitions in the Solvability Complexity Index

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We introduce the 'Markov' model of computation into the solvability complexity index hierarchy, which measures the difficulty of computational problems. Specifically, we explore the computational difficulty of solving spectral problems using Markov towers of algorithms. These algorithms accept a finite collection of Turing machines that describe the operator as input and produce a Turing machine that describes an output. Results in this model have implications for provability, such as determining whether the spectrum of an operator intersects a certain open ball. We also present a series of phase transitions, where, for some error tolerance \epsilon, we can compute the spectrum with accuracy ϵ, but cannot achieve this below this threshold. This leads to a zoo of classification results and a cascade of phase transitions through different levels of difficulty as \epsilon decreases. Applications to specific classes of operators, such as Schrödinger operators, will also be discussed.