An Analysis of the Styrian Parliamentary Elections in 2015 and 2019 Using Different (Theoretical) Approaches

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(based on joint work with Andreas Darmann, Julia Grundner and Manuela Puster)
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Mathematics of Voting and Representation
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Introduction

Does the electoral system matter?

- YES from a theoretical point of view
  - Saari (1994), Nurmi (1999)
- YES from an empirical point of view
  - Duverger (1951), Rae (1971), Lijphart (1994)
- this led to interest in electoral engineering
  - Riker (1986, 1988)
  - Taagepera and Shugart (1989) provide guidelines for justified changes in electoral systems
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- Behavioral Social Choice
  - Regenwetter et al. (2006)
  - data from actual elections tested against the negative predictions stemming from the theoretical literature
  - many of the theoretical problems were (often) not found in real-world decision problems
- experimental and survey studies
  - Baujard et al. (2020, 2018, 2014), Laslier and Sanver (2010), Laslier and van der Straeten (2008) on French elections
  - Roescu (2014) on Romanian elections
  - Wantchekon (2003) on Benin elections
  - Alos-Ferrer and Granic (2014) on German elections
  - Darmann et al (2017, 2019) and Darmann and Klamler (2023) on Austrian elections
  - McCune and McCune (2024) on various American ranked-choice elections
  - Blais and Degan (2019) and Stephenson et al (2018) on strategic aspects

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How could mathematics help us understand what goes on in elections?

Goal today is to introduce a particular mathematical approach to analyze voting situations

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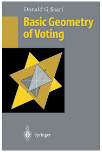

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- to say something about potential differences in outcomes and paradoxical situations

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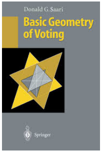

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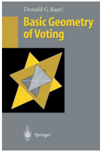

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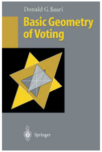

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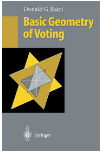

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- to say something about potential differences in outcomes and paradoxical situations

Austrian Elections

- Data collected via exit polls in front of several real polling stations during the Styrian parliamentary elections on 31 May 2015 and on 24 Nov 2019
- approximately 1000 respondents for each election
- Questions in particular on voters' preferences, e.g.,
  - full preference ranking of the parties
  - assignment of parties to pre-defined preference classes
  - approval preferences
  - points assigned on a scale from -20 to +20
  - but also on evaluation of parties on a left-right-political dimension
- eight parties in 2015 - six parties in 2019
  - used the weak-order model of Regenwetter et al. (2007) to receive complete rankings (for the 2015 election)
  - one third incomplete rankings in 2015 - 7% incomplete rankings in 2019

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Theoretical Considerations

- Theoretically situations as the following could occur:

5	4	3	2	1
a	e	d	c	b
b	b	e	d	c
c	c	b	e	d
d	d	c	b	e
e	a	a	a	a

- Assume that they provide (consistently) more detailed preference information

$A(B)$	5	$A(B)$	4	$A(B)$	3	$A(B)$	2	$A(B)$	1
30(20)	a	30(20)	e	70(10)	d	45(10)	c	35(10)	b
25(2)	b	25(5)	b	15(−1)	e	40(−1)	d	30(5)	c
20(1)	c	20(3)	c	10(−2)	b	10(−2)	e	25(−5)	d
15(−5)	d	15(−3)	d	5(−5)	c	5(−3)	b	10(−7)	e
10(−6)	e	10(−5)	a	0(−7)	a	0(−4)	a	0(−10)	a

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b	*b*	*e*	*d*	*c*
c	*c*	*b*	*e*	*d*
d	*d*	*c*	*b*	*e*
e	*a*	*a*	*a*	*a*

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- This leads to the following voting outcomes:

Plur	Runoff	STV	Borda	Cond	Appr	100 points	±20 points
a	e	e	b	$\emptyset$	c	d	a

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Empirical Results - outcomes 2015

- The results of our 2015 election were very consistent.

Voting rule	1st	2nd	3rd	4th	5th	6th	7th	8th
Plurality Rule	SP	VP	FP	GP	KP	NEOS	TS	Pir
Run Off	SP	VP	FP	GP	KP	NEOS	TS	Pir
STV	SP	VP	FP	GP	NEOS	KP	TS	Pir
Condorcet	SP	VP	GP	FP	KP	NEOS	TS	Pir
Approval	SP	VP	FP	GP	NEOS	KP	TS	Pir
Borda	SP	VP	GP	FP	NEOS	KP	TS	Pir
$\pm$20 Points	SP	VP	GP	KP	NEOS	FP	Pir	TS
100 Points	SP	VP	FP	GP	KP	NEOS	TS	Pir

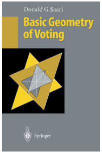
Empirical Results - outcomes 2019

- The results of our 2019 election showed more variation.

Voting rule	1st	2nd	3rd	4th	5th	6th
Plurality Rule	GP	VP	SP	FP	KP	NEOS
Run Off	GP	VP	SP	FP	KP	NEOS
STV	GP	VP	SP	FP	KP	NEOS
Condorcet	GP	NEOS	SP	VP	KP	FP
Approval	GP	NEOS	KP	VP	SP	FP
Borda	GP	VP	SP	NEOS	KP	FP
+/o/-	GP	NEOS	VP	SP	KP	FP
±20 Points	GP	NEOS	KP	SP	VP	FP
Anti-Plur	VP	SP	NEOS	GP	KP	FP

Saari's Geometric Approach

What determines the difference in outcomes?



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Formal Framework:

- $X = \{c_1, c_2, c_3, ..., c_n\}$ ... set of n candidates
  - $R \subseteq X \times X$ is a binary relation on X
  - $\mathcal{P}$ is the set of the $n!$ strict rankings of the candidates
- assume a finite number of m voters
- a profile is $p \in \mathcal{P}^m$
  - equivalently: $p \in \mathbb{R}^{n!}$, i.e., how many voters hold each of the $n!$ different strict rankings

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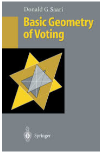

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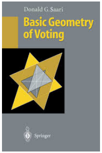

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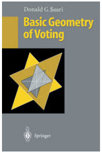

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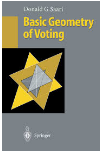

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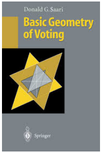

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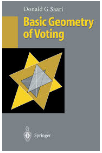

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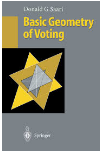

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Saari's Geometric Approach

Example with $n = 3$

type	ranking	type	ranking
1	$a \succ b \succ c$	4	$c \succ b \succ a$
2	$a \succ c \succ b$	5	$b \succ c \succ a$
3	$c \succ a \succ b$	6	$b \succ a \succ c$

- $p = (2, 0, 0, 4, 1, 0) \in \mathbb{R}^{n!}$ represents a profile
- $p' = (\frac{2}{7}, 0, 0, \frac{4}{7}, \frac{1}{7}, 0)$ is a normalized profile
- ... is a point in the $n! - 1$ dimensional simplex
  - already 5-dimensional for $n = 3$
- In the 2019 elections, $p \in \mathbb{R}^{720}$
  - only 227 rankings actually occurred
  - some "natural restrictions" of what are reasonable preferences

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  - some "natural restrictions" of what are reasonable preferences

Saari's Geometric Approach

Example with $n = 3$

type	ranking	type	ranking
1	$a \succ b \succ c$	4	$c \succ b \succ a$
2	$a \succ c \succ b$	5	$b \succ c \succ a$
3	$c \succ a \succ b$	6	$b \succ a \succ c$

- $p = (2, 0, 0, 4, 1, 0) \in \mathbb{R}^{n!}$ represents a profile
- $p' = (\frac{2}{7}, 0, 0, \frac{4}{7}, \frac{1}{7}, 0)$ is a normalized profile
- ... is a point in the $n! - 1$ dimensional simplex
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The following reduction in dimensions is, however, possible:

- take the $n-1$ dimensional simplex where each vertex represents a candidate
- each point in the simplex determines a ranking of the candidates based on the point's distance from the vertices
  - ... according to "the closer the better"
  - $a \succ b \succ c$
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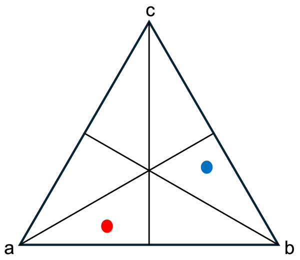

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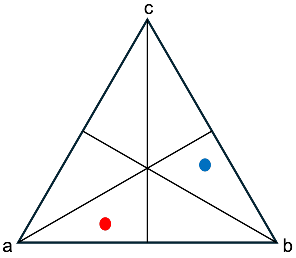

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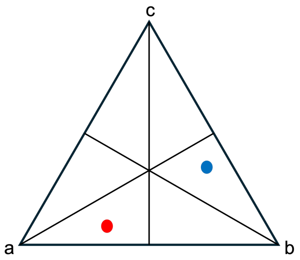

