RATIONALITY AND ARITHMETIC OF THE MODULI OF ABELIAN VARIETIES. THIS IS A JOINT WORK WITH DANIEL LOUGHRAN

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We study the rationality properties of the moduli space $\mathcal{A}g$ of princi- pally po- larised abelian g-folds over Q and apply the results to arithmetic questions. In particular we show that any principally polarised abelian threefold over Fp may be lifted to an abelian variety over Q. This is a phenomenon of low dimension: assuming the Bombieri-Lang con- jecture we also show that this is not the case for abelian varieties of dimension at least seven. About moduli spaces, we show that $\mathcal{A}g$ is unirational over Q for $g \leq 5$ and stably rational for $g = 3$. This also allows us to make unconditional one of the results of Masser and Zannier about the existence of abelian varieties over Q that are not isogenous to Jacobians.