

POINT-SPLITTING AND DIMENSIONAL REGULARIZATION IN QUANTUM MECHANICS

JAN DEREZIŃSKI

I will discuss Green-functions of the Laplace(-Beltrami) operator on the d -dimensional Euclidean space, sphere and hyperbolic space perturbed by a point potential. For $d=1,2,3$ they correspond to well-defined self-adjoint operators. For $d \geq 4$ they are not resolvents of well-defined operators. They are conjectured to describe asymptotically the resolvents of Schrödinger operators with perturbations of very small support.

I will explain two approaches to define these Green functions, both inspired by methods used in Quantum Field Theory. The first uses the point splitting method and differentiation in the energy. The second uses the so-called generalized integral--an idea that goes back to Hadamard and Riesz, and is similar to minimal subtraction and dimensional renormalization of Feynman diagrams.