Reaching the equilibrium: Long-term stable numerical schemes for SPDEs

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Partial Differential Equations appear in many applications, such as modeling of fluids, porous media, and superconductors, and are typically derived from physical principles. In extreme cases (e.g. at very fast or small scales), additional features need to be considered: Randomness takes into account model uncertainties that arise, for example, from thermal fluctuations and measurement errors. This motivates the use of SPDEs to improve the model's accuracy. In many cases, these models possess an energy structure, which characterises the stationary states of these systems. Unsteady solutions eventually tend to a steady state by dissipating their energy.

In this talk, we will explain how transport noise, a specific stochastic structure, enables full understanding of the energy dissipation of generalised Stokes equations and how this relates to the stochastic steady state -- an invariant measure. Moreover, we discuss how the energy dissipation can be preserved on the discrete level for a broad class of spatial discretisations. We conclude with numerical simulations.

The talk is based on joint work with Kim-Ngan Le (Monash University) and Jérome Droniou (CNRS & IMAG, Monash University).