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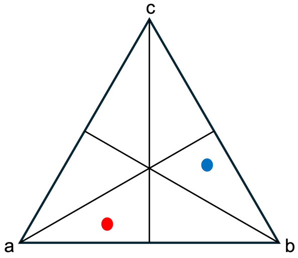

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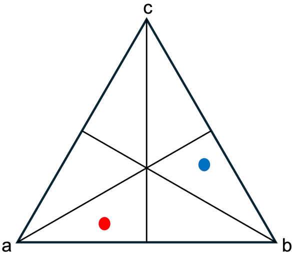

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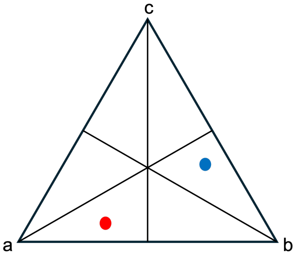

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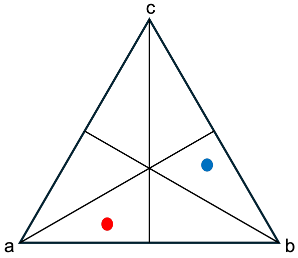

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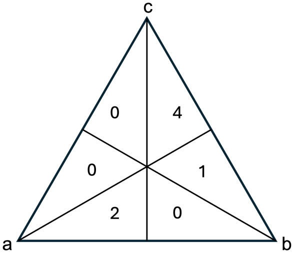
The profile $p = (2, 0, 0, 4, 1, 0)$ can now be presented in the triangle

In addition various voting outcomes can be determined:

- the numbers to the left and right of each line determine the pairwise majority outcome
- the numbers in the two areas closest to the vertices determine the plurality outcome
- the plurality numbers plus one half of the numbers in the areas next to that determine the Borda outcome



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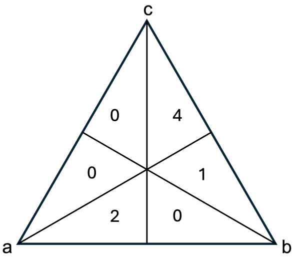

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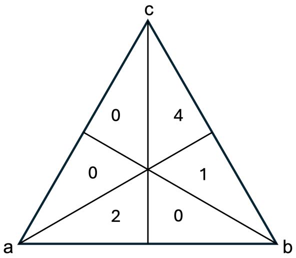

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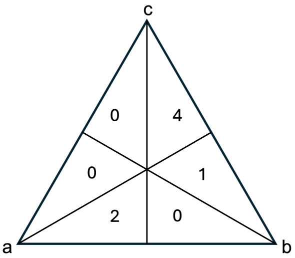

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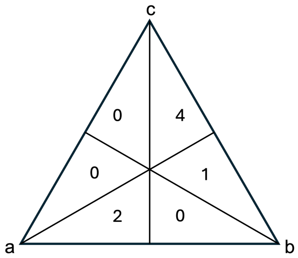

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Any rule that assigns scores to the candidates defines a point in the simplex.

- E.g., profile $p = (2, 0, 0, 4, 1, 0)$ leads to plurality scores of $(2, 1, 4)$ or, normalized, to $q^{Pl} = (\frac{2}{7}, \frac{1}{7}, \frac{4}{7})$.
  - red point in simplex
- plurality outcome $c \succ a \succ b$
- in general, we can plot the outcome for any scoring rule with (normalized) weights $w^s = (1, s, 0)$
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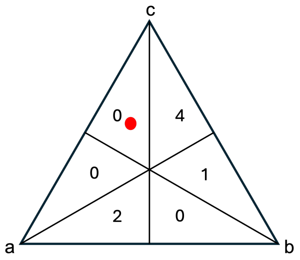

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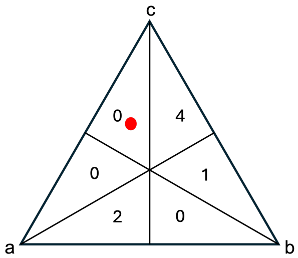

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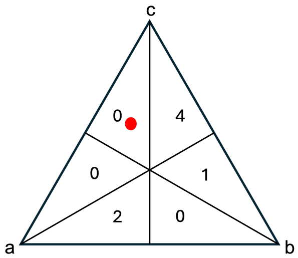

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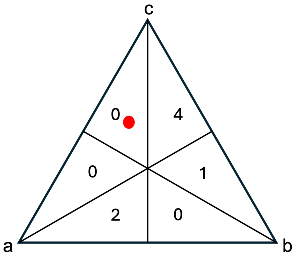

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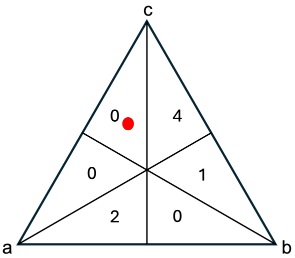

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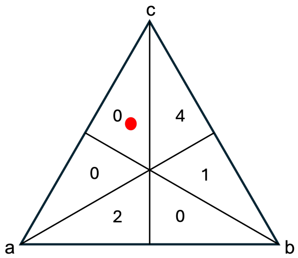

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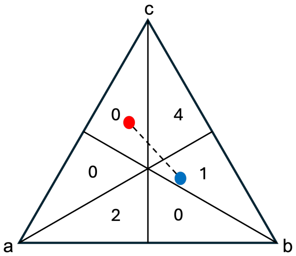
because for all $s \in [0, 1]$, w^s is a linear combination of $w^{Pl} = (1, 0, 0)$ and $w^{AP} = (1, 1, 0)$, all scoring rule outcomes must lie on a line from the Plurality outcome to the Anti-plurality outcome.

- Procedure Line
- from red (Plur) to blue (A-Plur)
- E.g., profile $p = (2, 0, 0, 4, 1, 0)$ leads to a procedure line indicating 5 different scoring rule outcomes (two of which contain indifferences).



For $n > 3$, the procedure line becomes the convex hull of the outcomes based on all k-Approval rules, i.e., the scoring vectors $(1, 0, 0, ..., 0)$, $(1, 1, 0, ..., 0)$, $(1, 1, 1, 0, ..., 0)$

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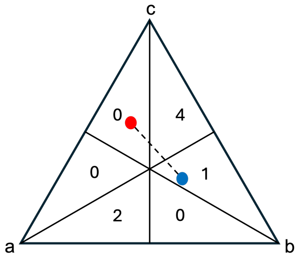

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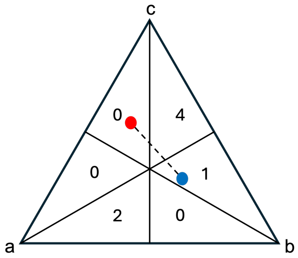

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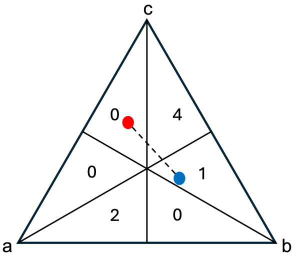

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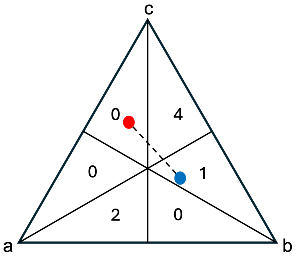

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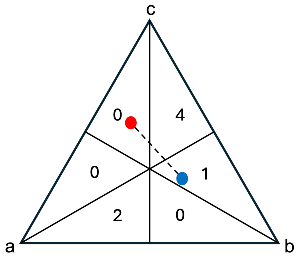

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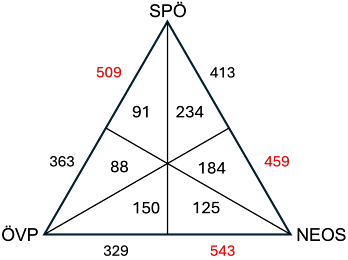
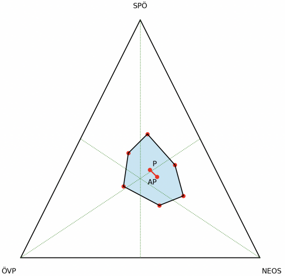
Let us apply this to the profile in the 2019 election restricted to the 3 parties SPÖ, ÖVP and NEOS.





