Eisenstein Hecke algebras in prime-square level

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Let p and N be primes at least 5. In Mazur's famous Eisenstein ideal paper, he shows that when N = 1 mod p, there is a cusp form of level N and weight 2 congruent to the Eisenstein series mod p. He also asks how many such cusp forms there are; in other words, what is the rank of the Eisenstein Hecke algebra? This question has since been addressed by work of Merel, Calgary—Emerton, Wake—Wang-Erickson, and Lecouturier, and its answer uncovers some deep arithmetic. There are also mod-p Eisenstein congruences in level N^2 when N = -1 mod p. We will discuss a recent observation that in this case, not only the rank but the Eisenstein Hecke algebra itself can be described explicitly and almost independent of N. This is joint work with Preston Wake.