EQUIVARIANT WEIERSTRASS THEOREM

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Consider a map Z from the space E of entire functions to the space D of discrete subsets of the complex plane that maps entire functions to their zero sets. The Weierstrass theorem asserts that this map is surjective. In the natural topologies, the spaces E and D are Polish, the complex plane acts continuously on both of them by translations, and the map Z is equivariant with respect to these actions. Does it admit an equivariant (right) inverse?

In a joint work in progress with Konstantin Slutsky and Aron Wennman, we prove the existence of an equivariant Borel inverse if the action of the complex plane on D is free. Such an inverse cannot be made continuous and no such inverse exists if the action is not free.