- Pairwise Majority: $N \succ S \succ O$
- Plurality Ranking: $S \succ N \succ O$
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- for most scoring rules: $N \succ S \succ O$
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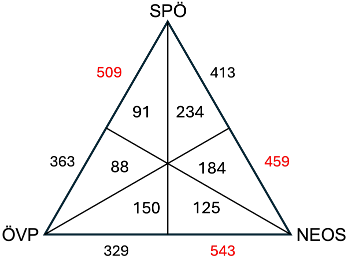

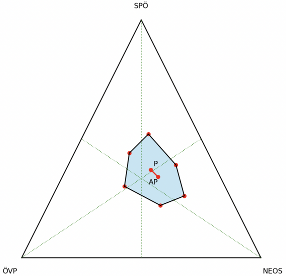

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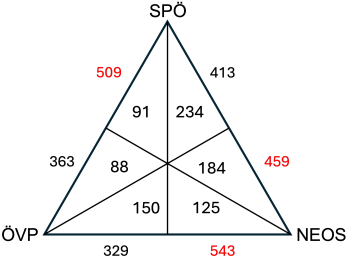

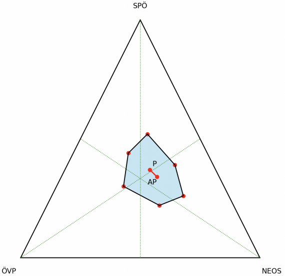

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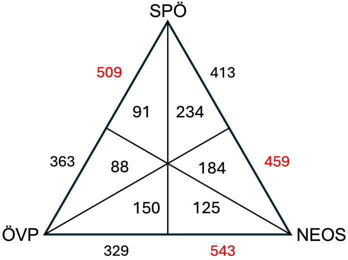

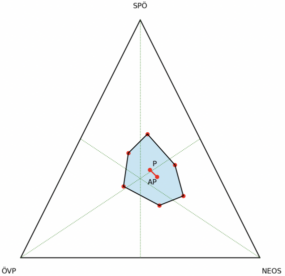

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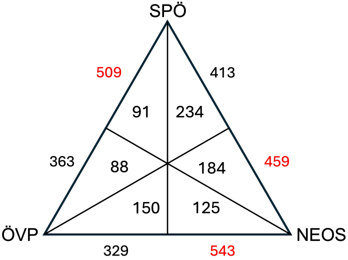

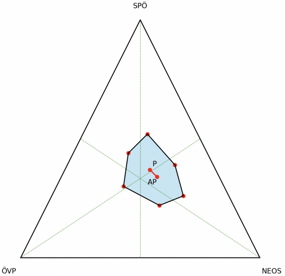

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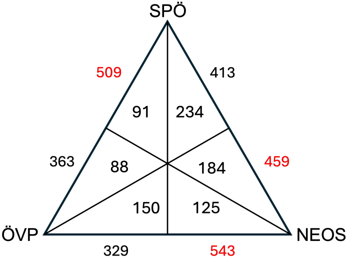

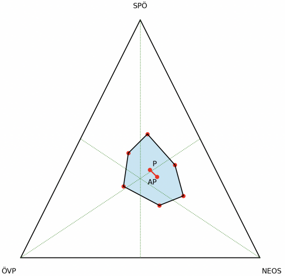

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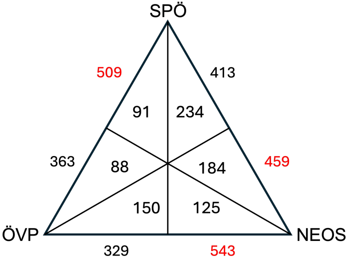

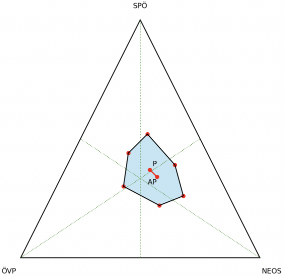

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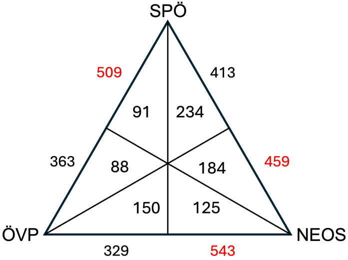

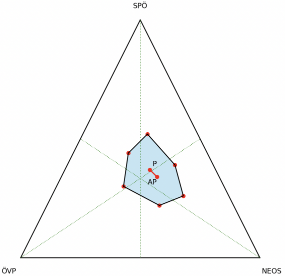

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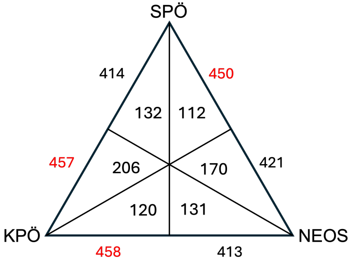

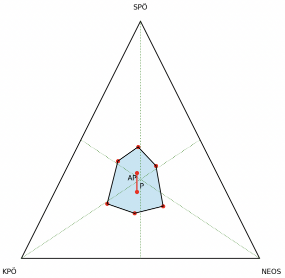

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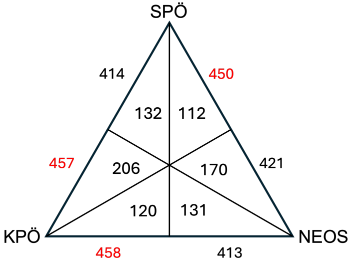

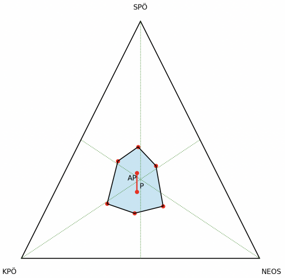

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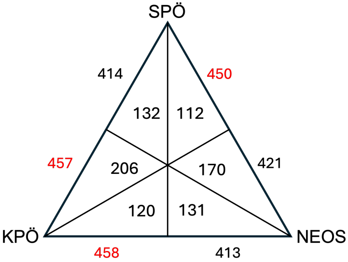

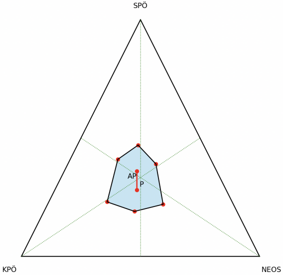

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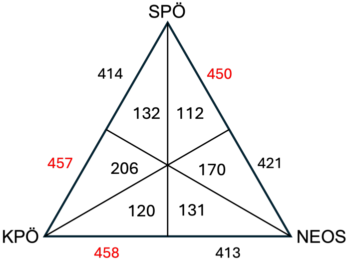

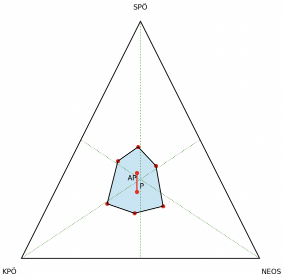

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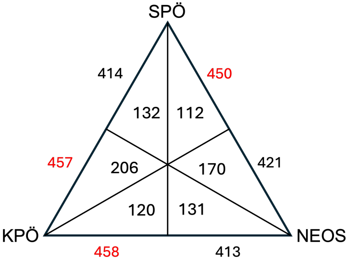

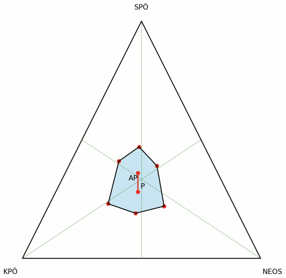

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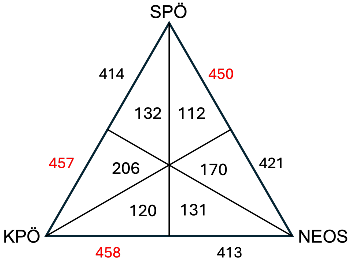

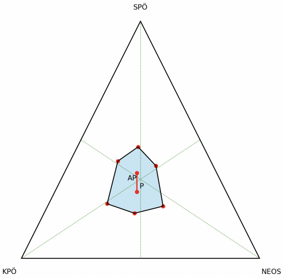

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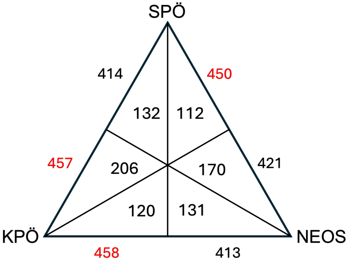

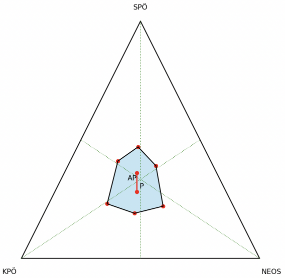

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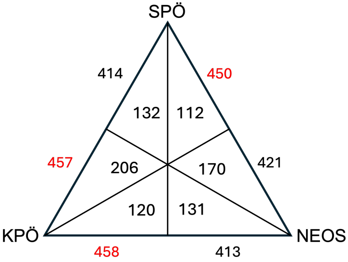

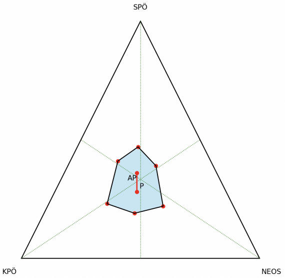

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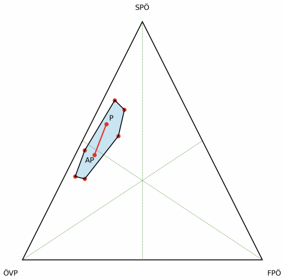
- only two different strict rankings as outcomes



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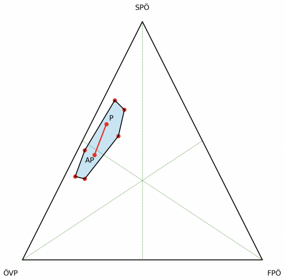

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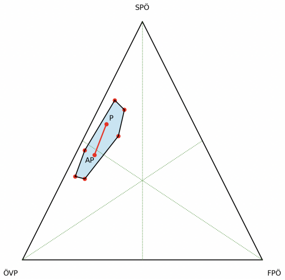

Profile decomposition

What actually drives the (potential) differences in the outcomes?

