EISENSTEIN COCYCLES AND THEIR SPECIALIZATIONS

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I will describe a relatively new construction of a relatively old explicit map from the homology of a modular curve to the second K-group of a cyclotomic integer ring, sending Manin symbols to Steinberg symbols of cyclotomic units. This map was conjectured to be, and has been largely proven to be Eisenstein, in the sense that it factors through the quotient of homology by an ideal in the Hecke algebra arising from coefficients of Eisenstein series. Venkatesh and I realized this map as a pullback at a torsion point of a GL_2-cocycle, which happens to be Eisenstein, valued in a limit of motivic cohomology groups of opens in the square of the multiplicative group over Q.

I'll give examples of other cases in which ""Eisenstein"" cocycles can be constructed and specialized to interesting maps. For instance, I'll mention a collection of cocycles for products of two CM elliptic curves, which specialize to a map from the homology of a Bianchi space to the second K-group of a ray class group of an imaginary quadratic field. The latter construction is joint with Lecouturier, Shih, and Wang.