Anomalies and Manipulation in Scottish Single Transferable Vote Elections

How common are voting anomalies in STV elections? An empirical analysis

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June 10, 2024

What is STV (in Scotland)?

- Voters rank candidates by preference (no ties) to select multiple election winners, n candidates running for S seats
- First-place rankings are totaled
- Candidates win seat after exceeding quota ($q = \frac{V}{S+1} + 1$), excess votes distributed proportionally ($d_i = \frac{e}{t} \cdot b_i$) to 2nd place
- If no candidate exceeds quota, lowest candidate eliminated and (whole) votes distributed to 2nd place
- Process continues until all seats filled–last seats <u>can</u> be filled by candidates who do not achieve quota
- STV to select 1 candidate is Instant Runoff Voting–our focus is on multiwinner elections

The STV data

- Local government council elections in Scotland 2007-2022
- 1100 elections, 30 IRV (1 seat) and 1070 multiwinner (2-5 seats)
- Most elections are for 3 or 4 seats
- Number of candidates ranges 3-14, most have 6-8
- Average ballot length is 3 to 4
- Almost all are "close" for at least one seat (98% have multiple rounds)
- Many political parties (SNP, Labour, Con, LD, Green, Ind)
- We use code to find (monotonicity) anomalies in the elections

Example STV election

$S = 2, \quad n = 4, \quad V = 501, \quad q = \frac{501}{2+1} + 1 = 168$

Votes	19	41	60	15	73	51	19	57	12	40	8	47	59
1st	A	A	A	A	B	B	B	C	C	C	D	D	D
2nd	B	B	C	D	C	A	D	A	B	D	A	C	B
3rd		C	D		A	D	C		A	B	C	B	
4th		D				C	A		D	A			

Quota =168

Candidate	Votes By Round			
A	135	**192**		
B	143	155	162.500	
C	109			
D	114	154	163.375	**233.375**

Note: $\frac{24}{192} \cdot 75 = 9.375$ excess votes distributed from A to D

(Monotonicity) anomaly types

- Committee-size anomaly: if you reduce number of candidates to be elected, and new winner set is NOT a subset of larger winner set

- Upward anomaly: raise a winning candidate's rank on some ballots, and that makes them LOSE their seat

- Downward anomaly: lower a losing candidate's rank on some ballots, and that makes them WIN a seat

- No-show anomaly: a group of voters gets a better result (more favored candidates win) if they do NOT vote

Real-world: Committee-size anomaly

Original election: 2017 Moray, Buckie Ward 3

$S = 3$, quota $= 786$

Candidate	Votes By Round		
Cowie (Ind)	673	**826.28**	
Eagle (Con)	**1060**		
McDonald (SNP)	691	701.60	710.27
Warren (SNP)	716	728.15	**734.36**

Modified: $S = 2$, quota $= 1047$

Candidate	Votes By Round		
Cowie	673	680.27	
Eagle	**1060**		
McDonald	691	691.50	**849.50**
Warren	716	716.58	830.26

Even worse committee-size anomaly

Original election: 2017 Moray, Buckie Ward 3

$S = 3$, quota $= 786$

Candidate	Votes By Round		
Cowie (Ind)	673	**826.28**	
Eagle (Con)	**1060**		
McDonald (SNP)	691	701.60	710.27
Warren (SNP)	716	728.15	**734.36**

$S = 1$, quota $= 1571$

Candidate	Votes By Round		
Cowie	673		
Eagle	1060	1390	1431
McDonald	691	791	**1462**
Warren	716	780	

Original election:

$S = 4$, quota $= 651$

Candidate	Votes By Round						
Finlayson	389	429.4	452.3	519	519.8	621.3	**777.1**
Fletcher	464	469.5	470.6	489.2	495	547.95	
MacInnes	324	332.2	336.5	367.4	367.9		
McCaffery	299	307.1	313.2				
Rattray	361	378.7	394.6	481.4	483.4	547.97	**631**
Rous	65	68.5					
Smith	584	604.7	606.9	**660.8**			
Wilson	**768**						

Winners: Wilson, Smith, Finlayson, Rattray.
Note razor-thin margin in round of three candidates.