Saari (1995) uses what he calls profile decomposition

- profile differential as the difference between two profiles with the same number of voters
  - e.g., $(0, -1, -3, 2, 1, 1)$, which sums up to zero and contains negative voters.
- can be made non-negative by adding a profile $\mathcal{K}$ in which there is one voter for each ranking
  - $(1, 1, 1, 1, 1, 1)$
  - all positional and pairwise voting rules have complete indifference over $\mathcal{K}$
- $(0, -1, -3, 2, 1, 1) + 3\mathcal{K} = (3, 2, 0, 5, 4, 4)$
- universal kernel $\mathbf{p}_\mathbf{K}$
- $n! - 2^{n-1}(n-2) - 2$ dimensional subspace of profile space $Si(n!)$

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Profile decomposition - Basic profile

Definition

The basic portion for a candidate X is the profile differential with one voter for each type where X is top-ranked and -1 voters where X is bottom-ranked.

type	ranking	type	ranking
1	$a \succ b \succ c$	4	$c \succ b \succ a$
2	$a \succ c \succ b$	5	$b \succ c \succ a$
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- e.g. $B_a = (1, 1, 0, -1, -1, 0)$ is the Basic vector for item a.
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- $\mathbf{p_B} = a_B B_a + b_B B_b + c_B B_c$ is the profile differential (for 3 items) coming from the Basic vectors
  - e.g., $4B_a + 2B_b + 1B_c$ determines the outcome $a \succ b \succ c$ for all pairwise and positional rules

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Profile decomposition - Condorcet profile

Definition

The Condorcet portion (for n=3) is the profile differential with one voter for each type in a cycle and -1 voters for each type in the opposite cycle.

type	ranking	type	ranking
1	$a \succ b \succ c$	4	$c \succ b \succ a$
2	$a \succ c \succ b$	5	$b \succ c \succ a$
3	$c \succ a \succ b$	6	$b \succ a \succ c$

- e.g. $C^3 = (1, -1, 1, -1, 1, -1)$ is the Condorcet portion strengthening the cycle $a \succ b \succ c \succ a$
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Profile decomposition - Reversal profile

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The reversal portion (for $n = 3$) for a candidate X is the profile differential with one voter for each type where X is top-ranked, one voter for each type where X is bottom-ranked, and -2 voters where X is middle-ranked.

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- e.g. $R_a = (1, 1, -2, 1, 1, -2)$ is the Reversal vector for item a.
- leads to complete indifference for pairwise methods and the Borda count but not the other positional rules
- $\mathbf{p_R} = a_R R_a + b_R R_b + c_R R_c$ is the profile differential (for 3 items) coming from the Reversal vectors

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All profiles can now be expressed as

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Example:

- $\mathbf{p_B} = 2B_a + 1B_b = (2, 1, -1, -2, -1, 1)$
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- hence we get $(10, -10, 7, -4, -2, -1)$
- Add $\mathbf{p_K} = 10\mathcal{K}$ to get $\mathbf{p} = (20, 0, 17, 6, 8, 9)$
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Profile decomposition - application

Consider the reduced profile for SPÖ, ÖVP and NEOS.

type	ranking	type	ranking
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Example:

- $\mathbf{p} = (91, 234, 184, 125, 150, 88)$
- if we subtract $88\mathcal{K}$ we get $(3, 146, 96, 37, 62, 0)$
  - has the same outcomes as the original profile
- Actually,
  $\mathbf{p} = -13.3B_{SP} - 51.6B_{VP} + 18.6R_{SP} + 28R_{VP} - 3.6C^3 + 145.3\mathcal{K}$
- Borda: $NEOS \succ SP \succ VP$ given the Basic portion
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Party types - definition

In Baujard et al. (2014) and Darmann et al. (2017) different party types have been defined

Which types could - in principle - exist?

- popular party
  - strong support from a specific segment of society and seen positively by a large proportion of society
- unpopular party
  - strong support from a small group and seen negatively by a large proportion of society
- medium party
  - acceptable to a large proportion of society and induces strong views only for small groups
- polarizing party
  - strong support from a certain, significantly large, part of society as well as strong negative support from another, significantly large, group

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In Baujard et al. (2014) and Darmann et al. (2017) different party types have been defined

Which types could - in principle - exist?

- popular party
  - strong support from a specific segment of society and seen positively by a large proportion of society
- unpopular party
  - strong support from a small group and seen negatively by a large proportion of society
- medium party
  - acceptable to a large proportion of society and induces strong views only for small groups
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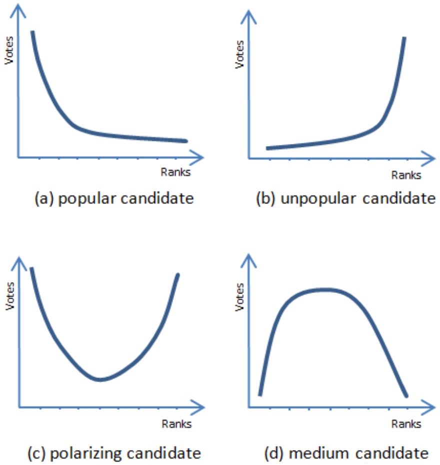

Figure: Distributions of votes over ranks for different types of candidates

Profile decomposition - application

Can also use Saari's approach to classify candidates

type	ranking	type	ranking
1	$a \succ b \succ c$	4	$c \succ b \succ a$
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- a large basic portion $B_a = (1, 1, 0, -1, -1, 0)$ indicates that a is a popular candidate
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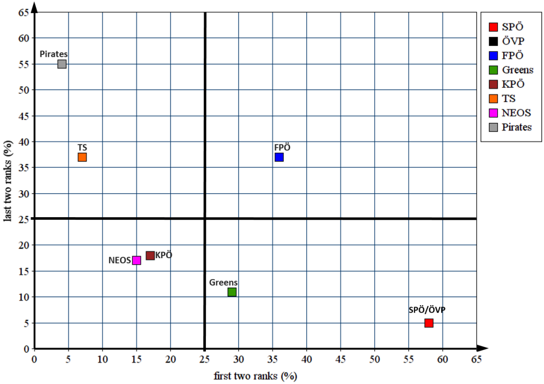

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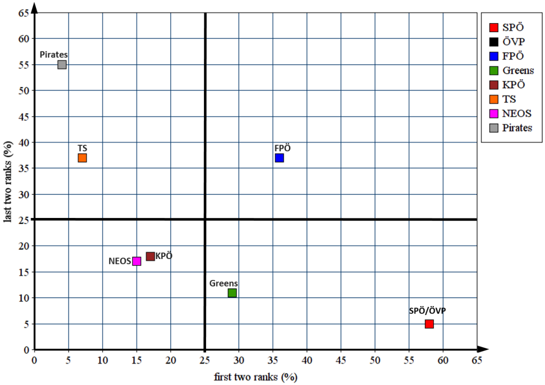

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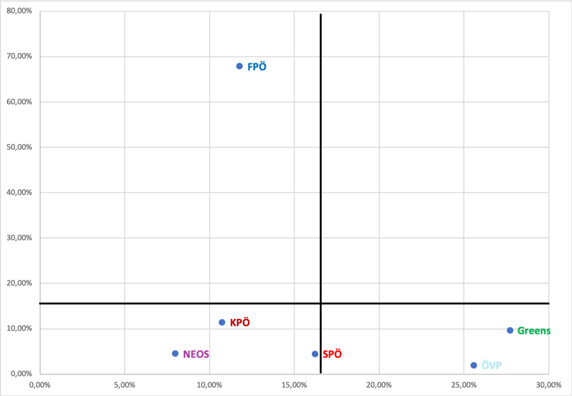

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Saari triangles - distances 2015

- could also ask how stable the outcomes are
  - points on the procedure line in the triangle indicate the distance to changes for rules with $w = (1, s, 0)$
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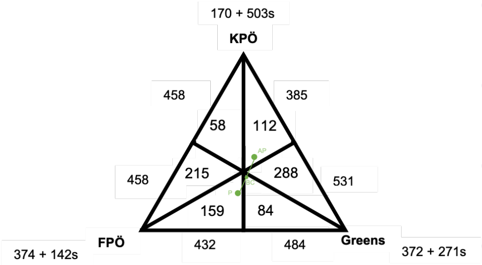

