WEAK REVIVALS IN TIME-EVOLUTION MODELS, A SPECTRAL THEORY POINT OF VIEW

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The phenomenon of revivals in linear dispersive equations was discovered first experimentally in optics, in around 1834, then rediscovered several times by theoretical and experimental investigations. While the concept has been used systematically and consistently by many authors, there is no consensus on a rigorous definition. Several have described it by stating that a given periodic time-dependent boundary value problem exhibits revivals at rational times, if the solution evaluated at a certain dense subset of times, is given by finite superpositions of translated copies of the initial condition. When this initial condition has jump discontinuities at time zero, these discontinuities are propagated and remain present in the solution at each rational time, but disappear completely at almost every irrational time. In this talk I will report on the presence of revivals, defined in the context of spectral theory, for three models of parabolic differential equations. • Nonlocal equations that arise in water wave theory and are defined by convolution kernels, [1]. • Schr odinger equations with different types of boundary conditions [2] or with complex potentials [3]. • Dislocated Laplacian time-evolution equations [4]. As we shall see, in all cases the solution is given explicitly by finite combinations of translations, dilations and scaling of the initial datum, plus additional regular terms. When they are present, these extra terms can be interpreted as a weak manifestation of the classical revivals phenomenon. The research has been conducted jointly with George Farmakis (London South Bank University), Peter Olver (University of Minnesota), Beatrice Pelloni (Heriot-Watt University) and David Smith (Yale NUS).

References

[1] Stud. Appl. Math. 147 (2021) p.1209.

[2] Proc. Royal Soc. A. 477 (2021) p.2251.

[3] Preprint ArXiV:2308.09961. To appear in ZAA (2024).

[4] Preprint ArXiV:2403.01117.