Gabor Frames of Totally Positive Functions and Estimates for their Frame Bounds

We consider totally positive functions g of finite type and the associated Gabor family

$$G(g,a,b) = (M_{kb}T_{ja}g;\ j,k \in \mathbb{Z}), \qquad M_{kb}T_{ja}g = e^{2\pi ikb}g(\cdot - ja).$$

It was proved in [1] that for all lattice parameters $a, b > 0$ with $ab < 1$, this family is a Gabor frame. The proof centers around the fact, that every bi-infinite matrix of the form

$$A = (g(x + ja - k/b))_{j,k\in\mathbb{Z}}$$

is totally positive. We aim at finding explicit expressions for lower frame bounds of the Gabor frame, as the lattice density tends to $ab = 1$.

[1] K. Gröchenig, J. Stöckler, Gabor frames and totally positive functions, Duke Math. J. 162 (2013).