- switch from winner F to G at $s = 0.015$
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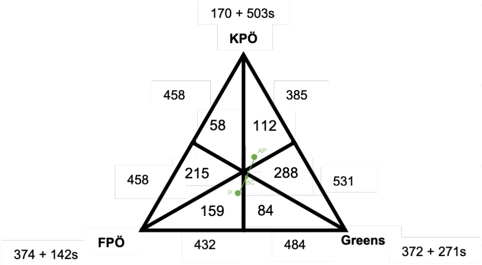

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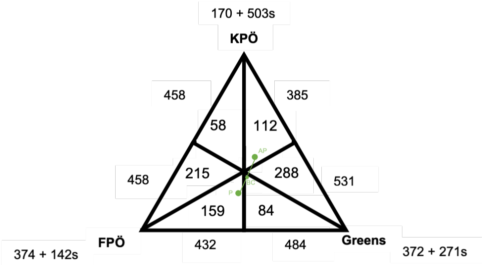

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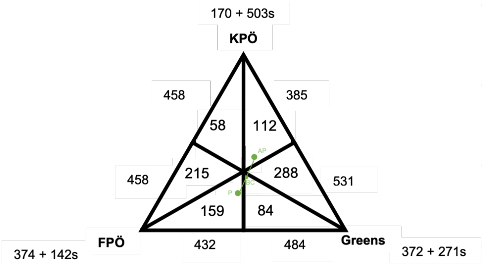

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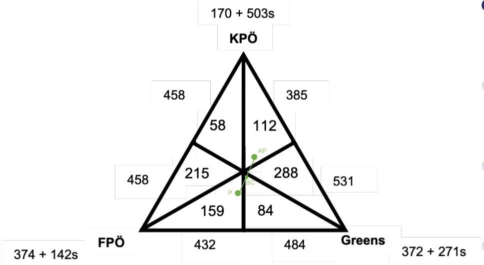

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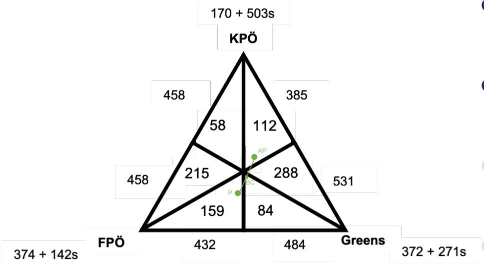

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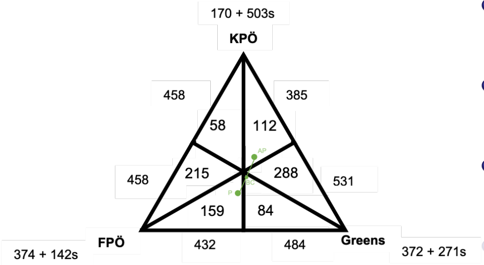

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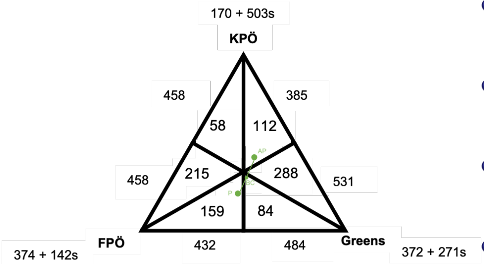

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- changes the procedure line
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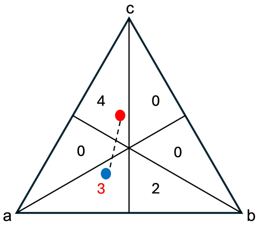

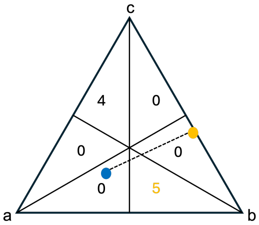

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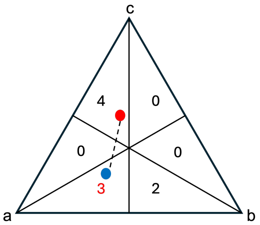

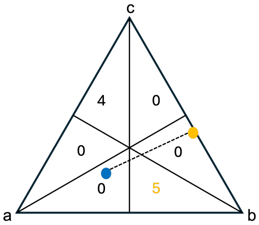

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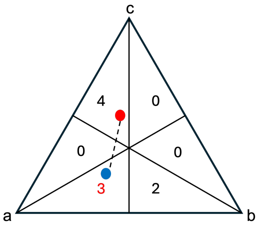

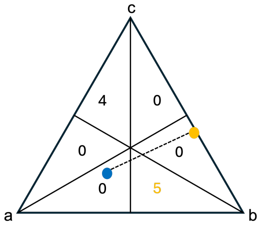

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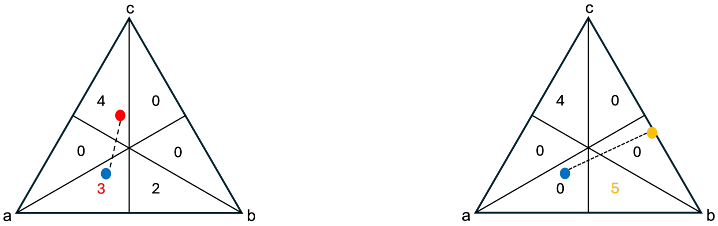

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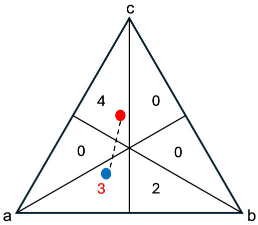

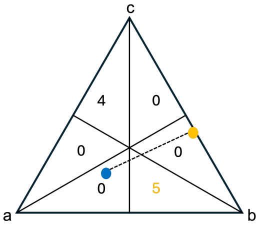

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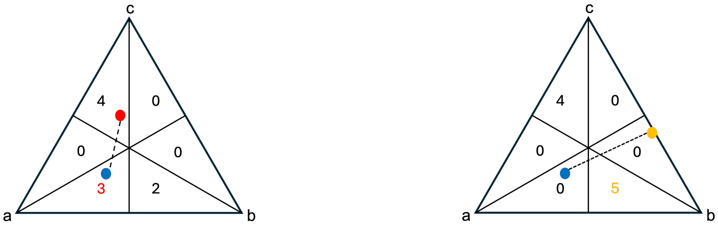

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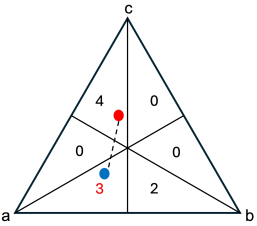

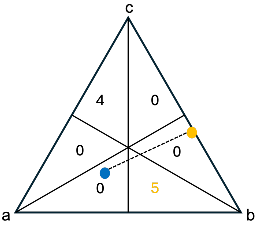

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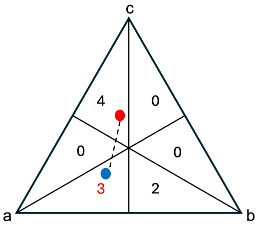

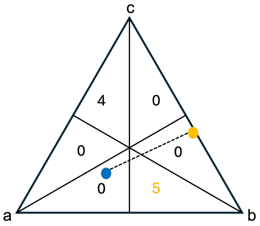

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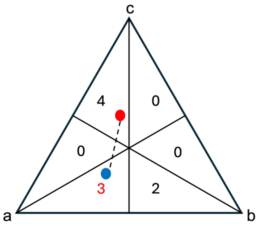

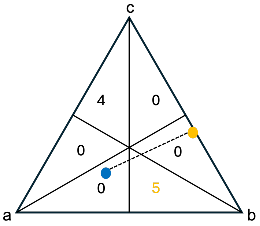

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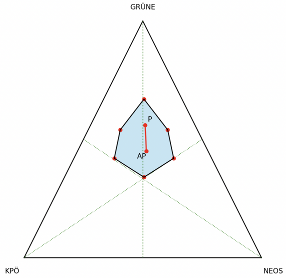
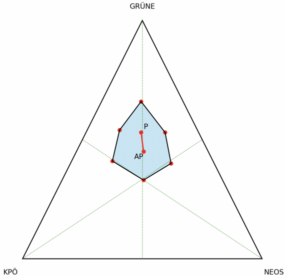
- various reasons for strategic votes were possible

For $n = 3$, we can see a change for (GRÜNE, KPÖ, NEOS)





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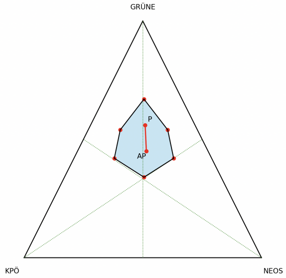

