We consider a system of classical particles, interacting via a smooth, long-range potential, in the mean-field regime, and we analyze the propagation of chaos in form of sharp estimates on many-particle correlation functions. While approaches based on the BBGKY hierarchy are doomed by uncontrolled losses of derivatives, we develop a non-hierarchical approach relying on discrete stochastic calculus with respect to initial data. This allows to rigorously truncate the BBGKY hierarchy and justify the so-called Bogolyubov corrections to the mean-field Vlasov description. As a by-product, for spatially homogeneous systems, we also discuss the justification of the Lenard-Balescu relaxation and of the Landau approximation. This is partly based on joint works with Armand Bernou, Laure Saint-Raymond, and Raphael Winter.