Eigenvalue Bounds for Perturbed Periodic Dirac Operators

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Consider the one-dimensional Dirac operator $H=H_0+V$ in $L^2(\mathbb R)^2$,
$$H = -i \sigma_2 d/dx + m \sigma_3 + q(x) + V (x) \qquad (x \in \mathbb R),$$
where σ_2, σ_3 are Pauli matrices, and $m \ge 0$ is the particle mass, q is real-valued and periodic of period a and V is a 2×2 matrix-valued function with entries in $L^1(\mathbb R)$. The free Dirac operator H_0 is self-adjoint and its spectrum has a band-gap structure, i.e. it is purely absolutely continuous and consists of a sequence of closed intervals in $\mathbb R$. The Dirac operator $H= H_0+V$ has the same essential spectrum as H_0 but can have additional eigenvalues in $\mathbb C$. We aim to establish regions in $\mathbb C$ which contain all eigenvalues of H. We have proved that $\lambda \in \mathbb C$ cannot be an eigenvalue if $$\|V\|_1 < \Gamma(M(\lambda)) \gamma_+ (\lambda) \gamma_- (\lambda),$$
where $M(\lambda)$ is the monodromy matrix of the periodic problem and Γ is a matrix function related to the angle between the eigenvectors of the matrix; $\gamma_+ (\lambda), \gamma_- (\lambda)$ relate to the size of Floquet solutions of the periodic problem. We then show that $\gamma_\pm (\lambda)$ are strictly positive and that $\Gamma(M(\lambda))$ is strictly positive except at the end-points of spectral bands, where it tends to 0.