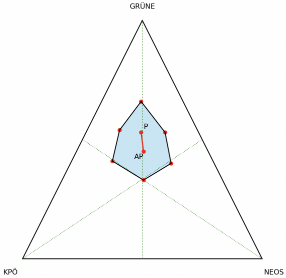

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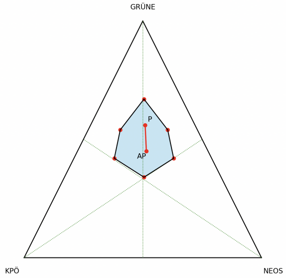

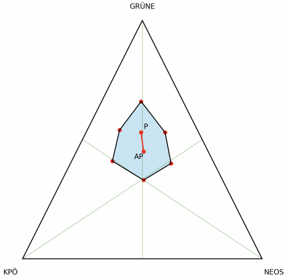

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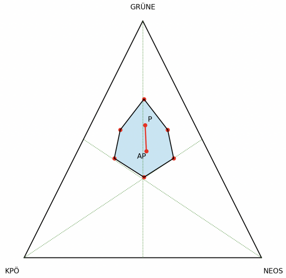

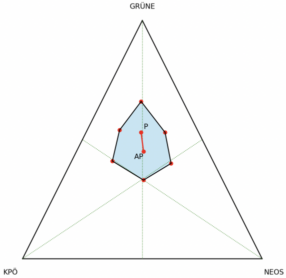

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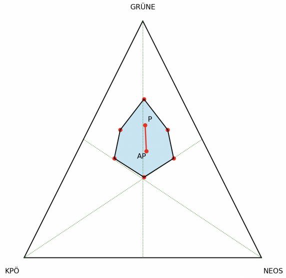

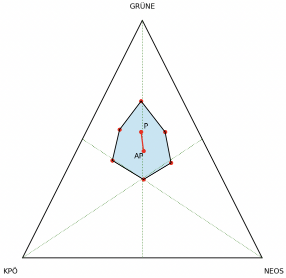

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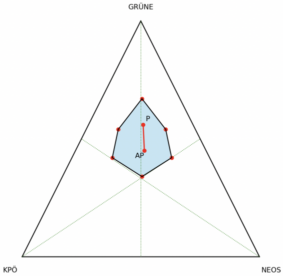

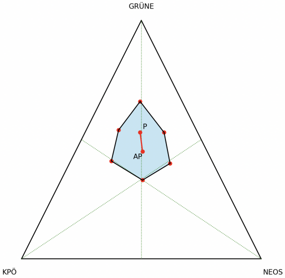

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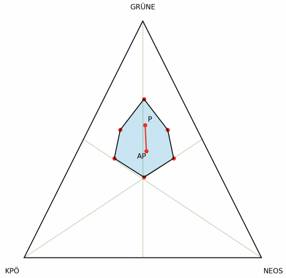
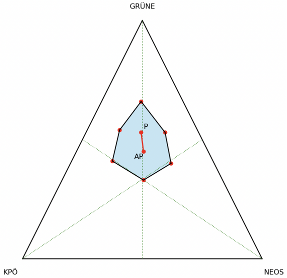
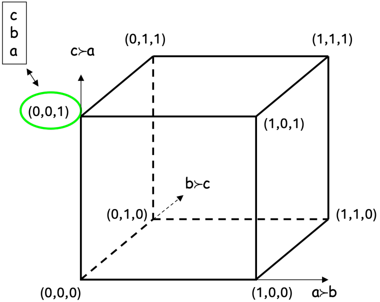
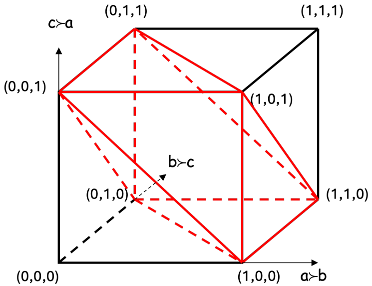




Cubes and Cycles

Saari also uses cubes to analyze pairwise majorities





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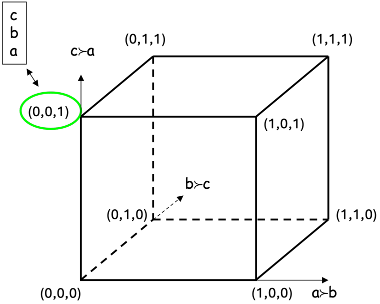

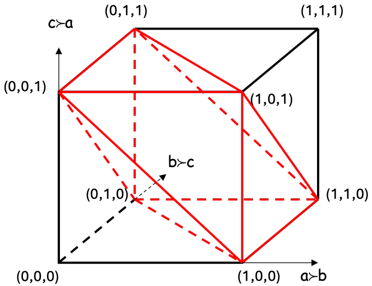

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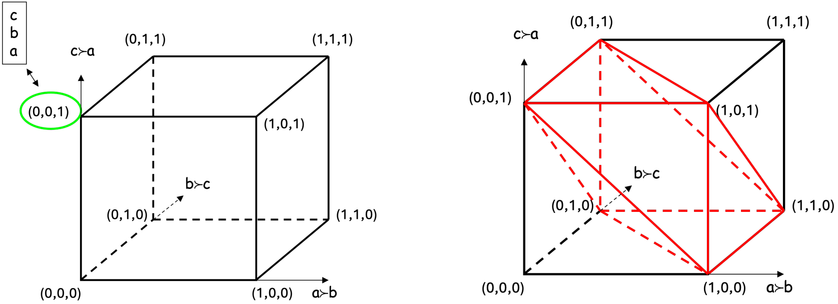

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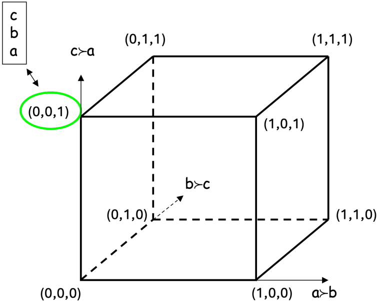

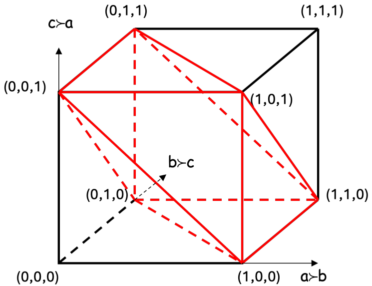

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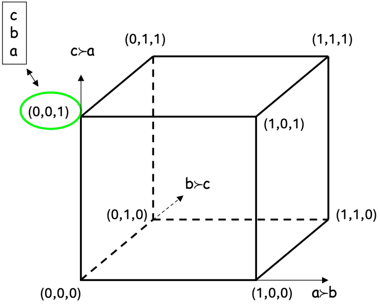

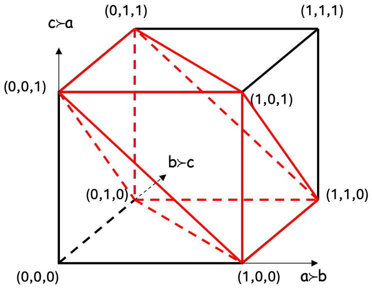

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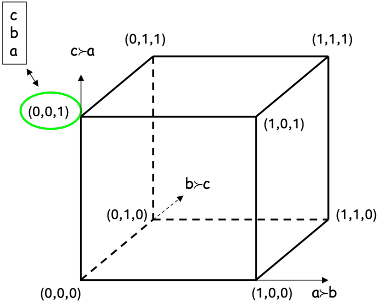

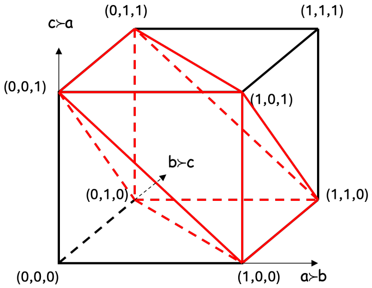

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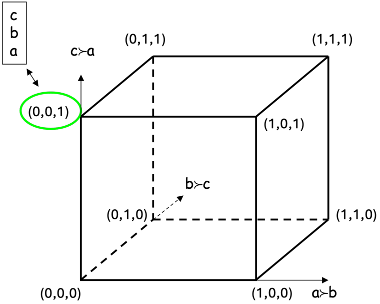

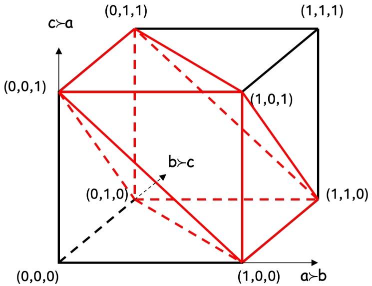

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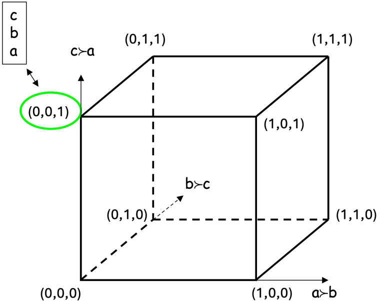

