Early Career Talk

Sharp quasi-invariance threshold for the cubic Szeg\H{o} equation

James Coe

The cubic Szeg\H{o} equation serves as a toy model for weakly-dispersive nonlinear Hamiltonian dynamics. We shall consider the flow of a family of Gaussian fields under these dynamics. We see that above a critical regularity, the measures are quasi-invariant under the flow, but below this regularity, quasi-invariance fails. In fact, the distribution at almost every future time is singular with respect to the initial distribution.
We will discuss a general heuristic of when we expect quasi-invariance or singularity in Hamiltonian systems and test this hypothesis for the Szeg\H{o} dynamics. In the high-regularity regime we prove standard energy estimates, and in the low-regularity setting we exhibit an instantaneous growth of Sobolev norms of the solution (at high frequencies), and then employ an abstract argument to show that such a growth cannot occur with positive probability at all times. This talk is based on joint work with Leonardo Tolomeo.