Theme 3: variational

Barycenters in unbalanced optimal transport

Maciej Buze

The theory of optimal transportation, dating back to Gaspard Monge's work in 1781, continues to develop at pace as one of the fundamental mathematical theories with an ever-growing list of diverse applications in fields such as economics, computer vision, image processing and machine learning. A central challenge in many applications concerns finding a representative, or barycentric (probability) distribution, which provides some average description of a given set of distributions. The basic optimal transport approach to this problem is to find the barycenter by minimizing the sum of weighted two-marginal optimal transport costs between the barycenter and each input distributions. In a seminal contribution by Agueh and Carlier [1], it was subsequently shown that an equivalent and computationally favourable approach is to instead solve a single least-cost multi-marginal optimal transport problem.

If the input distributions do not all have equal mass, an unbalanced barycenter can be found via a recourse to the emerging theory of unbalanced optimal transportation. This, however, can be done in a number of ways, depending on how one penalises mass deviations, what cost function is employed and whether one wishes to consider the conic formulation --- see the detailed discussion in [2].

In this talk, I will introduce the above ideas in an accessible manner, followed by presenting several results on how to recover the celebrated least-cost multi-marginal formulation of Agueh and Carlier in the unbalanced setting [3].

[1] M. Agueh and G. Carlier. Barycenters in the Wasserstein space. SIAM Journal on Mathematical Analysis 43.2 (2011), pp. 904–924.
[2] M. Liero, A. Mielke, and G. Savaré. Optimal entropy-transport problems and a new Hellinger–Kantorovich distance between positive measures. Inventiones mathematicae 211.3 (2018), pp. 969–1117.
[3] M. Buze. Constrained Hellinger-Kantorovich barycenters: least-cost soft and conic multi-marginal formulations. arXiv e-prints, 2402.11268, 2024 (to appear in SIAM Journal on Mathematical Analysis).