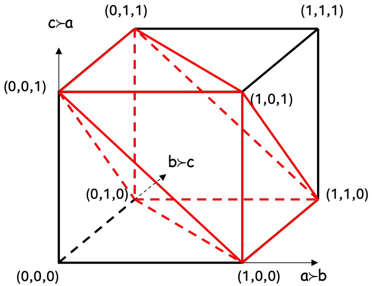

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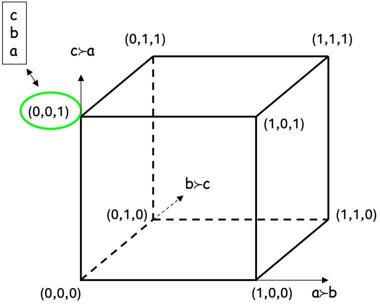

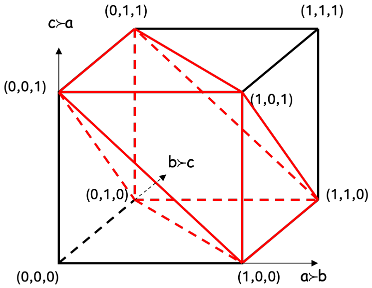

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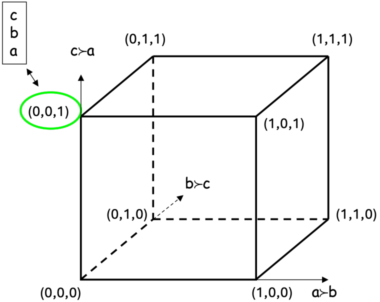

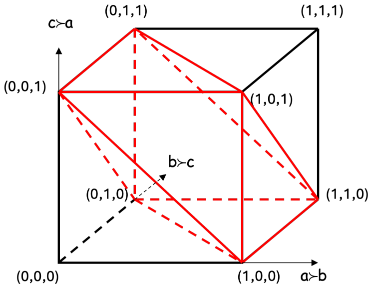

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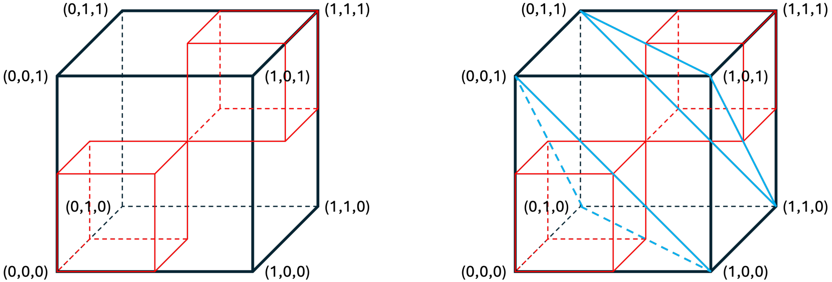
The majority outcome is the vertex of the subcube closest to the majority counts ... but two subcubes represent cyclic outcomes



- for $n > 3$ many more cyclic sub-polytopes
  - for $n = 4$ we jump to $2^{\binom{4}{2}} = 64$ vertices
  - of those $2^{\binom{n}{2}} - n! = 40$ are non-transitive
  - for $n = 6$ already 98% of the vertices are non-transitive

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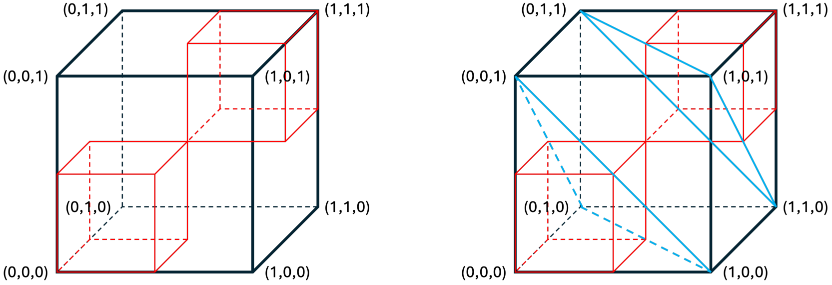

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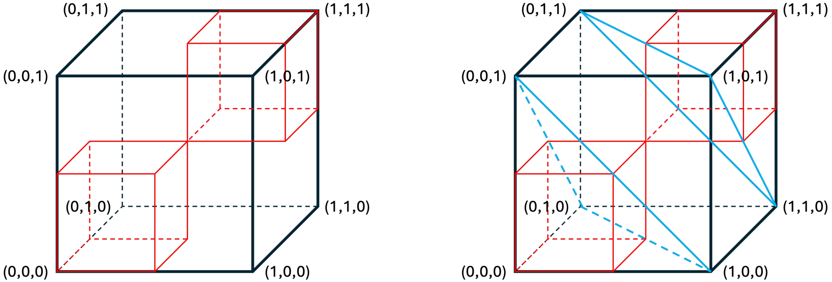

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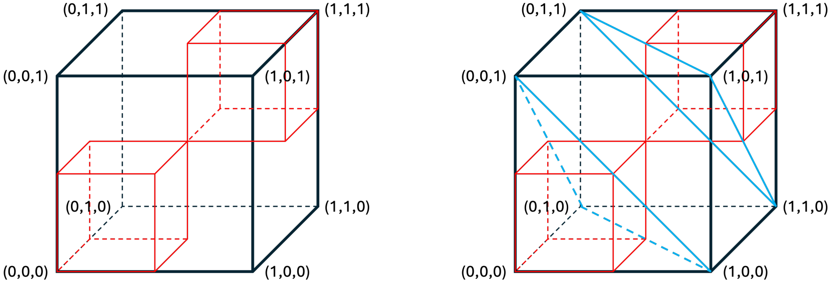

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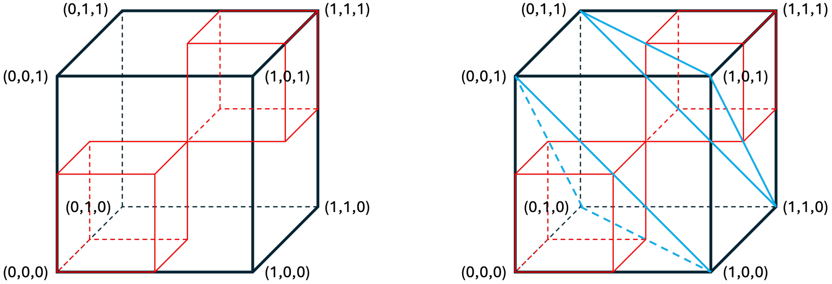

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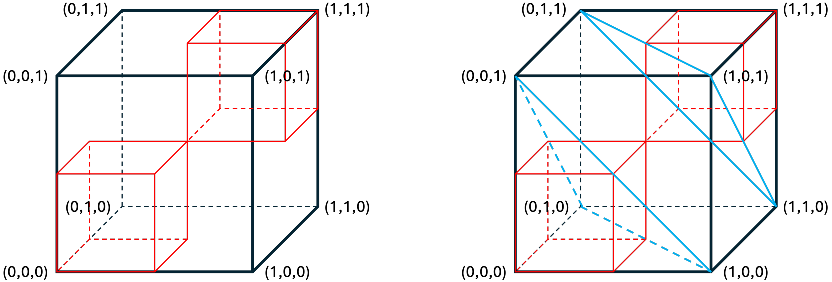

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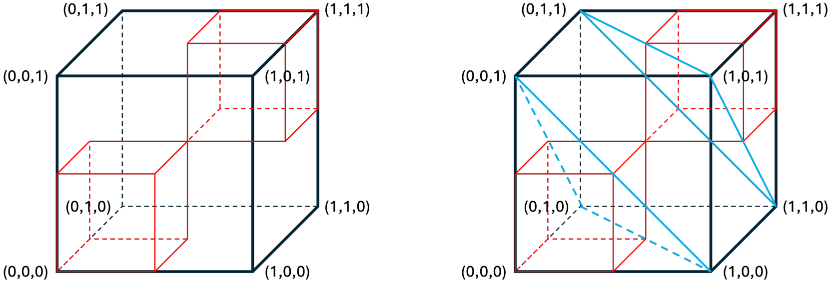

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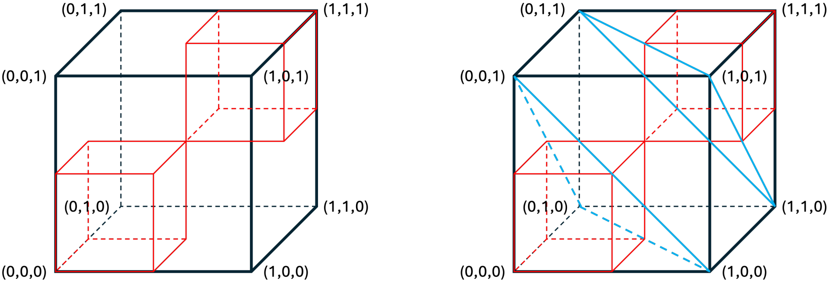