Original winners: Wilson, Smith, Finlayson, Rattray
Modified election: change 25 MacInnes>Rattray> . . . to
Rattray>MacInnes> . . ., and now Rattray does *not* win a seat:

$S = 4$, quota $= 651$

Candidate	Votes By Round						
Finlayson	389	429.4	452.3	524.3	614.0	618.5	**856.4**
Fletcher	464	469.5	470.6	506.1	530.0	559.2	**622.9**
MacInnes	299	307.2	*311.5*				
McCaffery	299	307.1	313.3	348.1			
Rattray	386	403.7	419.6	445.8	539.9	550.3	
Rous	65	68.5					
Smith	584	604.7	606.9	644.5	**701.7**		
Wilson	**768**						

Original winners: Wilson, Smith, Finlayson, Rattray.
Modified election: change 9 <u>Fletcher</u>>McCaffery> . . . votes to
McCaffery><u>Fletcher</u>> . . ., and 15 Fletcher bullet votes to
McCaffery><u>Fletcher</u>, and now <u>Fletcher</u> *wins* a seat

$S = 4$, quota $= 651$

Candidate	Votes By Round						
Finlayson	389	429.4	452.3	524.3	614.0	618.5	**856.4**
Fletcher	440	445.5	446.6	482.1	530	559.2	**622.9**
MacInnes	324	332.2	*336.5*				
McCaffery	323	331.1	337.2	372.1			
Rattray	361	378.7	394.6	445.8	539.9	550.3	
Rous	65	68.5					
Smith	584	604.7	606.9	644.5	**701.7**		
Wilson	**768**						

Note Strong vs Weak downward anomaly

Original winners: Wilson, <u>Smith</u>, Finlayson, Rattray.
Modified election: Remove 17 <u>Smith</u>><u>Fletcher</u> ballots, and now
<u>Fletcher</u> wins a seat (and <u>Smith</u> still wins a seat).

$S = 4$, quota $= 648$

Candidate	Votes By Round						
Finlayson	389	430.4	453.3	520.2	615.2	618.6	**856.0**
Fletcher	464	469.6	470.8	489.4	530.2	551.2	**614.3**
MacInnes	324	332.4	336.8	367.7			
McCaffery	299	307.3	313.4				
Rattray	361	379.1	395.1	481.9	540.5	548.3	
Rous	65	68.6					
Smith	567	588.25	590.4	*644.3*	**685.3**		
Wilson	**768**						

Anomaly results from 1070 multiwinner elections

- Scotland STV data includes political parties
- Can investigate anomaly percentages for individuals *or* parties
- 9 (0.9%) Committee-size events, party= 4
- 17 (1.6%) Downward monotonicity, 7 strong, 10 weak, party= 15
- 22 (2.1%) Upward monotonicity, party= 15
- 38 (3.6%) No-show, party= 29
- 21 of 60 overlap: 4 with upward, downward *and* no-show. Committee-size have no overlap
- Upward/downward anomaly % is approximately same as USA IRV (2.2%/1.6%) but no-show is much higher for STV (0.5% vs 3.6%). Perhaps due to seat order and quota changes (15 vs 23)?

Vote spreading for political parties

- *Party voter*: voter who lists all party members at top of ballot, eg: *ABC_*, *BAC_*, etc
- Note: cannot say for sure *party voter* is actually a party voter
- Can use party data to look for benefits to Vote Spreading within a party
- Specifically: Could a party do better (or worse) if party voters spread votes more equitably?
- Suppose *A* and *B* are in same party:
- 1300/400 votes for *A*/*B* vs 900/800
- If spreading votes results in a change, call it an "event", not an "anomaly"
- **Type I vote spreading event**: a party spreads votes, resulting in their winning *more* seats
- **Type II vote spreading event**: a party spreads votes, resulting in their winning *less* seats

Original election:

$S = 4$, quota = 893

Candidate	Votes By Round						
Bircham (Other)	27	28.5	32.7				
Black (Grn)	127	136.1	143.3	150.0			
Crawford (Lab)	**1279**						
McMahon (SNP)	641	656.4	659.1	660.1	626.1	743.2	**812.3**
Mochan (Lab)	295	577.5	599.7	605.1	626.1	736.8	
Owens (Ind)	371	390.9	411.4	418.5	461.3		
Todd (SNP)	731	740.7	743.0	745.0	773.7	831.6	**898.1**
Young (Con)	**991**						

Note Crawford (1279) has more votes than Mochan (295), both Labour candidates.

