Computing the Essential Spectrum of the Neumann–Poincaré Operator on Highly Oscillatory Boundaries

SIMON CHANDLER-WILDE

We investigate, by a mixture of analysis and rigorous computation, a conjecture of Carlos Kenig, that the essential spectral radius of the classical Neumann-Poincaré integral operator on the boundary of a general Lipschitz domain is $< 1/2$ on the Hilbert space of functions that are square-integrable with respect to surface measure. Our computations are for 2D domains that are piecewise analytic and locally dilation invariant in the sense of Chandler-Wilde, Hagger, Perfekt, and Virtanen, Numer. Math. (2023). This domain class permits infinitely many oscillations of arbitrarily large amplitude adjacent to singular points on the boundary. Such domains are promising candidates as counter-examples since it is known (Chandler-Wilde and Spence, Numer. Math. 2022) that, for these domains, the essential numerical radius can be arbitrarily large. Via localisation results and a Floquet-Bloch transform we reduce the computation of the essential spectral radius to a problem of estimating the spectral radii of a family of integral operators with periodic analytic kernels, for which the trapezium rule and associated error bounds are perfectly suited. Our methods of argument include results, that may be of independent interest, for estimating the spectral radius of any compact operator on a Banach space.

This is joint work with Raffael Hagger (Kiel), Karl-Mikael Perfekt (Trondheim), and Jani Virtanen (Reading).