Kinetic equations in global optimization and applications

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Optimization methods based on stochastic particle systems have a rich history and hold significant importance in various applications today, spanning from machine learning to optimal control. Many of these methods rely on metaheuristic algorithms, which often lack a rigorous mathematical foundation. Recently, leveraging tools inspired by statistical physics has enabled the description of these gradient-free algorithms through the lens of kinetic and mean-field PDEs. This approach provides convergence guarantees to the global minimum under mild assumptions on the objective function and allows for the introduction of novel enhancements to improve the algorithms' performance. In this short course, we will exemplify these concepts using popular algorithms like simulated annealing, genetic algorithms and particle swarm optimization, emphasizing how, in the limit of a large number of particles, these algorithms can be described by kinetic equations. We will also discuss the relations to other popular optimization algorithms, such as SGD and CBO, and explore some illustrative applications.