- Change 50 Crawford>Mochan votes to Mochan>Crawford

$S = 4$, quota $= 893$

Candidate	Votes By Round						
Bircham (Other)	27	28.4	32.5				
Black (Grn)	127	135.2	142.4	149.0			
Crawford (Lab)	**1229**						
McMahon (SNP)	641	654.9	657.6	660.1	626.1	743.2	
Mochan (Lab)	345	587.2	609.5	614.8	635.5	745.8	**790.2**
Owens (Ind)	371	389.0	409.5	416.6	459.4		
Todd (SNP)	731	739.7	742.1	744.1	772.7	830.4	**1432.5**
Young (Con)	**991**						

- Swap 31 to 909 Crawford>Mochan> ... votes to get *Type I* vote spreading event. Is vote spreading a "better" result? (1574 Labour vs 1372 SNP)

- Can also have *Type II*:

$S = 4$, quota $= 1415$, Original Election

Candidate	Votes By Round							
Aitken (Con)	1297	1355.9	1372.3	1390.4	1412.1	**1426.5**		
Allison (Grn)	220	243.8	249.2					
Convery (SNP)	1199	1242.0	1242.0	1330.2	1346.7	1357.2	1358.1	**2370.5**
Cunningham (Lab)	1020	1110.5	1113.5	1160.0	**1506.2**			
Devlin (Ind)	**1840**							
Ferguson (Other)	34	42.5						
Hay (Lab)	404	462.9	465.3	481.4				
Reilly (SNP)	1059	1125.1	1126.3	1167.1	1204.6	1218.1	1218.6	

$S = 4$, quota $= 1415$, 358 Cunn. $>$ Hay Ballots Modified

Candidate	Votes By Round						
Aitken (Con)	1297	1355.9	1372.3	1390.4	**1463.8**		
Allison (Grn)	220	243.8	249.2				
Convery (SNP)	1199	1242.0	1242.0	1330.2	1405.0	1407.0	**1564.4**
Cunningham (Lab)	662	752.5	755.5	801.9			
Devlin (Ind)	**1840**						
Ferguson (Other)	34	42.5					
Hay (Lab)	762	820.9	823.3	839.4	1214.3	1227.4	
Reilly (SNP)	1059	1125.1	1126.3	1167.1	1268.8	1269.8	**1454.7**

–Note Strong vs Weak Type II events

	Single Type I event	Single Type II event	Two Type I events	Both Types of events	Total number of elections	Total number of events
Only (2,1) situations	157	6	12	1	176	189
(3,2)/(3,1) situation	11	4	0	0	15	15
Multiple situations	XXXX	XXXX	2	1	3	6

Situation type	(2,0)	(2,1)	(2,2)	(3,0)	(3,1)	(3,2)	(3,3)	4 candidates
Frequency	12	673	563	0	9	71	26	0

Discussion of vote spreading results

- 194 elections with Vote spreading events (18.1%, or $194/855 = 22.7\%$)
- 210 total events out of 1354 situations, or $210/1354 = 15.5\%$
- Elections: 182 *Type I* only, 10 *Type II* only , 2 both
- 194 events in (2,1) situations.
- 3 cands: 13 from (3,2), 3 from (3,1): 14/4 split for Type.
- Generally, parties put up good number of candidates
- Parties should manipulate voters to spread votes, if possible
- Big winners: larger parties. Independents/Greens tend to lose
- SNP: 1324 total seats, could have 111 more seats if vote spread

Potential "fix" to vote spreading

- Allow "equal party" voting: votes for a party distributed equally to all permutations
- Example: suppose A, B, C all SNP candidates. Voter votes $SNP > D > E$
- Ballot does 1/6 $ABCDE$, 1/6 $ACBDE$, 1/6 $BACDE$, etc
- Rest of election runs the same
- Voters can vote for parties, or individuals, up to the voter
- We supposed all ABC, CBA, etc voters were party voters, and distributed their votes as above (note: NOT realistic)

Ex. EP election: 2017 E. Ayrshire, Cumnock ward

$S = 4$, quota $= 893$, Original election

Candidate	Votes By Round						
Bircham (Other)	27	28.5	32.7				
Black (Grn)	127	136.1	143.3	150.0			
Crawford (Lab)	**1279**						
McMahon (SNP)	641	656.4	659.1	660.1	626.1	743.2	**812.3**
Mochan (Lab)	295	577.5	599.7	605.1	626.1	736.8	
Owens (Ind)	371	390.9	411.4	418.5	461.3		
Todd (SNP)	731	740.7	743.0	745.0	773.7	831.6	**898.1**
Young (Con)	**991**						

$S = 4$, quota $= 893$, EP election

Candidate	Votes By Round						
Bircham (Other)	27	31.2	31.3				
Black (Grn)	127	134.2	135.3	141.4			
Crawford (Lab)	**926.5**						
McMahon (SNP)	685	687.7	689.5	690.5	714.1	770.7	
Mochan (Lab)	647.5	669.8	690.8	695.4	714.1	820.7	**860.2**
Owens (Ind)	371	391.5	393.9	400.9	443.0		
Todd (SNP)	687	689.4	690.5	692.5	719.9	776.2	**1419.7**
Young (Con)	**991**						

