

ON EISENSTEIN'S JUGENDTRAUM FOR COMPLEX CUBIC FIELDS

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In the early 2000's Ruijsenaars and Felder-Varchenko have introduced the elliptic gamma function, a remarkable youthful special function that comes from mathematical physics. It satisfies modular functional equations under the group $SL_3(\mathbb{Z})$ which make it a higher dimensional analogue of the Jacobi theta function. In this work, we unveil the place that this function and its avatars play in number theory. Our main thesis is that they play the role of modular units in extending the theory of complex multiplication to complex cubic fields. In other words we propose a conjectural solution to Hilbert's 12th problem for complex cubic fields, following a line of research actually initiated G. Eisenstein. We give a lot of numerical evidence that support this conjecture, and we will set the stage for the talks of Luis Garcia and Pierre Morain at this conference.

This is joint work with Nicolas Bergeron and Luis Garcia.

This talk is dedicated to the memory of Nicolas Bergeron.