GENERATIVE MODELLING WITH TENSOR TRAIN APPROXIMATIONS OF HAMILTON–JACOBI–BELLMAN EQUATIONS

ROBERT GRUHLKE

Sampling from probability densities is a common challenge in fields such as Uncertainty Quantification (UQ) and Generative Modelling (GM). In GM in particular, the use of reverse-time diffusion processes depending on the log-densities of Ornstein-Uhlenbeck forward processes are a popular sampling tool. In Berner et al. 2022 the authors point out that these log-densities can be obtained by solution of a \textit{Hamilton-Jacobi-Bellman} (HJB) equation known from stochastic optimal control. While this HJB equation is usually treated with indirect methods such as policy iteration and unsupervised training of black-box architectures like Neural Networks, we propose instead to solve the HJB equation by direct time integration, using compressed polynomials represented in the Tensor Train (TT) format for spatial discretization. Crucially, this method is sample-free, agnostic to normalization constants and can avoid the curse of dimensionality due to the TT compression. We provide a complete derivation of the HJB equation's action on Tensor Train polynomials and demonstrate the performance of the proposed time-step-, rank- and degree-adaptive integration method.