Plücker inequalities for weakly separated coordinates in the TNN Grassmannian

Prateek Kumar Vishwakarma, Indian Institute of Science

We show that the partial sums of the long Plücker relations for pairs of weakly separated Plücker coordinates oscillate around 0 on the totally nonnegative part of the Grassmannian. Our result generalizes the classical oscillating inequalities by Gantmacher–Krein (1941) and recent results on totally nonnegative matrix inequalities by Fallat–Vishwakarma (2024). In fact we obtain a characterization of weak separability, by showing that no other pair of Plücker coordinates satisfies this property.

Weakly separated sets were initially introduced by Leclerc and Zelevinsky and are closely connected with the cluster algebra of the Grassmannian. Moreover, our work connects several fundamental objects such as weak separability, Temperley–Lieb immanants, and Plücker relations, and provides a very general and natural class of additive determinantal inequalities on the totally nonnegative part of the Grassmannian. This is joint work with Daniel Soskin.