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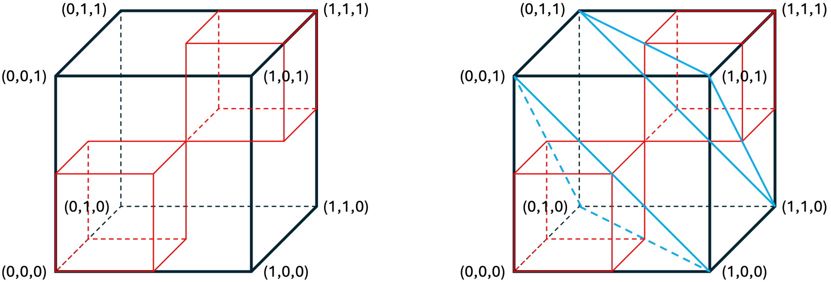

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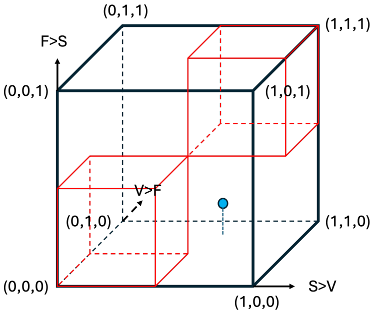
Could again think about stability of outcomes

- Euclidean distance to cycling subcubes shows closeness to cycles

Consider SP, VP and FP from the 2019 election



- $(S > V, V > F, F > S) = (0.58, 0.91, 0.14)$
- closest cyclical vertex: (1,1,1)
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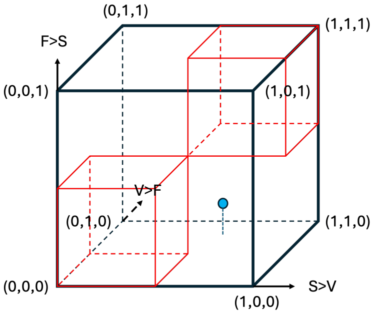

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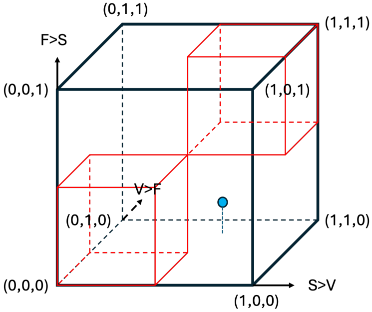

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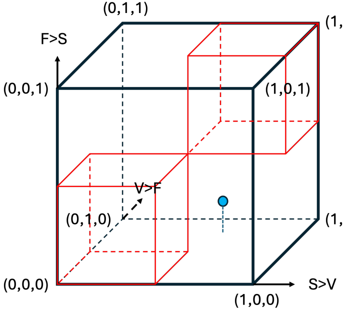

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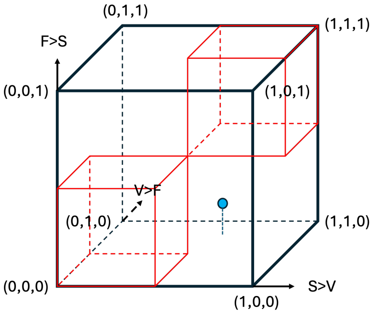

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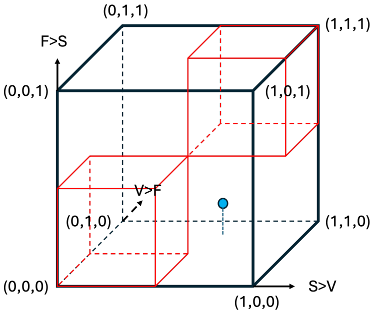

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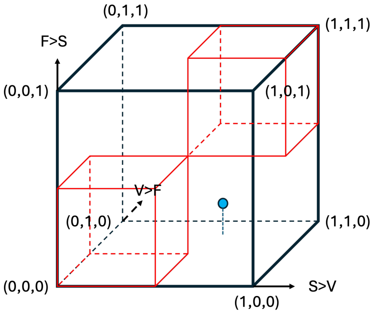

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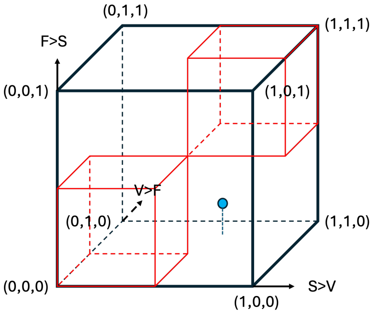

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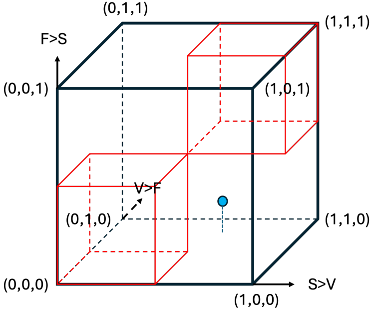

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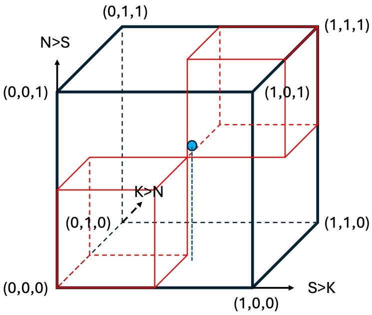

- $(S > K, K > N, N > S) = (0.516, 0.496, 0.526)$
- closest cyclical vertex: (1,1,1)
- distance 0.004
  - only takes 0.4% of the voters to change between N and K
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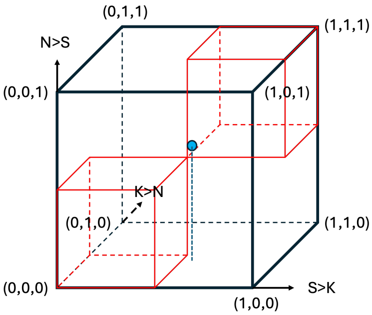
Cubes - applications

Consider SP, KP and NEOS from the 2019 election



- $(S > K, K > N, N > S) = (0.516, 0.496, 0.526)$
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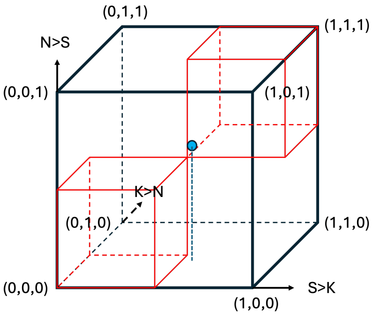

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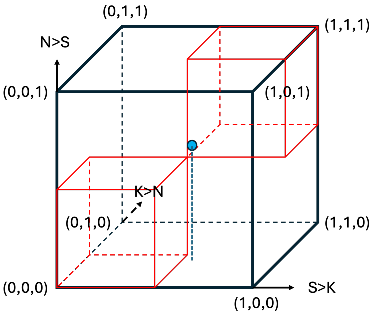

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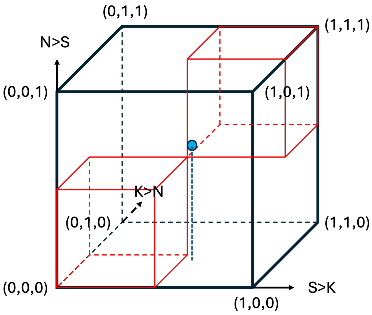

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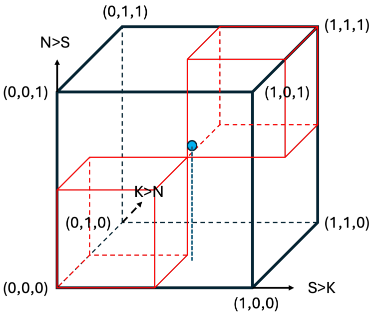

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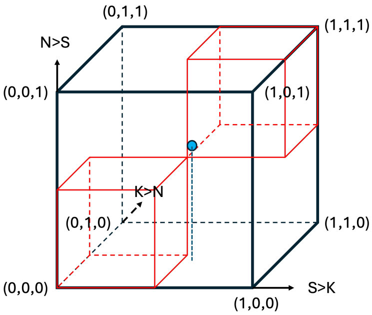

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Could also measure the distance from cycles for all 6 candidates in the 2019 elections

- there are $2^{\binom{6}{2}} - n! = 2^{15} - 6! =$ 32048 non-transitive vertices

What are the closest cycles?

- 3 cand.: SP, KP, NEOS with distance 0.004
- 4 cand.: SP, VP, KP, NEOS with distance 0.057
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- attempt to introduce Saari's geometric approach and apply it to data from two actual elections
- visualize in a simple way all differences in positional and pairwise voting rules for $n = 3$
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