RATE DISTORTION DIMENSION OF RANDOM BRODY CURVES

MASAKI TSUKAMOTO

Our main purpose is to propose an ergodic theoretic approach to the study of entire holomorphic curves. In particular, we develop a "holomorphic curve analogue" of the theory of Axiom A diffeomorphisms. Brody curves are one-Lipschitz holomorphic maps from the complex plane to the complex projective space. They naturally form a dynamical system, and “random Brody curves” refers to invariant probability measures on it. We study their geometric and dynamical properties. Our first main theorem claims that the rate distortion dimension of random Brody curves is bounded by the integral of a "potential function". This is analogous to the Ruelle inequality of smooth ergodic theory. Our second main theorem claims that there exists a rich variety of invariant probability measures attaining equality in this "Ruelle inequality for Brody curves". The proof is based on the "variational principle" for mean dimension with potential. This approach is motivated by the thermodynamic formalism for Axiom A diffeomorphisms.