–Get the "correct" winner without needing to do vote spreading

Discussion of Code

- All code in Python
- We are not really good at code
- Targeted vs Brute-force?
- Committee-size was easy (just change S)
- Targeted: Upward, Downward, No-show. At each stage, change elimination or election order–then check if new winner set. Need to double-check the targeted code
- Brute-force: Vote-spreading events. Also shows range of event
- All/most results were double-checked by hand

Future research

- Fill in truncated votes, any difference in anomalies?
- Compare winner set, anomaly % to other multiwinner methods (Some work has been done: (1) Chamberlin-Courant and vote spreading, (2) STV vs SRCV, (3) AQ STV, Meek, etc)
- New algorithms to find more anomalies? Use AI?
- Find theoretical conditions (necessary/sufficient) for anomalies
- Certainly many other things as well
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- And if we have time. . . anomaly hunting!

$S = 4$, quota $= 727$, Original election

Candidate	Votes By Round								
Doyle	125	126.9	136.1						
Glen-Lee	707	715.6	721.9	**732.02**					
MacDonald	361	372.7	380.0	410.8	411.1	437.8	476.8	542.6	**645.3**
MacIntyre	421	436.2	443.7	471.86	472.2	509.6	611.3	726.7	**811.4**
Mackay	222	231.3	239.0	243.1	243.3	264.3			
McIntosh	58	61.3							
Melville	410	415.9	420.0	445.1	448.8	471.0	517.2	541.9	
Neal	188	196.7	201.1	206.3	206.5				
Robertson	**807**								
Rutherford	332	340.1	343.3	353.5	353.6	384.4	409.2		

- Note: tiny margin (0.7 votes) in penultimate round
- Note: GL barely gets seat in round 4

$S = 4$, quota $= 727$, Change 5.1 GL>MacD to MacD>GL

Candidate	Votes By Round								
Doyle	125	126.9	136.1						
Glen-Lee	701.9	710.5	715.8	726.92	**746.7**				
MacDonald	366.1	377.8	385.1	415.9	440.6	441.9	480.9	546.8	
MacIntyre	421	436.2	443.7	471.86	506.2	507.6	609.4	724.9	**882.5**
Mackay	222	231.3	239.0	243.1	263.0	263.8			
McIntosh	58	61.3							
Melville	410	415.9	420.0	445.1	462.9	477.2	522.6	547.4	**666.8**
Neal	188	196.7	201.1	206.3					
Robertson	**807**								
Rutherford	332	340.1	343.3	353.5	384.3	384.7	409.5		

- Moving MacDonald UP in 5.1 ballots made MacDonald *lose* the election.
- Of course, can't change 5.1 ballots, only whole numbers

Slippery upward anomaly: Argyll-Bute, 2012, Ward 5

$S = 4$, quota $= 727$, Change 6 GL>MacD to MacD>GL

Candidate	Votes By Round								
Doyle	125	126.9	136.1						
Glen-Lee	701	709.6	714.9	726.02	**745.8**				
MacDonald	367	378.7	386.0	416.8	441.5	442.8	481.7	547.6	**650.3**
MacIntyre	421	436.2	443.7	471.9	506.2	507.5	609.3	724.8	**809.6**
Mackay	222	231.3	239.0	243.1	263.0	263.7			
McIntosh	58	61.3							
Melville	410	415.9	420.0	445.1	462.9	476.6	521.9	546.7	
Neal	188	196.7	201.1	206.3					
Robertson	**807**								
Rutherford	332	340.1	343.3	353.5	384.3	384.7	409.5		

- Moving MacDonald UP in 6 ballots does NOT make MacDonald lose the election.
- Help! Changing 5.03 to 5.5 GL>MacD to MacD>GL creates anomaly, but more or less doesn't work

Do 5 GL>MacD to MacD>GL, AND
2 Rob >GL>MacD to Rob >MacD>GL

$S = 4$, quota $= 727$

Candidate	Votes By Round								
Doyle	125	126.9	136.1						
Glen-Lee	702	710.4	716.7	726.8	**746.6**				
MacDonald	366	377.9	385.2	415.0	440.7	442.	481.0	546.9	
MacIntyre	421	436.2	443.7	471.9	506.2	507.6	609.4	724.9	**882.5**
Mackay	222	231.3	239.0	243.1	263.0	263.7			
McIntosh	58	61.3							
Melville	410	415.9	420.0	445.1	462.9	477.1	522.5	547.3	**666.6**
Neal	188	196.7	201.1	206.3					
Robertson	**807**								
Rutherford	332	340.1	343.3	353.5	384.3	384.7	409.5		

- Moving the Rob >GL>MacD gave enough fractional ballots to make MacDonald lose

Any questions?

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