Preserving Positivity and dimension reduction without sparsity

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Sparse positive definite matrices and operators have found extensive and widespread use in modern high dimensional statistics, probability and machine learning, especially in the field of probabilistic Markov Random Fields and graphical models. To this end, sparsity inducing positivity preservers have been studied extensively during the last decade, especially in the pure and applied mathematics context. This body of work has served to extend the classical theory previously developed by Schoenberg and several others. More specifically, recent work has established that so-called sparsity (zero) inducing functions can readily lead to the loss of positivity. In this talk we investigate the properties of dimension-reducing approaches which do not induce sparsity. We explore specific tractable approaches which aim to reduce dimension and simultaneously retain positivity, with ultimate goal being the development of mathematical theory and methods which are motivated by modern applications.